CHAPTER I

INTRODUCTION

1.1 The electromagnetic field due to a moving charge

In this thesis we shall study the problem of calculating the variations in time of the electromagnetic field at a fixed point in space due to an electric charge moving in the neigbourhood of that point.

Two formulas for the calculation are used. One formula taken from The Feynman Lectures on Physics* relates this electromagnetic field to information about the moving charge as seen from the fixed point. An attempt was made to calculate the magnetic field from the effect of the electric field on a moving test charge at the fixed point. This calculation was found to be too cumbersome and was abandoned. However, the method seems to be correct, and the incomplete calculation has been given in Chapter II

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* See Feynman, R.P.; Leighton, R.B. and Sands, M., The Feynman lecture on Physics Volume 1, equations (28.3) and (28.4)

The other formula used for calculating the electromagnetic field was taken from the Berkeley Physics Course, Volume Two(*). This formula applies only when the charge which is the source of the field is moving with a constant velocity. The variations in time of the electric field at the fixed point due to the moving charge as calculated by the Feynman formula and the Berkeley Physics Course formula were compared. The calculations were completed only in particularly simple examples, and a small discrepency between the two results was found.

Finally, a calculation was made by numerical methods to compare the two formulas, and agreement was obtained to within the accuracy used, which was however rather low.

1.2 Lorentz transformation

Consider two Euclidean reference systems S and S' with a constant relative velocity. In S we shall embed a set of rectangular axes with origin at θ and x-axis in the direction of the velocity v of S' relative to S. Next we shall take the set of all points in S' which move along the x-axis of S as the

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(*) See Purcell, Edward M. The electric and Magnetism, Berkeley Physics Course Volume 2, chapter 5.

x' - axis of a set of rectangular axes in S', orienting the x'-ais of S' in the same sense as the x-axis of S. The y' axis and the z' axis of S' we shall take as the set of those particles in S' which lie respectively on the y-axis and on the z-axis of S at the time $t = 0$ in S.

Then the origin θ' of $x' y' z'$ concides with the origin θ of xyz at time $t = 0$. We shall take the time t' of this event at θ' to be zero, thus synchronizing the clocks of θ and θ'.

The equations connecting the variables of the two systems are the Lorentz transformation equations[*]

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{xv}{c^2})$$

where $$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$ and

where v is the relative velocity of the two systems.

The unprimed quantities in terms of the primed quantities may be written as follows

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[*] See from any book on relativity.

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma(t' + \frac{x'v}{c^2})$$

1.3 Feynman's formula.

Consider the field propagated with speed c away from an electric charge. The field at any point P, at time t, must have originated from the charge at some earlier time, when the charge was at its retarded position R. The electric and magnetic fields take time $\frac{r_R}{c}$ to go from the retarded position R to the field point P, and the charge is at its retarded position at the retarded time $t - \frac{r_R}{c}$, see figure 1.1

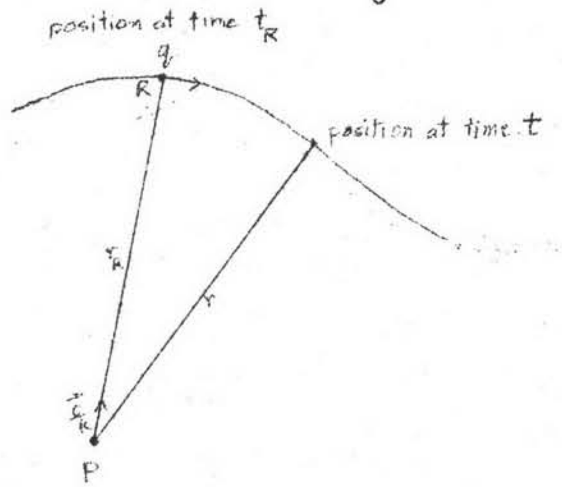

Figure 1.1 Retarded position of source.

