CHAPTER II

THEORY

The purpose of this chapter is to lay theoretical ground work to be used as basis for interpretation of the experimental results. Mathematical model of the system is described, solutions based on several special cases are presented. In section 2.1 a general description of system to be studied is given and important parameters of the system are identified. In section 2.2 mathematical models and solutions are described and discussed. And finally in section 2.3 an extraction efficiency is defined.

2.1 System Description and Dimensional Analysis

To obtain an optimal operating conditions in any storage system requires a good understanding of how the energy is distributed within the storage tank. In order to investigate this energy distribution, a general model of stratified hot water storage is being examined as shown in Fig 2. The water at temperature T_c is withdrawn from the bottom of the tank and circulated through the collector where it is heated and returned to the top of the tank with the temperature T_h. When the thermal energy is in demand, the hot water with temperature T_e will be extracted from the top of the tank to supply the load and returned to the bottom of the tank with lower temperature T_l.

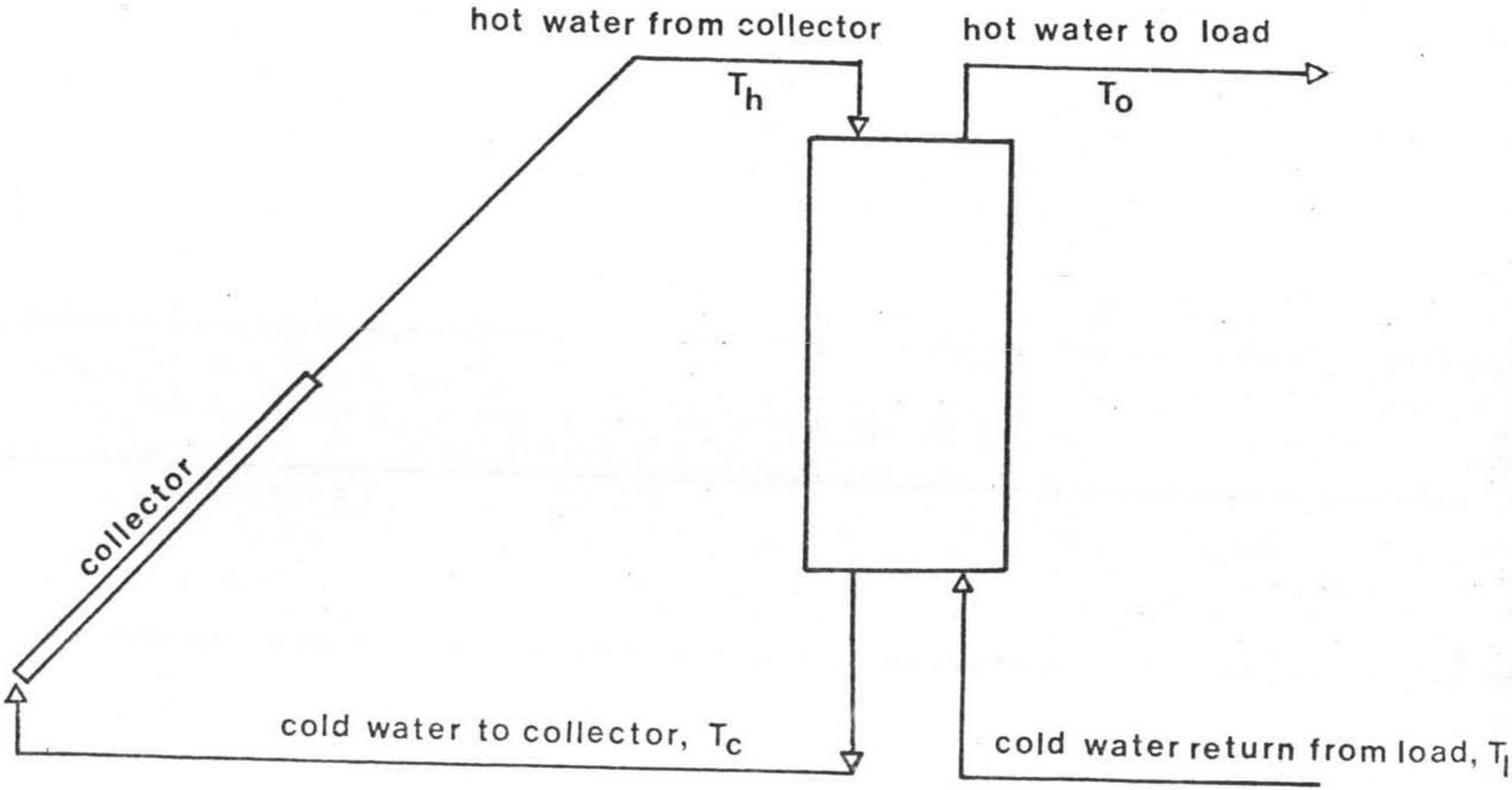

Figure 2 Hot water storage system with water circulation through collector to add energy, and through load to remove energy

For a good performance of the storage tank, the water in the tank should maintain steep thermoclines;ie. little mixing between the hot and incoming cold water. The experimental study is intended to concentrated on various effects of geometric and dynamic parameters which can influence the stratification of the hot water. These important parameters are the mass flow rate, length and diameter of the tank, diameter of inlet pipe line, inlet-outlet water temperature difference, and physical properties of the fluid. Using dimensional analysis, and assuming a steady-state flow these important parameters can be reduced to four dimensionless groups: Reynolds number, Re_d (based on inlet pipe diameter); length to diameter ratio, L/D; ratio of inlet diameter and diameter of tank, D/d; and a Grashof number, Gr_D (based on tank diameter).

A study has also been made on the effect of heat transfer by conduction through the tank wall. This effect of heat conduction is discussed later.

2.2 Mathematical Model of Stratified Sensible Heat Storage Tank

