

CHAPTER III



RESULTS AND DISCUSSIONS

Numerical work for Eq.(2.30) is carried out on the IBM 370-138 Computer at Chulalongkorn Computer Center. The radius ratio is varied from $a/b = 0.1$ to 0.8 , while the Poisson's ratio ν is fixed at 0.3 . The rigidity ratios considered in the computations are $k = 0.3, 1.0, 2.0, 4.0,$ and 6.0 . The variation of the pressure ratios is $0.0, 0.5,$ and 1.0 .

The critical buckling load, λ , is plotted against the radius ratio, a/b , and the curves are presented in Fig. 3.1 to Fig. 3.60. These curves exclude the buckling loads at 0.8 radius ratio and above, because the annular plate of those radius ratios behaves as a ring rather than a plate. The assumption that $b-a$ is much greater than its thickness, h , is no longer true. Therefore, the concepts of thin plates can not be used, and errors are surely created. The effect of the number of waves along the circumference of a plate on the buckling load will be discussed first, then the effect of the rigidity ratio on the buckling load. The last variable included in this discussion is the load effect or the pressure ratio.

The very first notice made on the number of waves along the circumference of the plate, n , is that the critical buckling load

can occur at any number of waves along the circumference of the plate. Mostly, the number of waves is not zero. That is the critical buckling loads are rarely restricted to the axisymmetric buckling. Therefore, the axisymmetric buckling load can not be used in the design with confidence. As the radius ratio getting bigger, the critical buckling load occurs at more number of waves along the circumference of the plate. In other words, the plate with a certain number of waves along its circumference can bear the critical buckling load within the proper range of the radius ratio of the plate. Beneath or above that range, the minimum load borne by the plate with the same number of waves along its circumference is not the critical buckling load any more. The plate with less number of waves and less radius ratio will buckle at lower load. It is clear that the buckling load parameter increases with the radius ratio which results in more number of waves in the circumferential direction. In case 1 and case 2 where the outer edges are clamped, the critical buckling loads occur at higher number of waves along the circumference of the plate than in case 3 and case 4 where the outer edges are simply supported, considering Fig. 3.1 and Fig. 3.31 for examples.

The influence of rigidity ratio is the next interest. At each pressure ratio, the rigidity ratio is varied from 0.3 up to 6.0 by the energy criterion that the rigidity ratio is not less than ν_c , where ν_c is here taken as 0.3. The increase in the rigidity ratio shows an obvious effect of increasing the critical buckling

load. Since the rigidity ratio is the squared root of the ratio of the tangential rigidity to the radial rigidity, therefore increasing the tangential rigidity will raise the critical buckling load. The variation of the rigidity ratio also brings out another interest which is the slopes of the curves. Taking Fig. 3.1 to Fig. 3.5 for examples, the curve of 0.3 rigidity ratio is flatter than the ones of higher rigidity ratio. The slope keeps on rising as the rigidity ratio increases. This is because the rigidity ratio implies how much the tangential rigidity dominates the radial rigidity. Enlarging the hole causes the plate more rigid in tangential direction. By the same increment of the radius ratio, the plate with higher rigidity ratio can bear larger increment of load than the one with lower rigidity ratio.

For the influence of the pressure ratio, it shows very slight effect on the critical buckling load. By increasing the pressure ratio from 0 to 0.5 and 1.0, the critical buckling load rises in small increments. One thing that can not be neglected is the slopes of the curves. The increase in the pressure ratio steepens the curves as shown in Fig. 3.1, Fig. 3.6, and Fig. 3.11.

