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ວິທຍານີພັນຫຼືນເປັນສ່ວນໜຶ່ງຂອງກາຮົກສາການທັກສູງປະເທດລາວ  
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ON LOCALLY CYCLIC DECOMPOSABLE GROUPS

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## ບຫກດຍຂອງ

ໂຄດສ່ົ້າສຶກລິກິດຮູບເປັນຮູບໃໝ່ທຸກ ๆ ກຽມຢ່ອຍທີ່ມີເນັນເນັດເຕືອນເປັນຈຳນານຈຳກັດ  
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 ເບັນຮູບແລະແສດງໃຫ້ເຫັນວ່າຮູບໃດ ຈະມີໂຄດສ່ົ້າສຶກລິກິດຄອມໄພສີ້ນີ້ໄດ້ຍ່າງມາກເພີ່ມ  
 ອ່າຍາງເຄີຍເຫັນນັ້ນ ຄືຄອມໄພສີ້ນີ້ກໍສົກເຊຫວອງແມາຊີມລົງໂຄດສ່ົ້າສຶກລິກິດຮູບຢ່ອຍທັງໝາຍ  
 ເອງ ເຮັດວຽກແສດງອັກວ່າວ່າ ດາ G ເປັນວິນິຍົງຮູບແລ້ວ G ຈະມີໂຄດສ່ົ້າສຶກລິກິດຄື-  
 ຄອມໄພສີ້ນີ້ ກໍຕ້ອມເມື່ອ ທອຮັນຮູບຢ່ອຍ tG ຈະເປັນໂຄດສ່ົ້າສຶກລິກິດຮູບນ້ອມໃຈນັ້ນ tG ຈະ  
 ເປັນຄິສຈອບທູ້ນີ້ເນັນຂອງແມິລືຂອງພື້-ໄສສຶກລິກິດຮູບຢ່ອຍ ເຫັນນັ້ນ ນອກຈາກນີ້ສໍາຫັນຮູບ  
 G ທີ່ໄໝເປັນວິນິຍົງຮູບ ເຮັດວຽກໄດ້ເພີ່ມວ່າສົ່ງທີ່ໄປນີ້ອີກວິວາເລັນທັນ

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- ຂ. G ເປັນຄິສຈອບທູ້ນີ້ເນັນຂອງແມິລືຂອງວິນິຍົງທີ່ໄດ້ເປັນທອຮັນໄຣຮູບຢ່ອຍ
- ຄ. ສໍາຫັນແຕລະຈຳນວນເຕັມບາກ n ທີ່ໄໝເປັນຄູ່ນົມ ແລະແຕລະ x, y  
 ທີ່ເປັນສາມາຊິກຂອງ G, ດາ  $x^n = y^n$  ແລ້ວ  $x = y$ .

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## ABSTRACT

A locally cyclic group is one in which every finitely generated subgroup is cyclic. A locally cyclic decomposable group is one which is a disjoint union of locally cyclic subgroups. In this thesis, we study the locally cyclic decomposable groups. In particular, we show that any group can have at most one locally cyclic decomposition and if such decomposition exists, it is just the collection of all the maximal locally cyclic subgroups of the group. We also show that an abelian group  $G$  is locally cyclic decomposable if and only if its torsion subgroup  $tG$  is either locally cyclic or else is the disjoint union of a family of  $p$ -cocyclic subgroups, for some prime  $p$ . However, for non-abelian group  $G$  we can show only that the following are equivalent :

- a.  $G$  is torsion-free locally cyclic decomposable.

b.  $G$  is a disjoint union of a family of abelian torsion-free subgroups.

c. For any non-zero integer  $n$ , for any  $x, y \in G$ ,  
 $x^n = y^n$  implies  $x = y$ .

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