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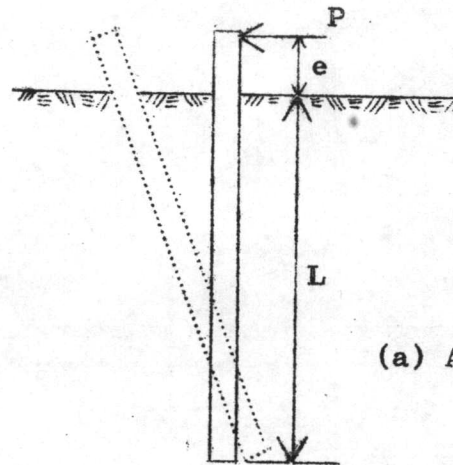


## APPENDIX A

Derivation of Deflection Equations for Short and Long Piles

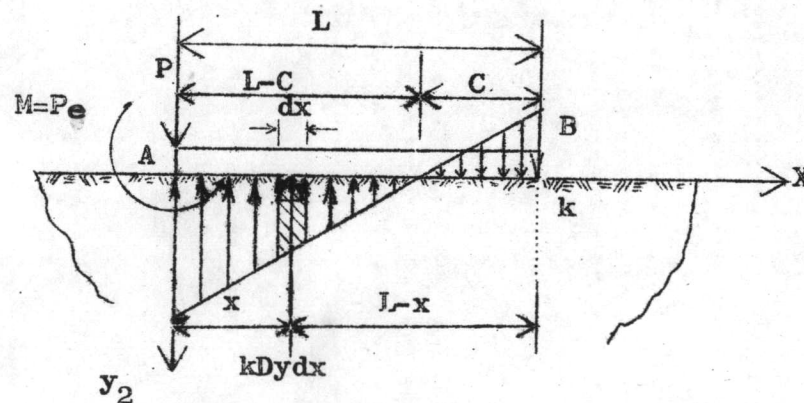
A Short Free-Headed Pile

For  $\beta L < 1.5$  (The pile is infinitely stiff)



(a) A Short Free-Headed Pile

In the following analysis the pile is treated as a rigid short beam resting on the surface of a semi-infinite, ideal elastic medium as shown in Fig. A-1b.



(b) A Short Beam Subjected to Edge Forces on Elastic Foundation

Fig. A-1 - Analysis of a Short Free-Headed Pile

Applying the static equation considering of vertical forces  $\sum F_y = 0$  we have

$$\int_0^L k y dx = P$$

$$\int_0^L y dx = \frac{P}{kD} \dots\dots\dots(A-1)$$

Substituting the straight line equation

$$y(x) = A + BX \dots\dots\dots(A-2)$$

into Eq. (A-1) leads to

$$AL + \frac{BL^2}{2} = \frac{P}{kD} \dots\dots\dots(A-3)$$

By taking the moments of the forces about point B (Fig. A-1b) one obtains

$$\int_0^L k y (L - x) dx - P(e + L) = 0$$

Substituting Eq. (A-2) and simplifying yields

$$AL + \frac{BL^2}{3} = \frac{2P(e + L)}{LkD} \dots\dots\dots(A-4)$$

Substituting constant A & B into Eq. (A-2) and simplifying yields

$$y(x) = \frac{2P}{kDL} \left[ \left( 2 + \frac{3e}{L} \right) - \frac{3x}{L} \left( 1 + \frac{2e}{L} \right) \right] \dots\dots(A-5)$$

$$y_A = y_0 = \frac{4P}{kDL} \left( 1 + \frac{1.5e}{L} \right) \dots\dots\dots(A-6)$$

$$y_B = \frac{-2P}{kDL} \left( 1 + \frac{3e}{L} \right) \dots\dots\dots(A-7)$$

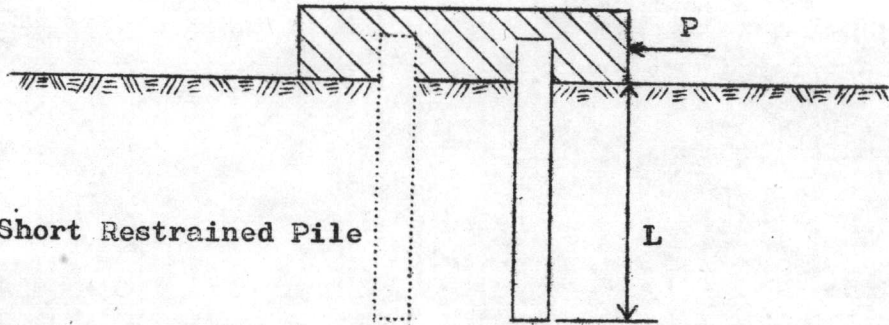


The moment at any point in the pile can be calculated from

$$\begin{aligned}
 M(x) &= \int_0^x kDy(L-x) dx - P(e+x) \\
 &= \frac{2P}{L} \int_0^x \left[ \left( 2 + \frac{3e}{L} \right) - \frac{3x}{L} \left( 1 + \frac{2e}{L} \right) \right] (L-x) dx - P(e+x), \\
 &\dots\dots\dots(A-8)
 \end{aligned}$$

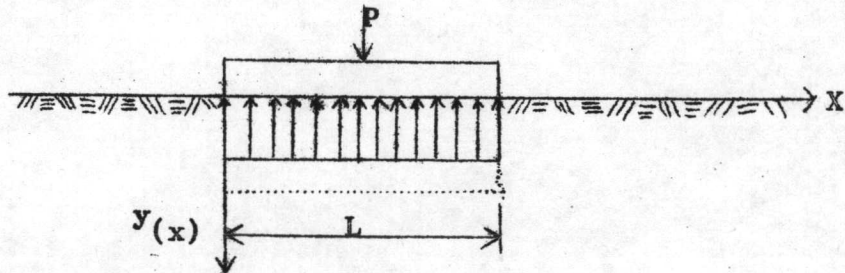


A Short Restrained Pile



(a) A Short Restrained Pile

In the following analysis, the pile is treated as a rigid short beam resting on the surface of semi-infinite, ideal elastic medium as shown in Fig. A - 2b.