The following model is developed for the system described in the previous section. The region of interest is the content of the cylindrical storage tank. The problem appears at first glance to be simply a convective heat transfer problem, however one must keep in mind that one is working with stratification problem where buoyancy force is a significant factor. This problem will not arise if we always keep bottom inlet condition at temperature lower than temperature of liquid in the tank and top inlet condition at temperature higher than liquid in the tank. However, one cannot guarantee such conditions in solar energy application. To keep the mathematic from being too sophisticated, we will keep the issues separated and be contented in looking at several different special cases, keeping in mind the intention of trying to understand physical reasoning behind experimental results rather than attempting to find a complete model of the system. We first look at the convective heat transfer model with an assumptions of negligible buoyancy effect, then attempt to analyze the effect of the assumption.

2.2.1 Convective Heat Transfer Model of the System

The thermal energy equation with the assumptions that flow is in axial (z) direction only, physical properties are constant, and temperature profile is symmetrical, reduces to

$$\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} = \alpha \left[\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) \right] \qquad (1)$$

where $\alpha = k/\rho c_p$.

Tanthapanichakoon and Laohakul (11) simplified equation (1) further and leads to

$$\frac{\partial T}{\partial t} + v_m \frac{\partial T}{\partial z} = \frac{2h(T_w - T)}{a \rho c_p} \qquad (2)$$

under the conditions that,

1. the fluid is flowing in the z direction with uniform velocity profile $v_z = v_m$ = constant for any $0 \leq r \leq R$

2. the boundary film at the wall essentially contains the entire temperature difference between the tank wall and the bulk of fluid, therefore all resistance to heat transfer between the fluid and the wall is concentrated in the film. Then the radial and axial terms can be replaced by $2h/a\rho c_p(T_w - T)$.

The required initial and boundary condition are

$$T(z, 0) = f(z) \quad \text{for} \quad 0 \leq z \leq L$$
$$T(0, t) = g(t) \quad \text{for} \quad t \angle 0 \tag{3}$$

where f and g are the given initial temperature of the fluid in the tank and the inlet temperature of the fluid, respectively. Equation (2) may be transformed into dimensionless form,

$$\frac{\partial \Theta}{\partial \theta} + \frac{\partial \Theta}{\partial \eta} = \Gamma(\Theta_w - \Theta) \tag{4}$$

where $\Theta(\eta, 0) = f^*(\eta)$ for $0 \leq \quad \leq 1$ (5)

