# CHAPTER II

# LITERATURE SURVEY

## Introduction

The problem of minimizing the cost of delivering power is not new. The classical examples relate to the problems of distributing a given total load among n units within a plant in which each unit has its individual characteristic( i.e., the relation of generated load and the energy input is not the same for all the units).

An early method for solving this problem called for supplying power from the most efficient plant until the point of maximum efficiency of that plant was reached, then the next most efficient plant would start to supply power to the system.[15] Today, it is known that this method fails to minimize cost.

The first application of a computerized economic dispatch was dated from the early 1960's.[12] This was preceded by some 18 years of theoretical work, mannual and semimanual economic dispatching operations. There are projects under way today that are planned to utilize more recent techniques, and there are indications that future applications will follow the more encompassing problem formulations.

## Background of Economic Operation Problems

The objective of the economic dispatching problem is to minimize the cost of generation satisfying at the same time the system constraints. There are several approaches to solve this problem.

SKROTZKI and VOPAT and STEVENSON Jr. (1962)[15] used a graphical method for solving a load distribution problem of n unit plant. Their method depended on a basic rule which stated that, for a minimum cost, all units within a plant had to operate at the same incremental fuel cost which was the slope of input-output curve.

DOMMEL and TINNEY (1968)[3] used the classical optimization method of Lagrangian multipliers to minimize costs. They improved the method by applying penalties to the system inequality constraints. The minimum and the penalty functions to account for dependent constraints are obtained by Newton's method and a gradient adjustment algorithm, respectively.

SASSON(1969)[10,11] formulated a very specific economic dispatching problem and used the nonlinear programming approach to solve this. In (3) and (4) penalties are also introduced for equality constraints which were required to satisfy the load demand. He also described various advantages of the sequential minimization procedure,

RAMAMOORTY and RAO (1970)[7] developed the SUMT funtion, which is used by SASSON, to the penalty function approach. They also used the Newton-Raphson technique for an initial solution athat satisfied the load demand. Their formulation reduced the dimension of the problem more than the earlier exact methods.

BILLINGTION and SACHDEVA (1972)[2] considered the four optimization problems; real power dispatch, reactive power dispatch, real-and reactive-power dispatch, and sequential real- and reactive-power dispatch. The method used to solve these problems was based on the nonlinear programming approach that had been used by SASSON.

ALSAC and STOTT (1974)[1] developed the Dommel-Tinney approach to give an optimal operating point of steady-state system, according to an economic objective.

HAPP (1974)[5] developed an economic dispatch procedure by the use of the incremental fuel costs. The major advantages of the procedure over other optimal dispatch procedures were its inherent simplicity and rapid convergence behavior which were the characteristics particularly important for on-line implementation. Nevertheless, it does not suit the system of this country, which is the semi-automatic economic dispatch operation.

SASSON and MERRILL (1974)[12] extended some applications of the optimization techniques which were used in economic dispatch to the other power system problems. They also gathered several methods and presented them in their paper.

## The Survey of Methodologies

### 1. The survey of load division methodologies

In solving the n-unit plant load distribution problem, there are three basic approaches:

(1) the incremental fuel cost approach;

(2) the nonlinear programming (NLP) appraach; and

(3) the penalty function approach.

All three approaches may be fiewed as the optimization or near optimization theory.

### 1.1 Incremental fuel zost approach

The fundamental concept of this approach was developed on the basis of the principal idea that all units within a power plant must operate at the same incremantal fuel cost for a minimum cost. This approach was widely used by STEVENSON Jr.[15], SKROTZKI and VOPAT.[13] HAPP[5] used this approach for the on-line implementation.

### 1.2 Nonlinear programming approach[4,16]

This is the most general case of mathematical programming. It is a problem to have at least one nonlinear constraint or to have a nonlinear objective function. Naturally, quadratic programming is a particular case of nonlinear programming.

The computational solution of an NLP problem is much more complicated..There is no computational algorithm which would guarantee a solution to any NLP problem in a finite number of steps. A number of isolated algorighms have been developed in the past, namely, the gradient approach; the method of feasible directions developed by G. Zoutendijk; the gradient projection method developed by J.B. Rosen; and Lastly, the sequential unconstrained minimization technique (SUMT) originally proposed by C.W. Caroll and then rigorously justified and developed into a working computer program by A.V. Fiacco and G.P. McCormick.

One of the main advantages of the last method (SUMT) is that it deals efficiently with nonlinear objective functions and constraints of both equality and inequality types. The convexity or concavity of the functions involved is not a necessary condition for the algorithm to work. However, if these conditions are not satisfied, the algorithm may converge to a local instead of a global extremum.

### 1.3 Penalty function approach (7,10,11)

This approach relies on studying the penalty factor of the penalty function used in SUMT. It is based on the transformation of the original constrained problem into an auxiliary unconstrained problem the minimum of which is the same as the minimum of the original problem. Unconstrained minimization techniques can then be used to obtain the optimum.