(b) A Short Pile Subjected to a Concentrated Load at the Middle on Elastic Foundation

Fig. A - 2 - Analysis of a Short Restrained Pile

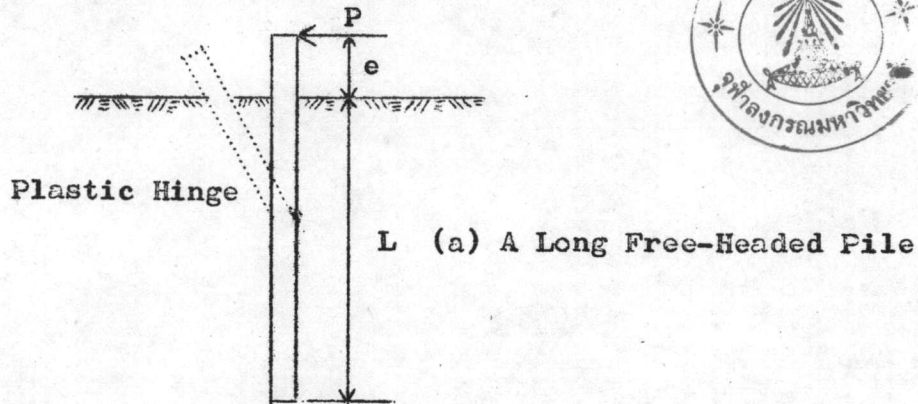
Applying the static equation considering of vertical forces

$\sum F_y = 0$  we have

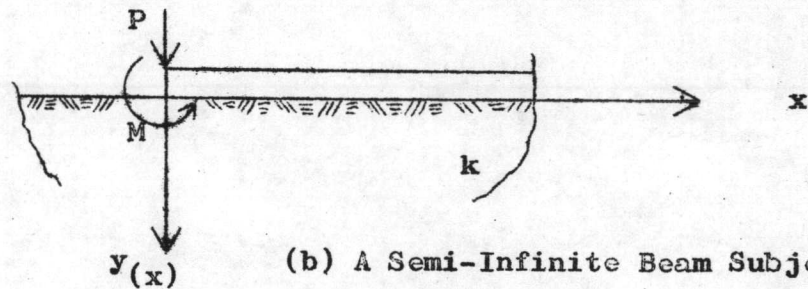
$$kDyL = P$$

$$y_0 = \frac{P}{kDL} \dots \dots \dots (A-9)$$

A Long Free-Headed Pile ( $\beta L > 2.5$ )



In the following analysis, the pile is treated as a semi-infinite beam resting on the surface of a semi-infinite, ideal elastic medium as shown in Fig. A - 3b.



**Fig. A-3 - Analysis of a Long Free-Headed Pile For Semi-Infinite Beam Subjected to Edge Forces**

General Solution :

$$y(x) = e^{-\beta x}(C \cos \beta x + D \sin \beta x) \dots\dots\dots(A-10a)$$

$$y'(x) = -\beta e^{-\beta x}(\cos \beta x (C - D) + \sin \beta x (D + C)) \dots\dots(A-10b)$$

$$M(x) = -2EI\beta^2 e^{-\beta x}(C \sin \beta x - D \cos \beta x) \dots\dots\dots(A-10c)$$

$$V(x) = -2EI\beta^3 e^{-\beta x}\{C(\cos \beta x - \sin \beta x) + D(\cos \beta x + \sin \beta x)\} \dots\dots\dots(A-10d)$$

Boundary Conditions :

$$\left. \begin{aligned} y'' \Big|_{x=0} &= \frac{M}{EI} \\ y''' \Big|_{x=0} &= \frac{P}{EI} \end{aligned} \right\} \dots\dots\dots(A-11)$$



and

$$y(x) = e^{-\beta x}(C \cos \beta x + D \sin \beta x) \quad \dots(A-12a)$$

$$y'(x) = \beta e^{-\beta x}(D \cos \beta x - C \sin \beta x) - \beta e^{-\beta x}(C \cos \beta x + D \sin \beta x) \dots(A-12b)$$

$$y''(x) = -2\beta^2 e^{-\beta x}(D \cos \beta x - C \sin \beta x) \quad \dots(A-12c)$$

$$y'''(x) = 2\beta^3 e^{-\beta x}(D \cos \beta x - C \sin \beta x) + 2\beta^3 e^{-\beta x}(C \cos \beta x + D \sin \beta x) \quad \dots(A-12d)$$

Substituting Eq. (A-12) into Eq. (A-11) yields

$$-2\beta^2 D = \frac{M}{EI}, \quad D = -\frac{M}{2\beta^2 EI}$$

$$2\beta^3 D + 2\beta^3 C = \frac{P}{EI}, \quad C = \frac{(P + \beta M)}{2\beta^3 EI}$$

Substituting C &amp; D into Eq. (A-10) yields

$$y(x) = \frac{e^{-\beta x}}{2\beta^3 EI} \left( P \cos \beta x + \beta M (\cos \beta x - \sin \beta x) \right) \quad \dots(A-13a)$$

