CHAPTER III

MODEL FORMULATION

Introduction

As mentioned in Chapter II, a nonlinear programming problem can be solved by any of the various methods or a combination of these. However, for the purposes of this study, only two methods will be used to solve this particular problem: Lootsma's method[10,11] for formulating penalty function and the Brooks-Rosenbrock's new direct search method[9] for obtaining the minimum. This study will attempt to find the load distribution of an n-unit plant for minimum energy input.

Load Division with Minimum Fuel Consumption Problem[13,15]

1. Station performance characteristics

The performance of any individual piece of power-plant equipment such as a boiler, turbine, pump, fan, or heat exchanger may be described by an input-output curve. Since a station is composed of several of these different forms of apparatus, their performance characteristics must be integrated so that the performance of the station as a whole may be expressed by a single input-output curve.

Fig. 3-1 shows the major curves for a steam station. The energy required to drive an auxiliary equipment of the station

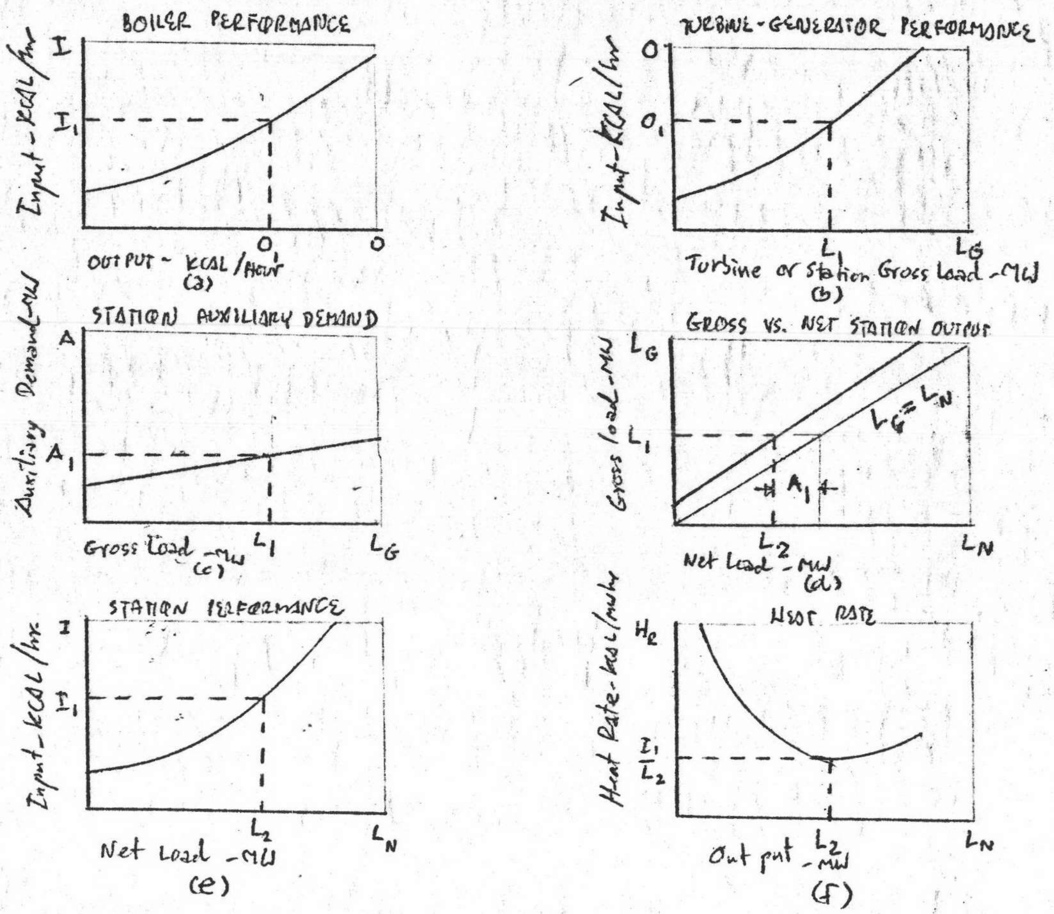

FIG. 3-1 Input-output curves of component station equipment and derivation of station input-output curve, and corresponding heat rate curve.