$\Theta(0, \theta) = g^*(\theta)$ for $\theta \geq 0.$ (6)

Here $\eta = z/L$, $\tau_1 = L/v_m$, $\theta = t/\tau_1 = tv_m/L$

$$\Theta_w = \frac{(T_w - T_i)}{\Delta T}$$

$$\Theta = \frac{(T(\eta, \tau) - T_i)}{\Delta T}$$

$$\Gamma = \frac{2hL}{a \rho c_p v_m}$$

$$= \frac{2h \tau_1}{a \rho c_p}$$

$$f^*(\eta) = \frac{(f(\eta) - T_i)}{\Delta T}$$

$$g^*(\eta) = \frac{(g(\theta) - T_i)}{\Delta T}$$

If the initial temperature of the fluid in the tank is equal to T_i then

$$\Theta(\eta, 0) = 0 \quad \text{for} \quad 0 \leq \eta \leq 1 \tag{7}$$

The Laplace transforms of equation (4) and (5) with respect to θ are

$$\frac{d\hat{\Theta}}{d\eta} + (s + \Gamma)\hat{\Theta} = \Gamma \hat{\Theta}_w \tag{8}$$

$$\hat{\Theta}(0, s) = \hat{g}^*(s) \quad \text{for} \quad s > 0 \tag{9}$$

The general solution of equation (8) subject to equation (9) is

$$\hat{\Theta} = (\hat{g}^*(s) - \frac{\Gamma}{s+\Gamma}\hat{\Theta}_w)e^{-(s-\Gamma)\eta} + \frac{\Gamma \hat{\Theta}_w}{s+\Gamma}$$

$$= \hat{g}^*(s)e^{-(s+\Gamma)\eta} + \frac{\Gamma \hat{\Theta}_w}{s+\Gamma}\left[1 - e^{-(s+\Gamma)\eta}\right] \tag{10}$$

The analytical solution of equation (4) is, then, given by taking the inverse Laplace transform of equation (10)

$$\Theta(\eta,\theta) = e^{-\Gamma\eta} g^{*}(\theta-\eta)U(\theta-\eta) + q(\theta) - e^{-\Gamma\eta} q(\theta-\eta)U(\theta-\eta) \qquad (11)$$

where $U(\theta)$ = unit step function

$g^{*}(\theta)$ = given dimensionless inlet fluid temperature

$$q(\theta) = \mathcal{L}^{-1}\left[\frac{\Gamma\hat{\Theta}_w}{s+\Gamma}\right] \qquad (12)$$

Equation (11) which has been derived by Tanthapanichakoon and Laohakul (11) is the general solution applicable for any storage tank model under the specified conditions.

For the case of our interest the inlet fluid temperature is a step function

$$g(t) = T_1 \qquad (13)$$

and in dimensionless form

$$g^{*}(\theta) = \begin{cases} 1 & 0 \leq \theta \\ 0 & 0 > \theta \end{cases} \qquad (14)$$

where $\triangle T$ is defined as $(T_1 - T_o)$.

Since θ is always greater then 0, equation (11) is reduced to

$$\Theta(\eta,\theta) = e^{-\Gamma\eta} U(\theta-\eta) + q(\theta) - e^{-\Gamma\eta} q(\theta-\eta)U(\theta-\eta)$$

$$\Theta(\eta, \theta) = e^{-\Gamma\eta} U(\theta-\eta)\left[1-q(\theta-\eta)+q(\theta)\right] \qquad (15)$$

For the temperature of the tank wall that needs to be evaluated, the two possibilities have been made:

1. The tank wall temperature is an average value over the tank length of the fluid temperature. That is

$$T_w = \frac{1}{L} \quad zT_1 + (L-z)\,T_0 \qquad (16)$$

2. The tank wall temperature is changed in the way such that

$$T_w - T = \begin{cases} -A & z/L \geq \theta \quad (= v_m t/L) \\ +B & z/L \geq \theta \end{cases} \qquad (17)$$