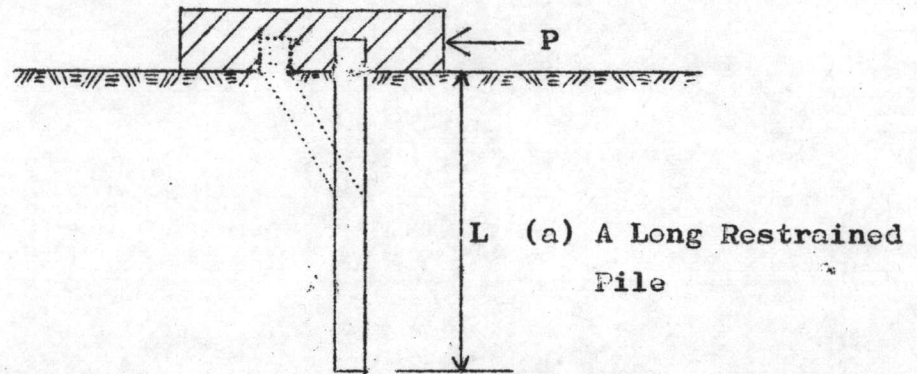
$$y'(x) = -\frac{P}{2\beta^2 EI} e^{-\beta x} (\cos \beta x + \sin \beta x) - \frac{M}{\beta EI} e^{-\beta x} \cos \beta x \quad \dots(A-13b)$$

$$M(x) = -EI y'' = -\frac{e^{-\beta x}}{\beta} P \sin \beta x + \beta M (\cos \beta x + \sin \beta x) \quad \dots(A-13c)$$

$$V(x) = -EI y''' = -P e^{-\beta x} (\cos \beta x - \sin \beta x) + \beta M e^{-\beta x} \sin \beta x \quad \dots(A-13d)$$

$$y_0 = \frac{2\beta P}{k_{\infty} D} (1 + \beta e) \quad \dots(A-14)$$

A Long Restrained Pile



In the following analysis, the pile is treated as a semi-infinite beam resting on the surface of a semi-infinite, ideal elastic medium as shown in Fig. A-4b.

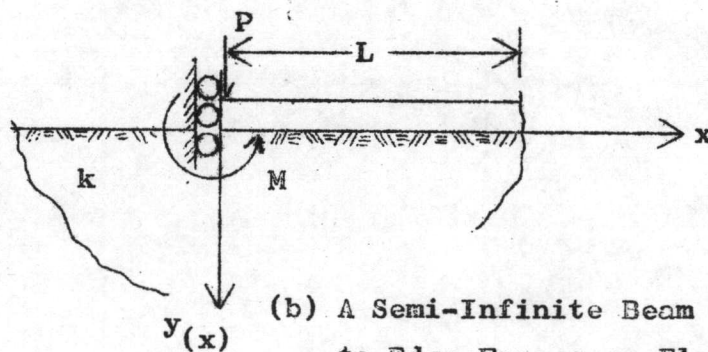


Fig. A-4 - Analysis of a Long Restrained Pile

Boundary Conditions:-

$$\left. \begin{aligned} y'(0) &= 0 \\ y''(0) &= \frac{P}{EI} \end{aligned} \right\} \dots\dots\dots (A-15)$$

Substituting Eq. (A-12) into Eq. (A-15) yields

$$\beta D - \beta C = 0, \quad C = D$$

$$2 \beta^3 D + 2 \beta^3 C = \frac{P}{EI}, \quad C = D = \frac{P}{4 \beta^3 EI}$$

Substituting C & D into Eq. (A-10) yields

$$y(x) = \frac{P\beta}{k_{\infty} D} e^{-\beta x} (\cos \beta x + \sin \beta x) \dots\dots\dots (A-16a)$$

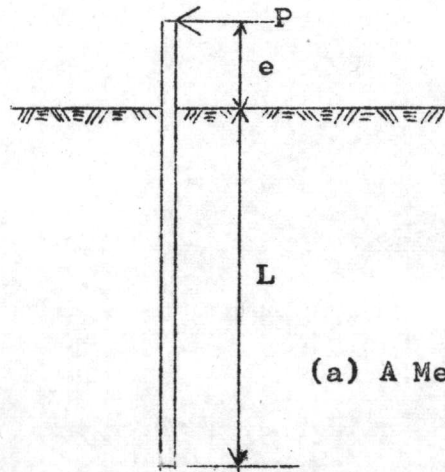
$$y'(x) = \frac{-P}{2 \beta^2 EI} e^{-\beta x} \sin \beta x \dots\dots\dots (A-16b)$$

$$M(x) = \frac{P}{2\beta} e^{-\beta x} (\cos \beta x - \sin \beta x) \dots\dots\dots (A-16c)$$

$$V(x) = -P e^{-\beta x} \cos \beta x \dots\dots\dots (A-16d)$$

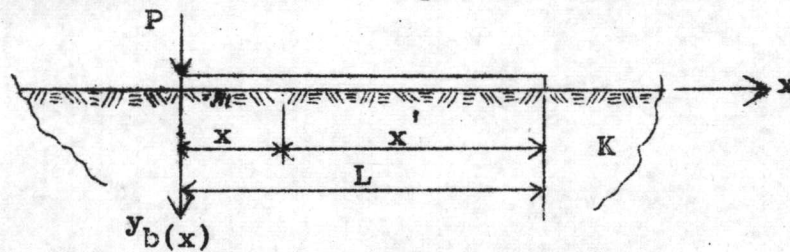
$$y_0 = \frac{P\beta}{k_{\infty} D} \dots\dots\dots (A-17)$$

A Medium Free Headed Pile

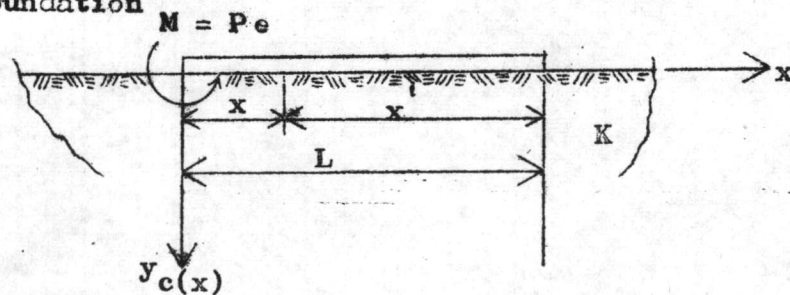


(a) A Medium Free-Headed Pile

In the following analysis, the pile is treated as a finite beam resting on the surface of a semi-infinite, ideal elastic medium as shown in Figs. A-5b and A-5c.