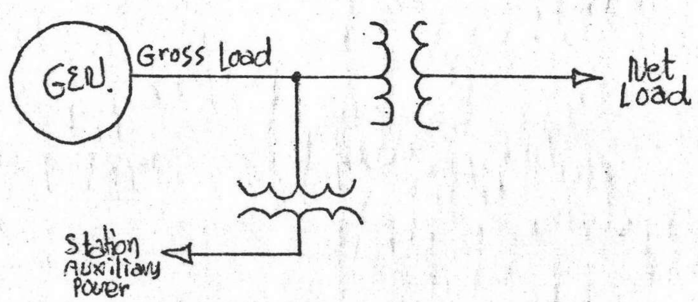

FIG. 3-2 Simplicated one-line diagram for each unit of power-plant.

varies with the station load. The relation is often of the simple $y = a + bx$ form, and the slope may be relatively flat (Fig.3-1c). This curve, and the relation of net load and gross load as shown in Fig. 3-2, may be readily converted into the form of the station net load plotted against the gross load as indicated in Fig.3-1d. Then, by plotting net load (MW) against input (KCAL per hour) in Fig. 3-1e, the performance characteristic of the station as an integral unit is determined.

From the basic input-output curve the more familiar heat-rate curve may be derived directly by taking from each load the corresponding input; then

$$H_R = \frac{I}{L} \text{ KCAL per MWhr} \qquad (3\text{-}1)$$

where I = the input energy in KCAL/hr, and

L = the gross load in MW

is plotted against the corresponding value of L as shown in Fig. 3-1f.

2. Load division

A system having more than one similar generating units has the proper division of load as a problem. Improper load division may appreciably decrease the thermal efficiency of the system as a whole. For maximum economy the total load should be divided among the units such that the combined input in minimum. It was shown in many texts[13,15] that, for a minimum combined input to carry a given combined output, the slopes of the input-output curves for each unit must be equal. This can shown as:

$$\frac{dI_a}{dL_a} = \frac{dI_b}{dL_b} = \frac{dI_c}{dL_c} = \cdots = \frac{dI_n}{dL_n} \qquad (3-2)$$

The slopes of these curves are the incremental rates, IR.

For simplicity without loss of generality, only two units in a plant are considered. An easy way for obtaining an economic operation can be found by plotting the incremental-rate curves for the two units, then the combined incremental-rate curve is plotted by adding up the loads at each incremental rate and plotting the sum against the particular incremental rate. The Load-division schedule with the aid of these curves may be presented. This method of Load division can be extended to include any number of the units.

This graphical method looks simple but it is very rough and cannot find out the exact numerical values for the given total load demand which is the computer application of the economic dispatching. Since the input I and incremental rate IR are nonlinear functions of load, it is very difficult to find load, L, for a given incremental rate, IR. This thesis will show how to solve these problems.

Mathematical Formulation[8,11,16]

1. Variables and their characteristics

In a power-generation system, the following variables are involved in the model:

n	the number of units of the plant;
x_i	the generated output of each unit of the plant where $i = 1,\ldots,n$;
$g_j(\bar{\underline{x}})$	the generation capacity of all units of plant where $j = 1,\ldots,2n$;
$y_i(x_i)$	the energy input to each unit, a function of the generated output of the i^{th} unit;
$F(\bar{\underline{x}})$	the summation of energy input of all the units; and,
L	the given total load demand, the summation of generated output of all units.

In fact, all the above variables must be positive numbers or zero.

2. Objective function

The performance of any plant can be accurately described by the input-output curve derived from tests of the individual equipment. In many cases, the curve is of the form defined by

$$y_i(x_i) = a_{io} + a_{i1}x_i + a_{i2}x_i^2 + a_{i3}x_i^3 + \cdots + a_{im}x_i^m \qquad (3\text{-}3)$$

$$i = 1,\ldots,n;$$

or

$$y_i(x_i) = \sum_{j=o}^{m} a_{ij}x_i^j \qquad i = 1,\ldots,n \qquad (3\text{-}4)$$

where the values of the coefficient a_{ij}, $i = 1,\ldots,n$ and $j = 0,\ldots,m$ are constant.