In the first case , the simplification of Eq. (16) suggested by Tanthapanichakoon and Laohakul (11) implies that the wall is of infinite conductivity materials that is the thermal conductivity of the tank wall is assumed to be very high, so the amount of heat should rapidly be distributed along the entire length of the wall and resulting in an average wall temperature. In the second case, the intention is to fit the experimental data with the theoretical model described above, if the assumption in case 1 is not valid. When the experimental measurement shows a definite temperature difference between the wall and the water, the second case can be used to see how well the model, without assumption regarding wall conduction, describes the system. When $z/L > v_m t/L$, the wall temperature is A degrees less than the water temperature; and when $z/L < v_m t/L$, the wall temperature is

B degrees higher than the water temperature. The later case implies that wall conductivity is quite low and the wall temperature is governed mainly by the temperature of fluid adjacent to the wall. With these two different conditions, the individual solution has been derived.

<u>The analytical solution for case 1</u>

since $$T_w = \frac{1}{L}\left[zT_1 + (L - z)\, T_o \right] \tag{18}$$

then $$\Theta_w = \frac{T_w - T_o}{\Delta T} = \frac{z}{L}\frac{(T_1 - T_o)}{\Delta T} + T_o - T_o$$

$$= \frac{z}{L} = \frac{v_m t}{L} = \theta \tag{19}$$

In term of a unit step function,

$$\hat{\Theta}_w = \theta - (\theta - 1)\, U(\theta - 1) \tag{20}$$

The Laplace transform of equation (20) is

$$\hat{\Theta}_w = \frac{(1 - e^{-s})}{s^2} \tag{21}$$

Substitution of equation (21) into equation (12) gives

$$q(\theta) = \mathcal{L}^{-1}\left[\frac{\Gamma(1 - e^{-s})}{s^2(s + \Gamma)} \right]$$

$$q(\theta) = \mathcal{L}^{-1}\left[\frac{(1-e^{-s})}{s^2} + \frac{\frac{1}{\Gamma}(1-e^{-s})}{s+\Gamma} - \frac{\frac{1}{\Gamma}(1-e^{-s})}{s}\right] \quad (22)$$

By taking the inverse Laplace transform of equation (22), we get

$$q(\theta) = \left[\theta - \frac{1}{\Gamma}(1-e^{-\Gamma\theta})\right] - \left[(\theta-1) - \frac{1}{\Gamma}(1-e^{-\Gamma(\theta-1)})\right]U(\theta-1) \quad (23)$$

Replacing equation (27) into equation (20), then

$$\begin{aligned}\Theta(\eta,\theta) = {} & e^{-\Gamma\eta}\,U(\theta-\eta)\Big[1 - \Big\{(\theta-\eta) - \frac{1}{\Gamma}(1-e^{-\Gamma(\theta-\eta)}) \\ & - \left[(\theta-\eta-1) - \frac{1}{\Gamma}(1-e^{-\Gamma(\theta-\eta-1)})\right]U(\theta-1)\Big\}\Big] \\ & + \left[\theta - \frac{1}{\Gamma}(1-e^{-\Gamma\theta})\right] - \left[(\theta-1) - \frac{1}{\Gamma}(1-e^{-\Gamma(\theta-1)})\right] \\ & U(\theta-1)\end{aligned} \quad (24)$$

Hence equation (24) is the general solution for case 1

<u>The analytical solution for case 2</u>

$$\text{If} \quad T_w - T = \begin{cases} -A, & \eta > \theta \\ +B, & \eta < \theta \end{cases} \quad (25)$$

then in dimensionless form

$$\frac{T_w - T}{\Delta T} = \Theta_w' = \begin{cases} -\dfrac{A}{\Delta T} & \eta \geq \theta \\ +\dfrac{B}{\Delta T} & \eta < \theta \end{cases} \tag{26}$$

or in term of a unit step function,

$$\Theta_w' = \frac{A + B}{\Delta T} U(\theta - \eta) - \frac{B - A}{\Delta T} \tag{27}$$

The general equation in dimensionless form as in equation (4) would be

$$\frac{\partial \Theta}{\partial \theta} + \frac{\partial \Theta}{\partial \eta} = \Gamma \Theta_w' (\eta, \theta)$$

$$= \Gamma \left[\frac{A + B}{\Delta T} U(\theta - \eta) - \frac{B - A}{\Delta T} \right] \tag{28}$$