(b) A Finite Beam Subjected to Concentrated Load on Elastic Foundation



(c) A Finite Beam Subjected to Concentrated Moment on Elastic Foundation

Fig. A-5 - Analysis of a Medium Free-Headed Pile



Using the following symbols:

$$A(x) = e^{-\beta x}(\cos \beta x + \sin \beta x)$$

$$B(x) = e^{-\beta x} \sin \beta x$$

$$C(x) = e^{-\beta x}(\cos \beta x - \sin \beta x)$$

$$D(x) = e^{-\beta x} \cos \beta x$$

$$E_I = \frac{1}{2(1 - D^2(L)) - (1 + C(L))(1 - A(L))} = \frac{e^{\beta L}}{2(\sinh \beta L + \sin \beta L)}$$

$$E_{II} = \frac{1}{2(1 - D^2(L)) - (1 - C(L))(1 + A(L))} = \frac{e^{\beta L}}{2(\sinh \beta L - \sin \beta L)}$$

$$x' = L - x$$

The governing equation of the problem of beam on elastic foundation is

$$\frac{d^4 y}{dx^4} + \frac{K y}{EI} = \frac{P(x)}{EI} \dots \dots \dots (A-18)$$

General solutions of infinite beam subjected to a concentrated load (Fig. A-6) are

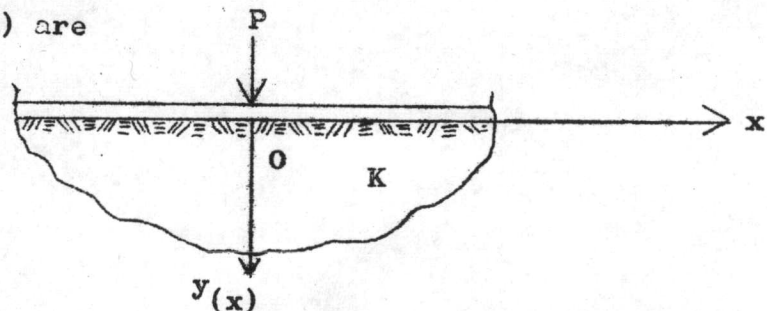


Fig. A-6 - Infinite Beam Subjected to a Concentrated Load

$$y(x) = \frac{P}{8\beta^3 EI} A(x) \dots\dots\dots (A-19a)$$

$$y'(x) = \frac{-P}{4\beta^2 EI} B(x) \dots\dots\dots (A-19b)$$

$$M(x) = \frac{P}{4\beta} C(x) \dots\dots\dots (A-19c)$$

$$D(x) = \frac{-P}{2} D(x) \dots\dots\dots (A-19d)$$

General solutions of infinite beam subjected to a concentrated Moment (Fig. A-7) are

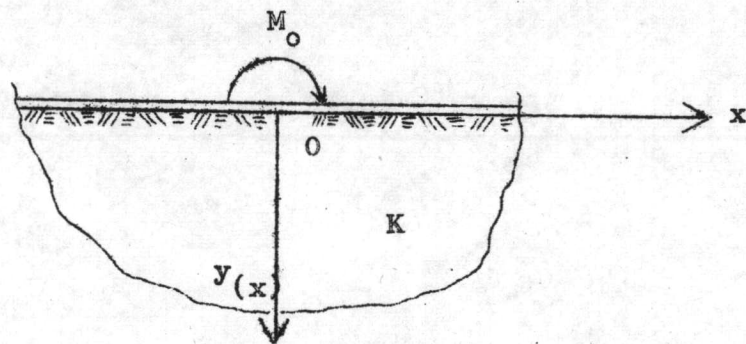


Fig. A-7 - Infinite Beam Subjected to a Concentrated Moment

$$y(x) = \frac{M_o}{4\beta^2 EI} B(x) \dots\dots\dots (A-20a)$$

$$y'(x) = \frac{M_o}{4\beta EI} C(x) \dots\dots\dots (A-20b)$$

$$M(x) = \frac{M_o}{2} D(x) \dots\dots\dots (A-20c)$$

$$V(x) = \frac{-M_o\beta}{2} A(x) \dots\dots\dots (A-20d)$$

From Fig. A-5b we get

$$\begin{aligned}
 y_b(x) = & \frac{P}{8\beta^3 EI} A(x) + \frac{P}{16\beta^3 EI} E_I \left\{ (1+C(L))(1-A(L)) + 2(1+D(L))^2 \right\} \{ \Lambda(x) + A(L-x) \} \\
 & + \frac{P}{16\beta^3 EI} E_{II} \left\{ (1-C(L))(1+A(L)) + 2(1-D(L))^2 \right\} \{ \Lambda(x) - A(L-x) \} \\
 & - \frac{P}{4\beta^3 EI} \left\{ E_I (1+C(L))(B(x)+B(L-x)) + E_{II} (1-C(L))(B(x)-B(L-x)) \right\} \\
 & \dots\dots\dots (A-21a)
 \end{aligned}$$

$$\begin{aligned}
 M_b(x) = & \frac{P}{4\beta} C(x) + \frac{P}{8\beta} E_I \left\{ (1+C(L))(1-A(L)) + 2(1+D(L))^2 \right\} \{ C(x) + C(L-x) \} \\
 & - \frac{P}{2\beta} E_{II} (1+C(L))(D(x)+D(L-x)) \\
 & + \frac{P}{8\beta} E_I \left\{ (1-C(L))(1+A(L)) + 2(1-D(L))^2 \right\} \{ C(x) - C(L-x) \} \\
 & - \frac{P}{2\beta} E_I (1-C(L))(D(x)-D(L-x)), \dots\dots\dots (A-21b)
 \end{aligned}$$

$$\begin{aligned}
 y_b(0) = & \frac{P}{8\beta^3 EI} + \frac{P}{16\beta^3 EI} E_I \left\{ (1+C(L))(1-A(L)) + 2(1+D(L))^2 \right\} \{ 1+A(L) \} \\
 & + \frac{P}{16\beta^3 EI} E_{II} \left\{ (1-C(L))(1+A(L)) + 2(1-D(L))^2 \right\} \{ 1-A(L) \} \\
 & - \frac{P}{4\beta^3 EI} B(L) \left\{ E_I (1+C(L)) - E_{II} (1-C(L)) \right\}. \dots\dots\dots (A-21c)
 \end{aligned}$$