The function to be minimized, known as the objective function of the cost function, is defined by

$$F(\underline{x}) = \sum_{i=1}^{n} y_i(x_i) \qquad (3\text{-}5)$$

In the power plant generation problem, F can be the fuel consumption which is the function of the load.

3. Formulation of constraints

There are two types of linear constraints.

3.1 Inequality constraints

Since every unit of the power plant has capacity and stability within its limitations, each can generate the output between the minimum and the maximum value. Therefore, the equation may be written as

$$x_{i\,max} \geq x_i \geq x_{i\,min} \qquad i = 1,\ldots,n; \qquad (3\text{-}6)$$

or

and

$$\left.\begin{aligned} x_{i\,max} - x_i &\geq 0 \\ x_i - x_{i\,min} &\geq 0 \end{aligned}\right\} \quad i = 1,\ldots,n. \qquad (3\text{-}7)$$

3.2 Equality constraint

Since the requirements of load in the system vary many times during the day, the Central Dispatching Center (CDC) will inform the power plant for the given total generation load. This given total load demand L, will be devided for n units within the plant.

Thus, it can be written as

$$\sum_{i=1}^{n} x_i = L \tag{3-8}$$

or

$$\sum_{i=1}^{n} x_i - L = 0 \tag{3-9}$$

Solution Procedure

The objective of this mathematical programming is to minimize the total energy input function, F (the fuel consumption, for example). From the equality constraint of Eq. (3-8), it can be seen that the total number of the unknown variables can be reduced from n to (n-1) by the equation

$$x_n = L - \sum_{i=1}^{n-1} x_i \tag{3-10}$$

The values of a_{ij}, i =1,...,n and j = 0,...,m, in Eqs. (3-3) and (3-4) can be found by the Least-Squares Curve Fitting method which will be shown in Chapter IV.

Nonlinear Programming Model

A general nonlinear programming problem can be stated in mathematical terms as follows:

$$\begin{aligned} &\text{Minimize} && F(\underline{x}) \\ &\text{Subject to} && \underline{g}(\underline{x}) \geq \underline{0} \end{aligned} \tag{3-11}$$

or in a symbolic form:

$$\min \left\{ F(\underline{x}) \mid g_i(\underline{x}) \geq 0,\ i=1,\ldots,q \right\} \tag{3-12}$$

wherein one or both functions of $F(\underline{x})$ and $g(\underline{x})$ must be nonlinear.

For the load distribution problem, $F(\underline{x})$ is derived from Eqs.(3-4), (3-5) and (3-10) and the final equation, with its corresponding derivations, is shown as follows:

$$y_i(x_i) = \sum_{j=o}^{m} a_{ij} x_i^j \qquad i=1,\ldots,(n-1) \tag{3-13}$$

$$y_n(\underline{x}) = \sum_{j=o}^{m} a_{nj} \left[L - \sum_{i=1}^{n-1} x_i \right]^j \tag{3-14}$$

$$F(\underline{x}) = \sum_{i=1}^{n-1} y_i(x_i) + y_n(\underline{x}) \tag{3-15}$$

or

$$F(\underline{x}) = \sum_{i=1}^{n-1} \sum_{j=o}^{m} a_{ij} x_i^j + \sum_{j=o}^{m} a_{nj} \left[L - \sum_{i=1}^{n-1} x_i \right]^j \tag{3-16}$$

which is nonlinear.

