CHAPTER I

INTRODUCTION

In this thesis, we are primarily interested in the following two questions :

Question 1 . Which groups are locally cyclic decomposable ?

Question 2. If a group is locally cyclic decomposable, is the decomposition unique ?

In chapter II, we recall some relevant notions and facts from group theory. Particularly, we prove that a p-primary abelian group is (direct sum) indecomposable if and only if it is isomorphic to a cyclic group of order a power of p or else to a group of type p^{∞}.

In chapter III, locally cyclic groups are studied and they turn out to be just the groups isomorphic to subgroups of the additive rationals $\mathbb{Q}$ or of the additive group $\mathbb{Q}/\mathbb{Z}$, the rationals modulo $\mathbb{Z}$.

In the remainder of the thesis, we devote our full attention to the locally cyclic decomposable group. We show in Chapter IV, that any group, abelian or not, can have at most one locally cyclic decomposition, and that if such a decomposition does exist , it is precisely the collection of all maximal locally cyclic subgroups of the given group. Thus the question of uniqueness is completely answered; moreover, in the process, we have found how prominent a role is played by the maximal locally cyclic subgroups.

In chapter V, we show that the locally cyclic decomposability of a torsion-free groups G is equivalent to several other conditions; particular to the simple condition that for any non-zero integer n, and for any x, y $\in$ G, $x^n = y^n$ implies $x = y$.

For abelian groups, the question 1 is completely settled in a very satisfactory manner. Namely, in Chapter VII, we show that an abelian group G is locally cyclic decomposable if and only if its torsion subgroup tG is, and (by chapter VI) this is so if and only if either tG is locally cyclic or else tG is the set-theoretical union of a disjoint family of p-cocyclic subgroups (for a fixed prime p).

In conclusion, this thesis contains an affermative answer to Question 2; partial but very satisfactory answers to Question 1. Equally important is the following question which is left untouched.

Question 3. Given a locally cyclic decomposable group G, can we reconstruct G from the known subgroup structures of the decomposition ?