HEFENYI (1946) simplified these equations into the forms:

$$y_b(x) = \frac{2P\beta}{K} \left[ \frac{\text{Sinh } \beta L \text{ cosh } \beta x \text{ Cosh } \beta x' - \text{sin } \beta L \text{ Cosh } \beta x \text{ cos } \beta x'}{\text{Sinh}^2 \beta L - \text{sin}^2 \beta L} \right] \dots\dots\dots (A-22a)$$

$$M_b(x) = \frac{-P}{\beta} \left[ \frac{\text{Sinh } \beta L \text{ sin } \beta x \text{ Sinh } \beta x' - \text{sin } \beta L \text{ Sinh } \beta x \text{ sin } \beta x'}{\text{Sinh}^2 \beta L - \text{sin}^2 \beta L} \right] \dots\dots\dots (A-22b)$$

$$y_{b(0)} = \frac{2P\beta}{KJ} \left( \frac{\sinh \beta L \cosh \beta L - \sin \beta L \cos \beta L}{\sinh^2 \beta L - \sin^2 \beta L} \right) \dots\dots\dots(A-22c)$$

From Fig. A-5c, we get

$$y_c(x) = \frac{-M}{4\beta^2 EI} B(x) + \frac{M}{4\beta^2 EI} E_I (1-A(L)) (A(x) + A(L-x))$$

$$\frac{-M}{8\beta^2 EI} E_I \left\{ (1+C(L))(1-A(L)) + 2(1-D(L))^2 \right\} (B(x) + B(L-x))$$

$$+ \frac{M}{4\beta^2 EI} E_{II} (1+A(L)) (A(x) - A(L-x))$$

$$\frac{-M}{8\beta^2 EI} E_{IK} \left\{ (1-C(L))(1+A(L)) + 2(1+D(L))^2 \right\} (B(x) - B(L-x)),$$

\dots\dots\dots(A-23a)

$$M_{c(x)} = -\frac{M}{4} (D(x) + D(L-x)) + \frac{M}{2} E_I (1-A(L)) (C(x) + C(L-x))$$

$$- \frac{M}{4} E_I \left\{ (1+C(L))(1-A(L)) + 2(1-D(L))^2 \right\} (D(x) + D(L-x))$$

$$- \frac{M}{4} (D(x) - D(L-x)) + \frac{M}{2} E_{II} (1+A(L)) (C(x) - C(L-x))$$

$$- \frac{M}{4} E_{II} \left\{ (1-C(L))(1+A(L)) + 2(1+D(L))^2 \right\} (D(x) - D(L-x)), \dots\dots(A-23b)$$

$$y_c(0) = \frac{M}{4\beta^2 EI} E_I (1-A^2(L)) - \frac{MB(L)E_I}{8\beta^2 EI} \left\{ (1+C(L))(1-A(L)) + 2(1-D(L))^2 \right\}$$

$$+ \frac{M}{4\beta^2 EI} E_{II} (1-A^2(L)) + \frac{MB(L)E_{IK}}{8\beta^2 EI} \left\{ (1-C(L))(1+A(L)) + 2(1+D(L))^2 \right\}.$$

\dots\dots\dots(A-23c)

HETENYI (1946) simplified these equations into the forms:



$$y_c(x) = \frac{-2M\beta^2}{KJ} \left\{ \frac{\text{Sinh } \beta L (\text{Cosh } \beta x' \sin \beta x - \text{Sinh } \beta x' \cos \beta x) + \sin \beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L} \right. \\ \left. (\text{Sinh } \beta x \cos \beta x' - \text{Cosh } \beta x \sin \beta x') \right\} \dots\dots\dots (A-24a)$$

$$M_c(x) = -M \left\{ \frac{\text{Sinh } \beta L (\text{Sinh } \beta x' \cos \beta x + \text{Cosh } \beta x' \sin \beta x) - \sin \beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L} \right. \\ \left. (\text{Sinh } \beta x \cos \beta x' + \text{Cosh } \beta x \sin \beta x') \right\} \dots\dots\dots (A-24b)$$

$$y_c(o) = \frac{2M\beta^2}{KJ} \left\{ \frac{\text{Sinh}^2 \beta L + \sin^2 \beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L} \right\} \dots\dots\dots (A-24c)$$

Superpositions:-

$$y_a(x) = y_b(x) + y_c(x)$$

$$M_a(x) = M_b(x) + M_c(x)$$

$$y_a(o) = y_o = y_b(o) + y_c(o)$$

By superposition method, we get

$$y_a(x) = \frac{2P\beta}{K} \left\{ \frac{\text{Sinh } \beta L \cos \beta x \text{ Cosh } \beta x' - \sin \beta L \text{ Cosh } \beta x \cos \beta x'}{\text{Sinh}^2 \beta L - \sin^2 \beta L} \right\} \\ - \frac{2M\beta^2}{K} \left\{ \frac{\text{Sinh } \beta L (\text{Cosh } \beta x' \sin \beta x - \text{Sinh } \beta x' \cos \beta x) + \sin \beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L} \right. \\ \left. (\text{Sinh } \beta x \cos \beta x' - \text{Cosh } \beta x \sin \beta x') \right\} \dots\dots\dots (A-25a)$$