There is no way to tell how accurate the results are. What can be done here is to compare to solutions reached by other methods as the Rayleigh-Ritz solution by Vijayakumar and Rao[14], and the same Galerkin's method for isotropic plates by Wiwat[17]. Since Vijayakumar and Rao's results assumed an axisymmetric buckling, where $n=0$, thus the results at $n=0$ found in this study are given

in the figures as well as Vijayakumar and Rao's for comparing at 0.5 radius ratio of plates of 0 and 1.0 pressure ratios. In case 1 and case 2, at zero pressure ratio the calculated critical buckling loads are less than that from Vijayakumar and Rao's, but opposite conclusion is found for a unit pressure ratio. For the rest two cases, the calculated critical buckling loads nearly coincide with that from Vijayakumar and Rao's. By comparing the calculated results of a unit rigidity ratio to the isotropic case by Wiwat, which are also given in Fig. 3.2 and Fig. 3.17, they agree very well to each other, except where there are load jumps. The coincidence between the radius ratio and the Poisson's ratio causes load jumps in the case that the inner edge is simply supported. There is no problem of load jumps in Wiwat's work, because his deflection functions for this case do not get along with all boundary conditions.

This research yields the results in agreement with some available prediction by others. Therefore, they can be used in checking the design of such a structural element with some confidence. However, one should be aware that the selected deflection function in the form of truncated polynomials will constrain the annular plates, thus the theoretical critical load is expected to be higher than the actual one. Experimental verification of the accuracy of the results contained herein is needed for future work.

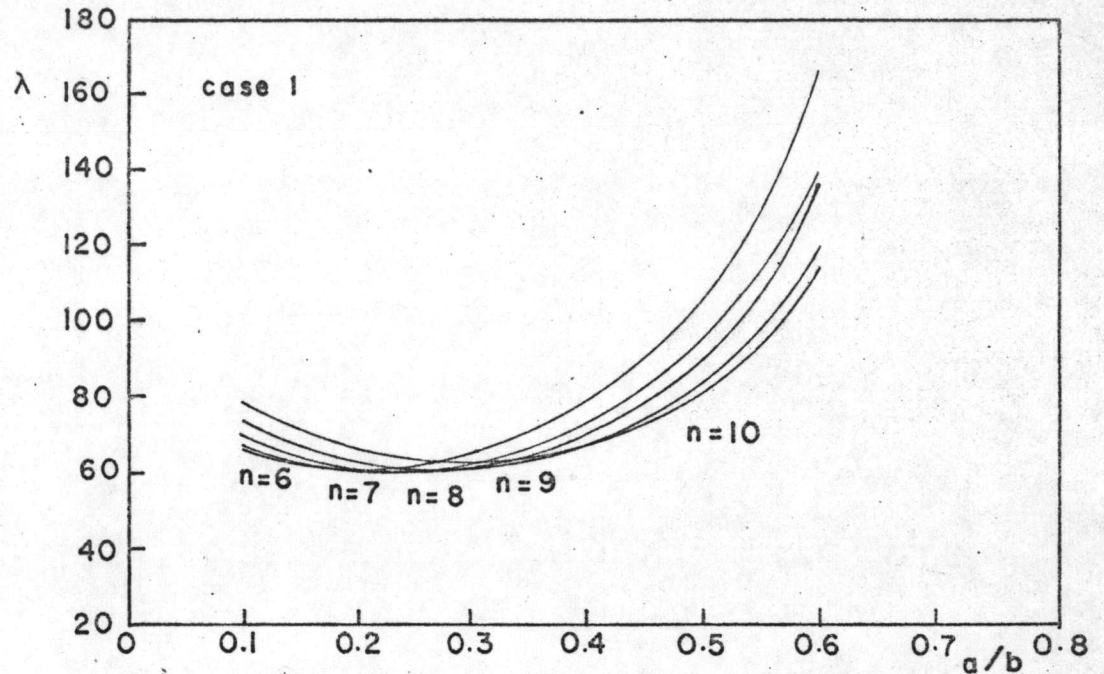


Fig. 3.1 Buckling of polar orthotropic annular plates
for $P_i/P_o = 0.0$ and $k = 0.3$