The inequality constraints $g(\underline{x})$ are linear as expressed in Eq.(3-17) and will be changed, using Eq.(3-10), into the following form:

$$\left.\begin{aligned} x_{i\,max} - x_i &\geq 0 \\ x_i - x_{i\,min} &\geq 0 \end{aligned}\right\} \quad i=1,\ldots,(n-1) \tag{3-17}$$

and

$$x_{n\,max} + \sum_{i=1}^{n-1} x_i - L \geq 0 \tag{3-18}$$

$$L - \sum_{i=1}^{n-1} x_i - x_{n\,min} \geq 0 \tag{3-19}$$

The Sequential Unconstrained Minimization Technique (SUMT)[7-12,16]

The basic idea of the SUMT method is the transformation of the constrained mathematical problem in Eq.(3-11) or (3-12) into a series of unconstrained minimization problem by using certain penalty functions for the constraints. To achieve this, one defines a modified objective function of the following form:

$$P(\underline{x},r) = F(\underline{x}) - r \sum_{i=1}^{q} \ln g_i(\underline{x}), \qquad (3\text{-}20)$$

where r is a weighting coefficient. The summation term is the Lootsma's penalty function. The SUMT algorithm proceeds in the following way:

1. An initial point $\underline{x}_o$ within the region defined by the inequality constraints,

$$g_i(\underline{x}_o) > 0, \qquad i=1,\ldots,q, \qquad (3\text{-}21)$$

   is chosen.

2. An initial value of r, i.e., r_o is chosen. The actual value of r_o is immaterial; one may start practically with any r_o, provided that it does not make the last term of Eq.(3-20) too small at the beginning. The value of r_o used in this study is chosen to make the last term equal to 10 per cent of $F(\underline{x})$.

3. A minimization method which will be described in the next section is used to minimize the function $P(\underline{x},r_o)$, starting from point $\underline{x}_o$ in the direction of decreasing the value of $P(\underline{x},r_o)$. The minimum of $P(\underline{x},r_o)$ is assumed to occur at point $\underline{x}_1$.

a variety of unconstrained minimization methods may be used.[9,16] However, for this study, the Brooks-Rosenbrock's direct search method will be used for minimization of the function $P(\underline{x},r)$.

4. A new value of r is computed in the following way:

$$r_1 = \frac{r_o}{C}, \quad C > 1 \tag{3-22}$$

Usually, one chooses C in the neighborhood of 4 or 5. It is not advantageous to choose a C which is too large, say $C > 10$, since it tends to decrease the last term of $P(\underline{x},r)$ too much prematurely. The new coefficient r_1 is substituted into Eq. (3-20) , and $P(\underline{x},r_1)$ is minimized starting from point $\underline{x}_1$.

5. Step 4 is repeated for smaller and smaller values of r, obtained by the formula

$$r_{k+1} = \frac{r_k}{C} \tag{3-23}$$

The computation is terminated when the final convergence criterion is satisfied; i.e., when

$$\left| P(\underline{x},r_{k-1}) - P(\underline{x},r_k) \right| < \epsilon, \quad \epsilon > 0 \tag{3-24}$$

where ϵ is a prescribed small number fepending on the precision required.

Looking at Eq.(3-20), one can see that as r becomes very small and the constraints of the problem remain satisfied,

$$P_{min}(\underline{x},r) = F_{min}(\underline{x}) + \delta \tag{3-25}$$

where δ is a very small number which may be made as small as desired. As it was pointed out, the algorithm always proceeds along the points $\underline{x}_k$ inside the region defined by $g_i(\underline{x}) > 0$, i=1,...,q. If even one of the inequality constraints $g_i(\underline{x})$ comes too close to the feasible boundary, i.e. $g_i(\underline{x})$ comes too close to the feasible boundary, i.e. $g_i(\underline{x}) \to 0$, one can see from Eq.(3-20) that $P(\underline{x},r) \to \infty$, which is contrary to the minimization procedure. Therefore, as long as $P(\underline{x},r)$ is being minimized, the inequality constraints will be satisfied automatically.

New Direct Search Method to Solve Unconstrained Minimization Problem [9,16]

The SUMT method described in the last section, requires numerous intermediate solutions of unconstrained extremization problem. One of the possibilities of performing unconstrained extremization is through direct search. The most elementary version of the direct search method is to search sequentially along each of the n coordinate directions and to repeat the cycle till convergence is obtained. But this method is highly oscillatory and has poor convergence properties.