By taking Laplace transform, equation (28) becomes

$$\frac{\partial \hat{\Theta}}{\partial \eta} + s\hat{\Theta} - \theta = \left[\frac{A + B}{\Delta T} \frac{e^{-\eta s}}{s} - \frac{B - A}{(\Delta T)s} \right] \tag{29}$$

Rearranging equation (29) on the right hand side and taking $\theta = 0$

then $$\frac{\partial \hat{\Theta}}{\partial \eta} + s\hat{\Theta} = \frac{\Gamma}{\Delta T} \left[\frac{(A+B)e^{-\eta s} - (B-A)}{s} \right] \tag{30}$$

This is an ordinary first order differential equation which can be solved by using the integrating factor method. Hence the solution for equation (30) becomes

$$\hat{\Theta} = \frac{\Gamma}{\Delta T}(A+B)\eta\left(\frac{e^{-s\eta}}{s}\right) - \frac{(B-A)\Gamma}{\Delta T}\frac{1}{s^2} + \frac{(B-A)\Gamma}{\Delta T}\frac{e^{-s\eta}}{s^2} + \frac{e^{-s\eta}}{s} \qquad (31)$$

Taking the inverse Laplace transforms of equation (31), we get

$$\Theta = \frac{(A+B)\Gamma}{\Delta T}U(\theta-\eta) - \frac{(B-A)\Gamma}{\Delta T}\theta U(\theta) + \frac{(B-A)\Gamma}{\Delta T}\theta U(\theta-\eta) + U(\theta-\eta)$$

Under rearranging, the equation above becomes

$$\Theta = \left[\frac{(A+B)\eta\Gamma}{\Delta T} - \frac{(B-A)\Gamma\theta}{\Delta T} + 1\right] U(\theta-\eta) - \frac{(B-A)\Gamma}{\Delta T}\theta U(\theta) \qquad (32)$$

Hence equation (32) is the general solution for case 2.

The general solutions for both cases will be used to investigate the effect of wall conduction on the stratified storage system under the assumptions made from Eq. (16) and Eq. (17), and will also be used to see the validity of these assumptions.

2.2.2 Investigation of Buoyancy Effect

It is imperative to investigate a case where the bottom inlet temperature is higher than the temperature of stored fluid in the tank or the top inlet temperature is lower than the temperature of stored fluid in the tank. In such cases buoyancy effects must be investigated and one needs to midify the convective term, $v_z \frac{\partial T}{\partial z}$, of equation (1). When buoyancy effect predominates v_z will have to be modified and the driving force for equation of motion is $g\Delta\rho$ (see chapter 9 of Bird, Steward, and Lightfoot (2)). One may imagine that the fluid rise or fall according to the law of mechanics and the velocity of the fluid may be much larger than v_m which was used in the previous section. As a rough approximation, we may assume that when there is a large density different between the entering fluid and the fluid in the tank, fluid will break into drops and each drop moves due to bouyancy and gravity force and neglects friction drag which may occur. If one further assumes that heat transfer process between the fluid particle and the body of surrounding fluid is very slow then

$$\rho_1 \frac{dv}{dt} = g\Delta\rho = g(\rho_1 - \rho_o) \tag{33}$$

where ρ_1 is the density of fluid entering the system containing fluid of ρ_o. Integration of equation (33) using initial condition that the inlet fluid is v_o , then

$$\frac{dz}{dt} = v = \frac{g(\rho_1 - \rho_o)\, t}{\rho_1} + v_o$$

and

$$z = \frac{1}{2}\frac{g(\rho_1 - \rho_o)\, t^2}{\rho_1} + v_o t \tag{35}$$

where z is the position of the liquid entering the tank at time t = 0. Thus the time that the fluid remains in the tank is

$$t = \sqrt{\frac{2L \quad 1}{g|\rho_1 - \rho_o|}} \tag{36}$$

Eq. (36) may be used as a very rough order of magnitude estimate for certain special case. One must noted however that very bold assumptions have been made, with an intention to isolate certain experimental case where the model described earlier will definitely fail due to the velocity profile assumption alone.