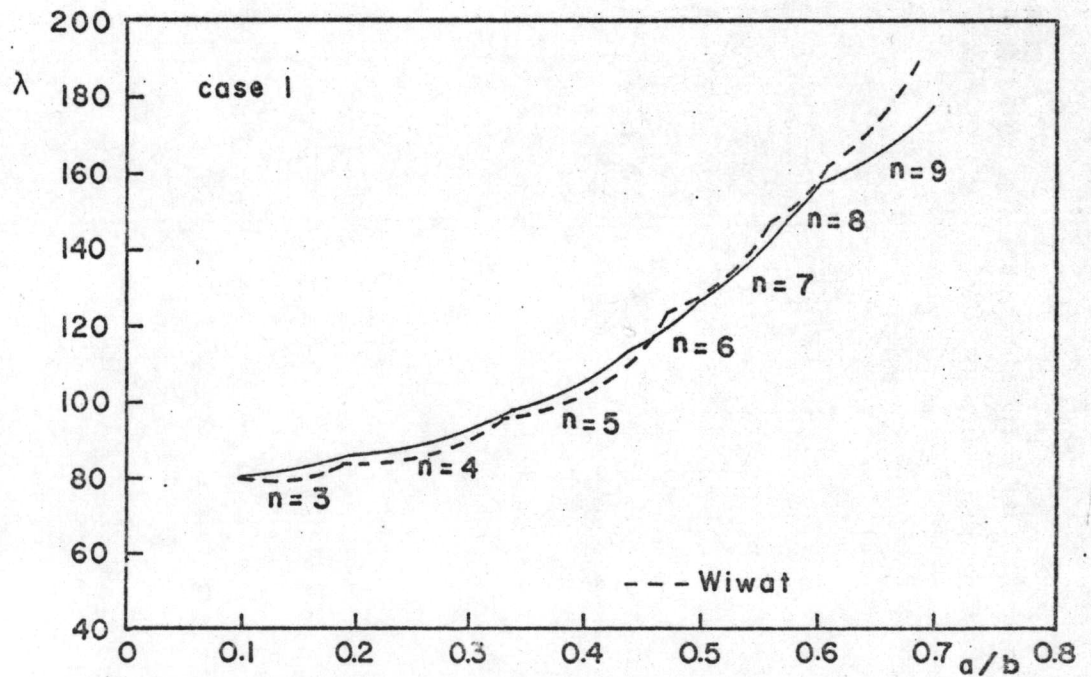


Fig. 3.2 Buckling of polar orthotropic annular plates
for $P_i/P_o = 0.0$ and $k = 1.0$