1. Modified Direct Search Method

Rosenbrock, Hooke and Jeeves found that the line joining the first and last points of search in the above method is a useful search direction[9] since the oscillatory is reduced and the convergence property is better than an ordinary one.

The method is illustrated for a two dimensional case in Fig.3-3. An iteration consists of two parts. Starting from an initial approximation $\underline{x}^o$, each coordinate direction is searched sequentially. This part is known as the exploratory move. If a positive displacement of any point does not yield a reduction of objective function the opposite direction is tried. When an improvement is noted in a direction, the movement is continued till a minimum is located in that direction. At the end of the series of exploratory moves we come to a point $\underline{x}^1$. Now a search is made along $\underline{x}^1 - \underline{x}^0$ and is continued till a minimum is located in that direction. This search is called the Pattern Search. At the end of the pattern search the procedure is restarted.

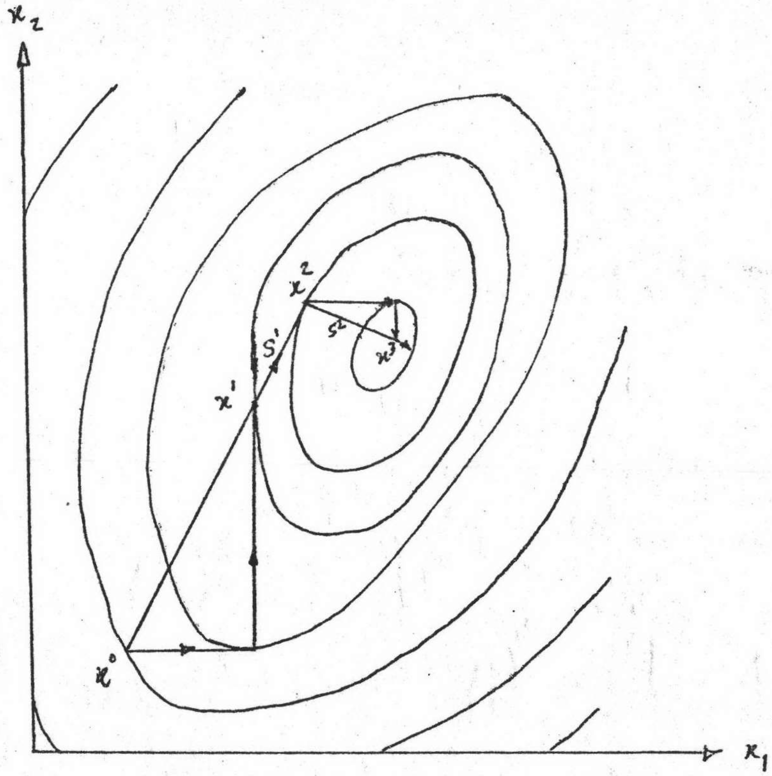

FIG. 3-3. New direct search method

2. Random Search Method

Another very interesting method is the Random Search method made by Brooks. This method is described below.

The direction of search at any point is selected purely in a random manner. Firstly, a unit vector for independent variables is chosen. The maximum step length at any move will not exceed the quantity of this unit vector and hence the maximum allowable step change can be fixed from practical consideration. Let $\underline{U} = (u_1, \ldots, u_n)$, be the step size vector. Now a sequence of n random numbers $r_1, \ldots, r_n$ are generated. These random numbers between ±1 are non-repetitive and have a flat frequency distribution. The components of the search direction $\underline{S}$ are given by $(u_1 r_1, \ldots, u_n r_n)$. If the function does not show improvement in this direction the opposite direction is tried. Once a decrease is noticed in a direction, movement is taken along the direction till a minimum is boxed and then the iteration is started again.

If at a point both a random direction and its opposite direction do not show any decrease in the objective function, a new random direction is generated. If (5×n) random directions together with their opposites do not yield improvement the step length is reduced. The procedure is terminated when the step size is below a specified accuracy and the objective function accuracy and the objective function accuracy is also achieved.