2.2.3 Axial Dispersion Model

Another model which may be of interest is the axial dispersion model where analogy is drawn to the non-idealization behavior of tubular reactors. This model characterizes mass transport in the axial direction by diffusivity, $\mathcal{D}_L$, that is superimposed on the plug flow velocity. This model assumes that the fluid velocity and concentration of fluid with a given temperature are constant across the tube diameter. The magnitude of the dispersion is assumed to be independent of position within the tank. That is we assume that deviation from plug flow arises from mixing of the two fluids by molecular diffusion processes and by turbulent eddies and vertices. The dispersion processes can be characterized by one effective dispersion coefficient, $\mathcal{D}_L$. This term is practically grouped into a dimensionless form, $\mathcal{D}_L/v_mL$ and is called a disperaion number. It is the parameter which measures the extent of axial dispersion. Thus if $\mathcal{D}_L/v_mL$ approaches zero, the dispersion is negligible, and the model is characterized by a plug flow behavior. If $\mathcal{D}_L/v_mL$ approaches infinity, it results a very large mixing effect. Levenspiel (7) has suggested various methods of calculating the value of dispersion number. The practical one is to find its variance from the experimental curve and inserted into the equation

$$\frac{\mathcal{D}_L}{v_m L} = \frac{\sigma^2}{2} \tag{37}$$

Hence the dispersion number, $\mathcal{D}_L/v_mL$ can be evaluated and the effect of mixing is determined.

2.3 Extraction Efficiency

To compare the optimum performance of various sizes of the storage tank, the term extraction efficiency is being introduced which is the ratio of the extracted energy at various time devided by the total energy stored in the water. The higher extraction efficiency means the better performance of the storage tank. In practical application, the extraction efficiency is compared within the specified range of temperature requirement. The assumptions for the derivation of the extraction efficiency are as follow:

1. The reference temperature is T_1.
2. The density and heat capacity of water are independent of temperature.
3. The flow rates of cold and hot water are constant.
4. No heat of mixing.
5. No heat loss to the surrounding.

If E_a is the total energy stored in the water, then

$$E_a = \rho V c_p (T_h - T_1) \tag{38}$$

and E_c is the total energy that flows out with the hot water at various time

$$E_c = \int_{t=0}^{t=\text{final}} \rho Q c_p (T_o - T_1)\, dt$$

$$E_c = \rho Q c_p \int_{t=0}^{t=\text{final}} (T_o - T_l)\, dt \qquad (39)$$

Let ε = extraction efficiency of the storage tank

therefore
$$\varepsilon = \frac{E_c}{E_a}$$

$$= \frac{\rho Q c_p \int_{t=0}^{t=\text{final}} (T_o - T_l)\, dt}{\rho V c_p (T_h - T_l)}$$

$$= \frac{Q}{V} \int_{t=0}^{t=\text{final}} \frac{(T_o - T_l)}{(T_h - T_l)}\, dt \qquad = \frac{Qt^*}{V}$$

$$= \int_{\tau=0}^{\tau_f = Qt_f/V} \frac{(T_o - T_l)}{(T_h - T_l)}\, d\left(\frac{Qt}{V}\right) \qquad (40)$$

Then the experimental results are given in terms of an extraction efficiency defined as $\varepsilon = Qt^*/V$ where

$$t^* = \int_{t=0}^{t=\text{final}} \frac{(T_o - T_l)}{(T_h - T_l)}\, dt.$$

In general the thermal process system requires the energy at high temperature. If the extracted temperature is below the specific demand, the rest of energy stored in the tank is too low and not applicable for the process. For this reason, the extraction efficiency of various storage tanks is compared at the inlet - exit temperature differences, ΔT, has dropped to some preassigned value. This value in the experiment is shown to correspond to a 10 percent drop of the initial ΔT, ie. $(T_e - T_1)/(T_h - T_1) = 0.90$. Hence $\varepsilon_{90} = Qt^*_{90}/V$. The extraction effeciency at ε_{50} is defined similarly.