Carrol first proposed a SUMT method that Fiacco and McCormick later developed and gave rigorous proof. This method deals with the case of a nonlinear problem which can be stated as " minimize f(x) subject to the constraints

$$g_i(x) \geqslant 0 \qquad i = 1,\ldots,m \tag{2-1}$$

where x is an n-dimensional vector and $m > o$." The method is to minimize the unconstrained function

$$P(\underline{x},r_k) = f(\underline{x}) + r_k\sum_{i=1}^{m}\frac{1}{g_i(\underline{x})}, \qquad g_i(\underline{x}) > 0. \tag{2-2}$$

Lootsma searched for a function $g_i(\underline{x})$ which is

$$P(\underline{x},r_k) = f(\underline{x}) - r_k \sum_{i=1}^{m} \ln g_i(\underline{x}), \quad g_i(\underline{x}) > 0. \qquad (2\text{-}3)$$

A necessary condition of both methods before commencing the first minimization is that an initial feasible point has to satisfy all inequality constraints $\underline{g}(x) > \underline{0}$. As $r_k$ approaches zero, P and f coincide.

Lastly, a method due to Zangwill was presented,

$$P(\underline{x},r_k) = f(\underline{x}) + \frac{1}{r_k} \sum_{i=1}^{m} (g_i(\underline{x}))^2, \quad g_i(x) \geq 0. \qquad (2\text{-}4)$$

A sequence of $r_k$ values are chosen, but there is no need for an initial feasible point with Zangwill's method.

Even though Zangwill's method has the advantage over the other two in that it can handle equality constraints as well as inequality constraints and does not require an initial interior point. Sasson proved in his paper[11] that with Zangwill's method an undesired effect can also be produced with an unsuitable choice of the initial r factor. If large values are selected, the process becomes a very lengthy one; if too small, premature convergence to a sub-optimal point can occur. In both cases, there is a tendency to involve a very large number as $r_k$ approaches zero, making the minimizations very difficult.

In this study, the Lootsma's penalty function is used.

## 2. The survey of unconstrained minimization methodologies [9-10,16]

Some of the techniques described in Sec.2, and in particular the SUMT method, require numerous intermediate solutions of unconstrained extremization problems. One of the possibilities of performing unconstrained extremization is through a direct search method. Many efficient search methods have been developed; i.e., the steepest descent method, parallel tangents method, Powell's method, Fletcher and Reeves' method of conjugate gradients, Davidon's method, Fletcher-Powell method, the generalized Newton-Raphson method, and so on. Several comparative studies(9,16) have been made on these methods.

## 2.1 The steepest descent method

The steepest descent method is based on the fact that the gradient of a function at a point shows the direction of the greatest rate of increase of the function at the point. Thus, by moving in the negative gradient direction, the function is decreased at the maximum rate.

Although the logic of the steepest descent method is easy to comprehend and simple to implement, the method suffers from serious disadvantages. Firstly, the method does not guarantee convergence after a definite number of iterations even for simple and well-behaved functions. For instance, even when the isocost contours of the objective function P are a family of ellipses, the convergence depends to a large extent on the starting point. Secondly, when P is complicated and exhibits ridges, oscillations occur and the convergence is impaired. In addition, when P is complicated, the evaluation of the gradient is itself a laborious process.

4.2 Davidon-Fletcher-Powell method

This method belongs to the category of variable matric methods which are derived from the well-known Newton-Raphson or second order gradient technique. Instead of moving along the negative gradient ($\nabla P$) the movement is made along a mapped gradient ($-H^i \nabla P$), where $H^i$ is a $(n \times n)$ positive definite matrix.

The procedure is started with an initial approximation $X^o$ and any positive definite matrix $H^o$, usually the unity matrix $I_n$. Thus, the first move is similar to the steepest descent move. Subsequently, $H^i$ is updated and tends to $G^{-1}$ where G is a matrix of the second-order partial derivatives of P at the minimum. This method locates the minimum of a quadratic objective function of n variables in n iterations.

2.3 Powell's method of conjugate directions

Two vectors X and Y are said to be conjugate with respect to a matrix A if $X'AY=0$. If $X_1, X_2 \ldots X_n$ are n vectors conjugate to each other, it can be shown that a quadratic function $F = X'AY + B'X + C$ is minimized by searching once along each of the n conjugate directions. The theorem is the basis of all conjugate direction methods. There are several variations of the method depending on the way in which the conjugate directions are generated.

The Fletcher and Reeves method and the method of parallel tangents make use of the gradient of the function to generate conjugate directions. Another method due to Powell achieves the same without the use of gradients.

2.4 Direct search methods.

These methods make use of only function evaluations and are reviewed in detail by Fletcher. The most elementary version of this is to search sequentially along each of the n coordinate directions and to repeat the cycle till convergence is obtained. But this method is highly oscillatory and has poor convergence properties. Details will be described in Chapter III.

4.5 Random search methods

These methods belong to an entirely different class of minimization techniques. They do not require derivatives and rely only on direct function evaluations. A review of the random search methods has been made by Brooks, and it will be described in Chapter III too.

Ramarathnam, Desai and Rao commented in their paper [9] that the last two methods (Sec.4.4 and 4.5) work very well for the functions used to test the efficiency of the algorithms. The combination of these two methods will be used in this study.