3. New Direct Search Method

The new direct search method which is used in this study is the combination of those two methods described in the last two sections. The technique in the random search method is used for finding optimum value but, instead of a random manner as the direction of search, the modified direct search method is used. With the step length, s_o, the step-size vector $\underline{S} = (s_1, s_2, \ldots, s_n)$ has $s_i \leqslant s_o$. Starting from an initial approximation $\underline{x}^o$, each coordinate direction is searched sequentially with $\underline{S}$ to a point $\underline{x}^1$ by exploratory moves. Then, a pattern search is made along $\underline{x}^1 - \underline{x}^o$ and the procedure is restarted. If both the exploratory move and the pattern search do not yield improvement, the step length s_o is reduced. As in random search method, the procedure is terminated when the step size and the decreasing of objective function is smaller than the specified accuracy.

The Decision Rule and Minimization Algorithm

The new direct search algorithm described here is the combination of Lootsma's method and the Brooks-Rosenbrock's new direct search method. The Lootsma's method is a technique for formulating the penalty function as described in Eq.(3-20) of SUMT which used Brooks-Rosenbrock's new direct search method described in the last section for seeking the optimum point. The nonlinear programming problem as formulated in the 'Nonlinear Programming Model' section will be solved.

The result of this algorithm will be the load distribution operation of the power plant. The following variables are defined:

$\underline{x}_1$	a feasible solution before exploratory move with dimension n;
$\underline{x}_2$	a feasible solution after exploratory move with dimension n;
$P(\underline{x})$	the augmented objective function;
r_k	the penalty factor;
$\underline{S}$	the step size vector which has (n-1) elements;
S_o	a step length of step size vector; and
S_m	the maximum element of $\underline{x}_2-\underline{x}_1$

Then, the step by step procedure for this minimization algorithm may be described as follows:

Step 1. Read all the input data: the number of units of plant(n), the given total gross load (L), the capacity of plant which is the minimum generation ($x_{i\ min}$) and the maximum generation ($x_{i\ max}$), the maximum degree of the polynomial equation (m), coefficients (a_{ij}) of the polynomial equation which is the characteristics of input-output equation of each unit of the plant.

Step 2. Set the starting point of generation load distribution ($\underline{x}_o$) within the capacity constraints, i.e., the region $\underline{g}(\underline{x}_o) > \underline{0}$ or

$$x_{i\ max} > x_{io} > x_{i\ min}$$

and

$$L - x_{n\ min} > \sum_{i=1}^{n-1} x_{io} > L - x_{n\ max}$$

Step 3. Compute the initial value of penalty factor, r_o, to make the penalty term equal to ten per cent of the objective function F. That is,

$$r_o \sum_{i=1}^{q} \ln g_i(\underline{x}_o) = 10\% \text{ of } F(\underline{x}_o) = \frac{F(\underline{x}_o)}{10.}$$

or

$$r_o = \frac{F(\underline{x}_o)/10.}{\sum_{i=1}^{q} \ln g_i(\underline{x}_o)}$$

where

$$F(\underline{x}_o) = \sum_{i=1}^{n-1} \sum_{j=o}^{m} a_{ij} x_{io}^{j} + \sum_{j=o}^{m} a_{nj} \left(L - \sum_{i=1}^{n-1} x_{io}\right)^{j}$$

and

$$\underline{g}(\underline{x}_o) = \begin{bmatrix} x_{1\,max} - x_{1o} \\ \vdots \qquad \vdots \\ x_{(n-1)max} - x_{(n-1)o} \\ x_{1o} - x_{1\,min} \\ \vdots \qquad \vdots \\ x_{(n-1)o} - x_{(n-1)min} \\ x_{n\,max} + \sum_{i=1}^{n-1} x_{io} - L \\ L - \sum_{i=1}^{n-1} x_{io} - x_{n\,min} \end{bmatrix}$$

Step 4. Select the step length S_o. The step length $S_o = 1$ is used herein.

Step 5. Set the step-size vector $\underline{S}$ for exploratory moves, i.e.,

$$S_i = \begin{cases} S_o & \text{when } i = \text{number of moving co-ordinate} \\ 0 & \text{when } i = \text{other co-ordinate.} \end{cases}$$

The number of moving co-ordinate is increased until the last co-ordinate has been moved. It is the end of exploratory moves.