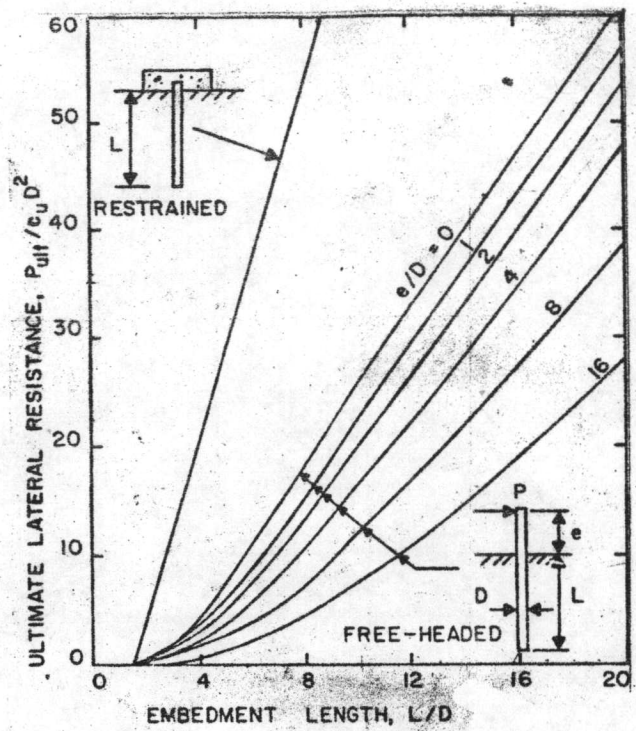
$$M_a(x) = -\frac{P}{\beta} \left\{ \frac{\text{Sinh } \beta L \sin \beta x \text{ Sinh } \beta x' - \sin \beta L \text{ Sinh } \beta x \sin \beta x'}{\text{Sinh}^2 \beta L - \sin^2 \beta L} \right\} \\ - M \left\{ \frac{\text{Sinh } \beta L (\text{Sinh } \beta x' \cos \beta x + \text{Cosh } \beta x' \sin \beta x) - \sin \beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L} \right. \\ \left. (\text{Sinh } \beta x \cos \beta x' + \text{Cosh } \beta x \sin \beta x') \right\} \dots\dots\dots (A-25b)$$

$$y_0 = \frac{2P\beta}{K} \left( \frac{\sinh \beta L \cosh \beta L - \sin \beta L \cos \beta L}{\sinh^2 \beta L - \sin^2 \beta L} \right) + \frac{2M\beta^2}{K} \left( \frac{\sinh^2 \beta L + \sin^2 \beta L}{\sinh^2 \beta L - \sin^2 \beta L} \right).$$

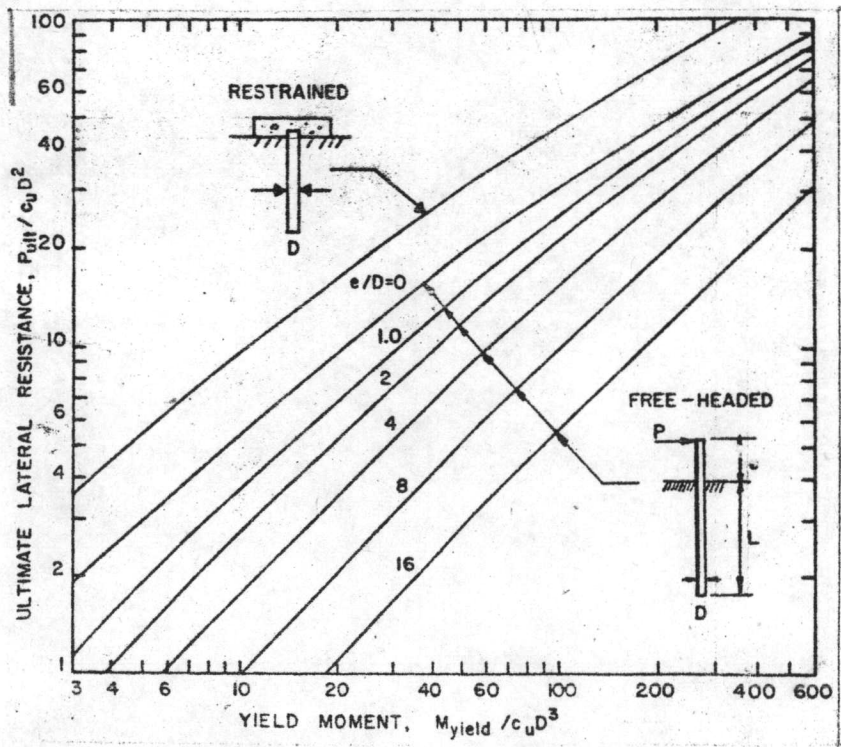
.....(A-25c)

APPENDIX B

Ultimate Lateral Resistance of Cohesive Soils



B - 1 Short Pile (After BROMS, 1964a)



B - 2 Long Pile (After BROMS, 1964a)

Fig. B - Cohesive Soils-Ultimate Lateral Resistance



## APPENDIX C

Displacement Influence Factor for Horizontal and Moment Loads

(AFTER POULOS, 1973)