Feynman's formula for the electric field is

$$\vec{E} = - q \left[\frac{\vec{e}_R}{r_R^2} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{\vec{e}_R}{r_R^2} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \vec{e}_R \right] \qquad (1.3.1)$$

which is the electric field at a point P in terms of a charge q as seen from P.

Feynman's formula for the magnetic field is

$$\vec{B} = - \vec{e}_R \times \vec{E} \qquad (1.3.2)$$

1.4 The Berkeley Physics Course formula

A charge moves with velocity v in S. parallel to x-axis, see figure 1.2

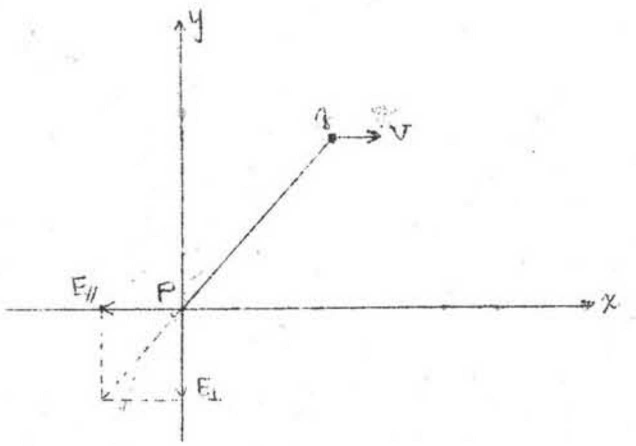

Figure 1.2. The charge q moving in S frame.

Let S' be the frame of reference in which the charge q is at rest, see figure 1.3. Suppose at time $t' = t = 0$, P is at the origin P' in S'

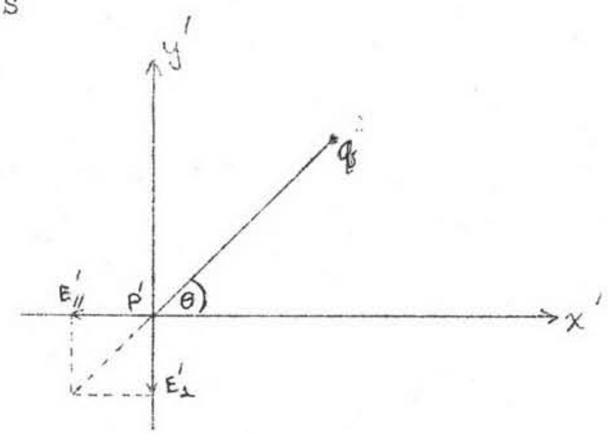

Figure 1.3 The charge q stationary in S' frame.

Therefore the electric field at P' due to q is given by Coulomb's law in S'

$$E'_{//} = - E'_x = \frac{q \cos \theta}{r'^2} = \frac{qx'}{(x'^2 + y'^2)^{3/2}} \qquad (1.3.3)$$

$$E'_{\perp} = - E'_y = \frac{q \sin \theta}{r'^2} = \frac{qy'}{(x'^2 + y'^2)^{3/2}} \qquad (1.3.4)$$

The relation between the coordinates of an event in the two frames is given by the Lorentz transformation

$$x' = \gamma (x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma (t - \frac{v}{c^2} x),$$

$$\text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substitute these values in (1.3.3) and (1.3.4).

The x-componant of the electric field in the frame in which the charge is moving is equal to the electric field in the rest frame of the charge, and the y-component of the electric field in the frame in which the charge is moving is equal to γ times the electric field in the rest frame of the charge⊛ . Therefore, we have

$$E_x = E'_x = \frac{-q\gamma x}{\left[(\gamma x)^2 + y^2\right]^{3/2}} \tag{1.3.5}$$

$$E_y = \gamma E'_y = \frac{-q\gamma y}{\left[(\gamma x)^2 + y^2\right]^{3/2}} \tag{1.3.6}$$

⊛ See from Purcell, Ed ward M. The Electric and Magnetism Berkeley Physics Course Volume 2,chapter 5.