Step 6. Move $\underline{x}$ with the step size $\underline{S}$ from $\underline{x}_{old}$ to $\underline{x}_{new}$, i.e.,

$$\underline{x}_{new} = \underline{x}_{old} + \underline{S}$$

Step 7. Check P which is the function of $\underline{x}_{old}$ and $\underline{x}_{new}$ for the success of decreasing value. If $P(\underline{x}_{new}) < P(\underline{x}_{old})$ moves $\underline{x}$ to $\underline{x}_{new}$, then go to step 6; otherwise go to step 8(a).

Step 8. If $P(\underline{x})$ fails to decrease in the first move, go to step 8(b), if it is not the first move go to step 9.

(b) Reverse the direction of step size vector

$$\underline{S} = -\underline{S}$$

then go to step 6 to move $\underline{x}$ again.

Step 9. Check for exploratory move exit, then go to step 10(a), otherwise go to step 5 to set $\underline{S}$ for moving $\underline{x}$ of the next co-ordinate.

Step 10. (a) If the pattern search has never been set up after exploratory moves, go to step 10(b); otherwise go to step 11(a).

(b) Set the step size vector $\underline{S}$ for pattern search i.e.,

$$\underline{s} = \frac{S_o}{S_m}(\underline{x}_2 - \underline{x}_1)$$

where S_o = Step length;

S_m = maximum value of elements in $(\underline{x}_2 - \underline{x}_1)$,

$\underline{x}_1$ = feasible solution before exploratory moves; and,

$\underline{x}_2$ = feasible solution after exploratory moves.

Theh go to step 6.

Step 11. (a) If $P(\underline{x})$ fails to decrease in both the exploratory moves and the pattern search then go to step 11(b), if there is some success go to step 11(c).

(b) If the step length is smaller than the specified accuracy, then go to step 12(a); otherwise go to step 11(d).

(c) If the decreasing value of $P(\underline{x})$ before exploratory moves and after the pattern search is smaller than the specified accuracy, then go to step 12(a); otherwise go to step 11(e).

(d) Reduce the step length, i.e.,

$$S_{o\ new} = \frac{S_{o\ old}}{C}$$

where

$$C > 1.$$

The value of C chosen should not be too large; otherwise the number of iterations will be greatly increased.

(e) Restart the first move of exploratory moves, i.e., setting the number of moving co-ordinate = 1, then go to step 5.