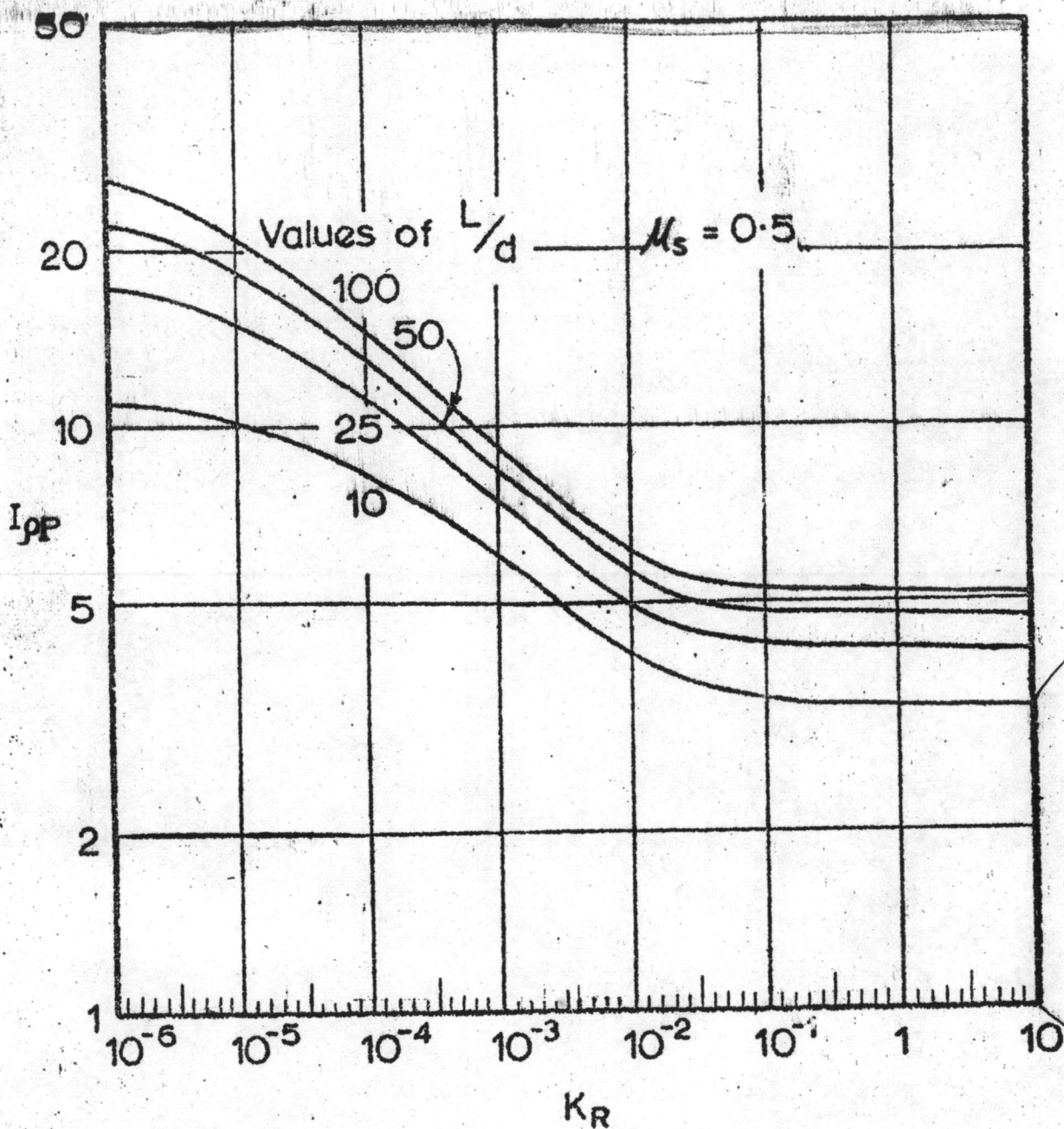


Fig. C-1 Values of  $I_{pp}$  - Free Head Pile.  
Constant Soil Modulus.

(AFTER POULOS, 1973)