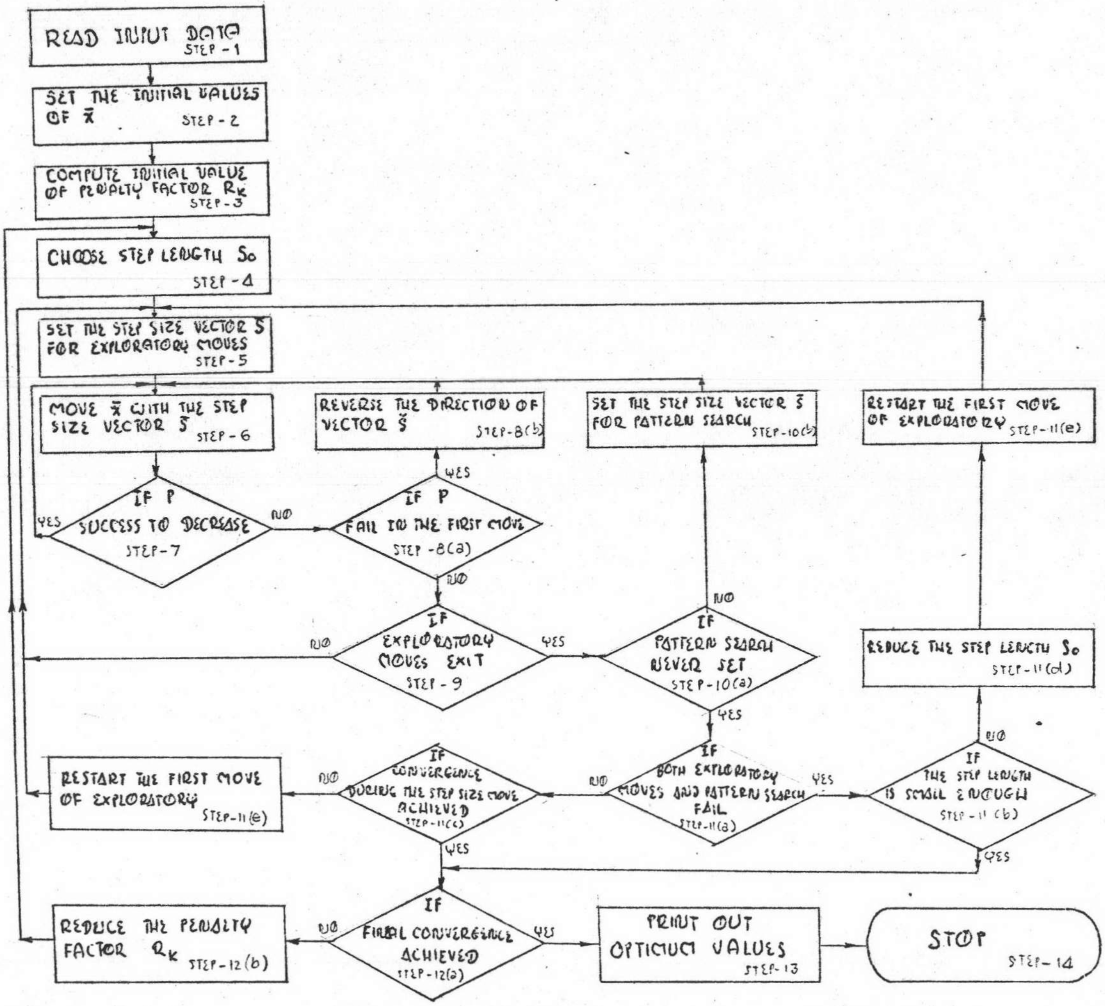

FIG. 3-4. THE FLOW CHART DIAGRAM OF n-UNIT PLANT LOAD DISTRIBUTION PROBLEM.

Step 12. (a) If the final convergence is achieved, i.e.,

$$\left| P(\underline{x}, r_{k-1}) - P(\underline{x}, r_k) \right| < \epsilon,$$

where the specified accuracy $\epsilon > 0$, the computation is finished; otherwise go to step 12(b).

(b) Reduce the penalty factor r_k, i.e.,

$$r_{k+1} = \frac{r_k}{C}$$

$C > 1$, then go to step 4.

Usually C = 4 or 5. If C > 10, it tends to decrease the penalty term too much prematurely.

Consideration of Emergency Conditions of Power Plant

Sometimes the generation load cannot be varied as required, since there are some emergency conditions to limit the generation load. For steam power plant there are two kinds of abnormal situations.

1. Routine

The machine must be tested periodically to find abnormal conditions. The generation load must keep constant in this condition. The machine which must be tested frequently and very carefully is the steam turbine. Usually, the output is reduced to half generation load.

Sometimes the periodic testing of machines, for example, the generator, boiler and air heater, could be carried out when a unit of the power plant is shutdown for the purpose of checking.

Another example is the condenser backwashing which reduces the efficiency of steam condensing, so that the generating load must be reduced to the safety limit; otherwise a low condenser vacuum is shown. This is an emergency condition.

2. <u>Emergency</u>

In case of emergency, the generated load must be reduced imm[illegible] to the safety region, or unit is shutdown, if necessary. Examples of emergency cases which may occur are low condenser vacuum, burner tripping and defects in some important parts of the power plant.

Both routine and emergency conditions above must keep the output within a limit or the unit shutdown if necessary, both of which affect the calculation of the optimum load distribution. For the unit shutdown case the unknown variables are reduced to the numbers of the unit remains. For the limitation of generation load within the safety region, the inequality constraints have to be reset. Subsequently, the algorithm used is in the same manner , both in the 'Decision Rule and the Minimization Algorithm'.