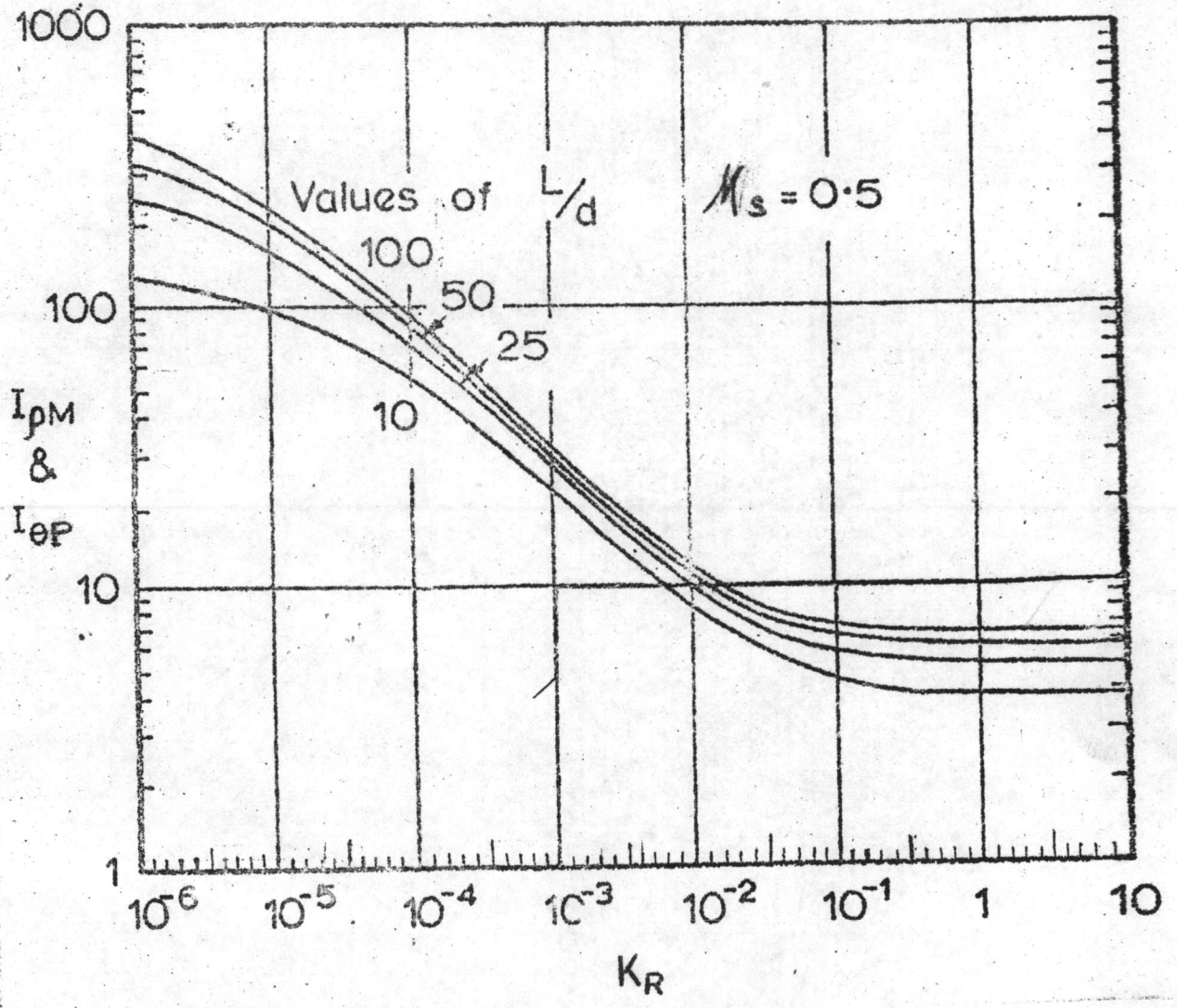
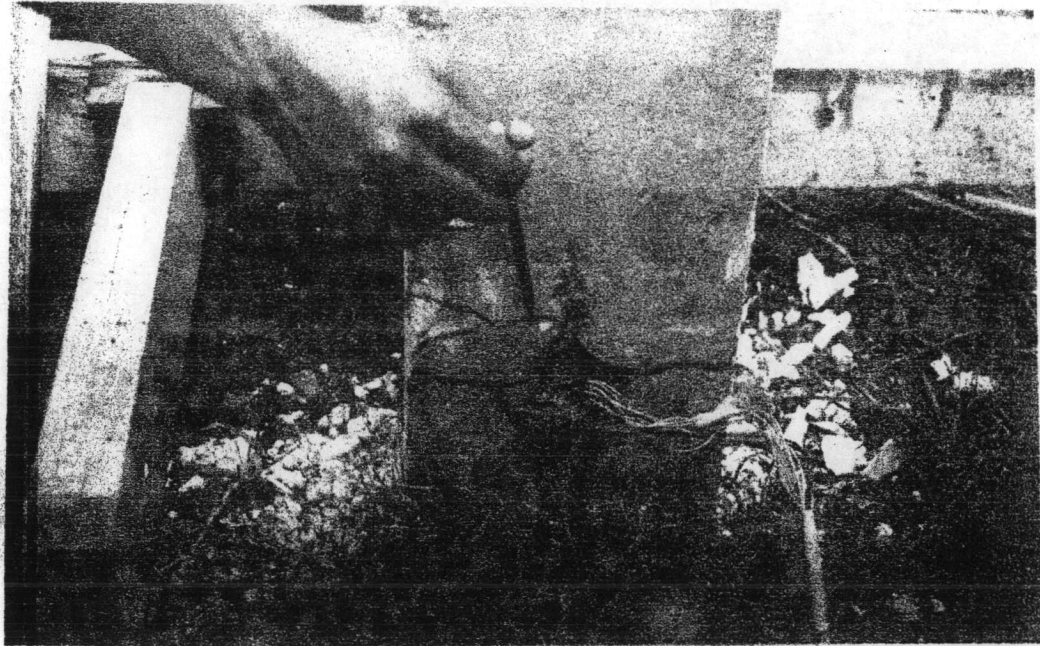


Fig. C-2 Values of  $I_{pM}$  and  $I_{\theta P}$  - Free Head Pile.  
Constant Soil Modulus.

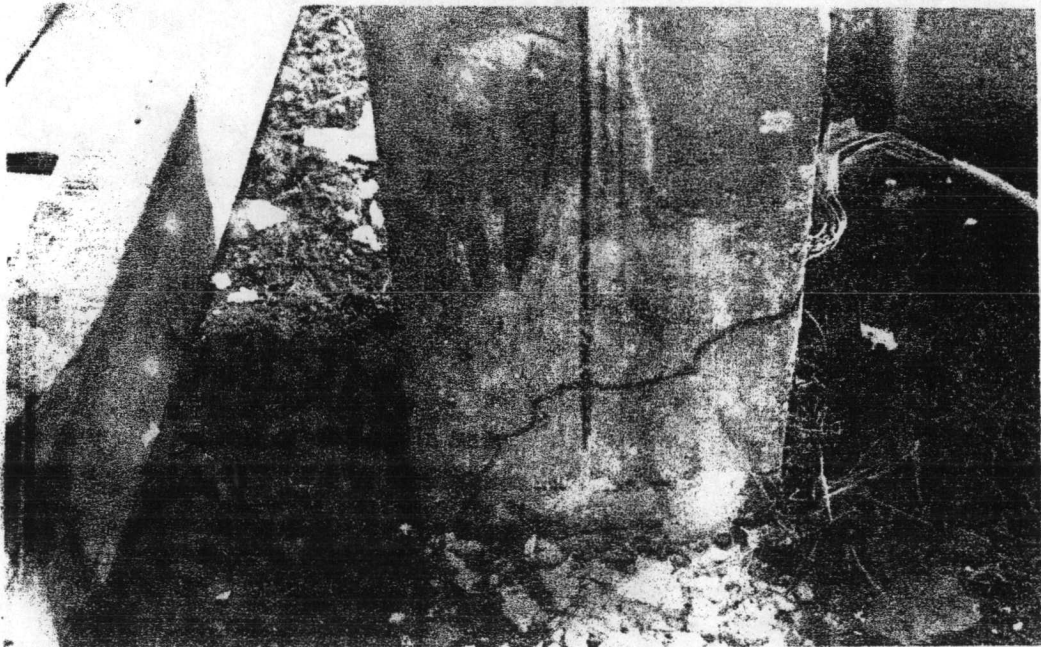


## APPENDIX D

## Cracking Characteristic and Position of a Long Pile



(D-1) Cracking in the front of the Pile



(D-2) Cracking in the Side of the Pile

Fig. D - Showing Cracking of Pile No. 5

## VITA

Mr. Suphachai Sithilertprasit

B. Eng. Khon Kaen University, 1969.

He joined the Faculty of Engineering, Khon Kaen University after graduation as a University lecturer up to now.