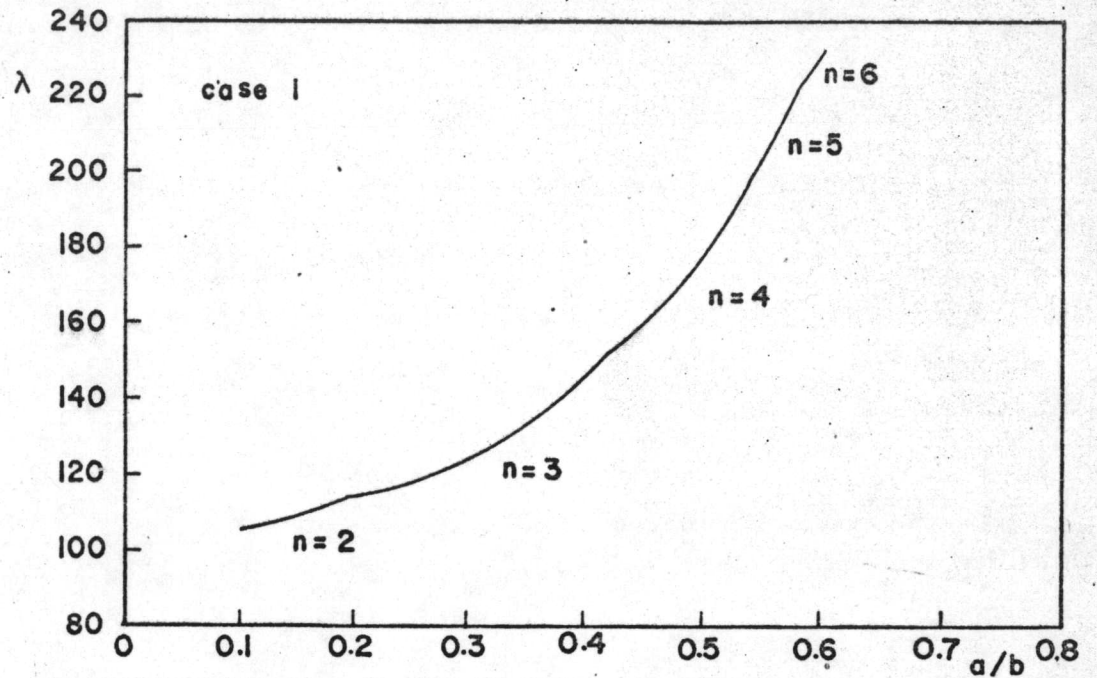


Fig. 3.3 Buckling of polar orthotropic annular plates
for $P_i/P_o = 0.0$ and $k = 2.0$

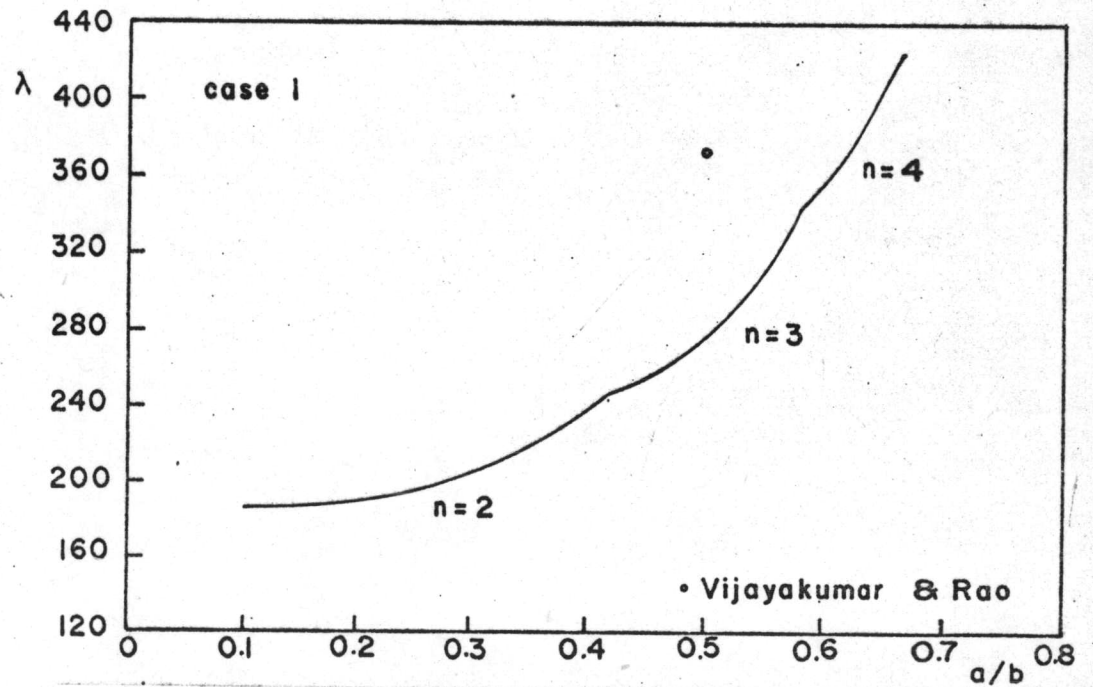


Fig. 3.4 Buckling of polar orthotropic annular plates
for $P_i/P_o = 0.0$ and $k = 4.0$

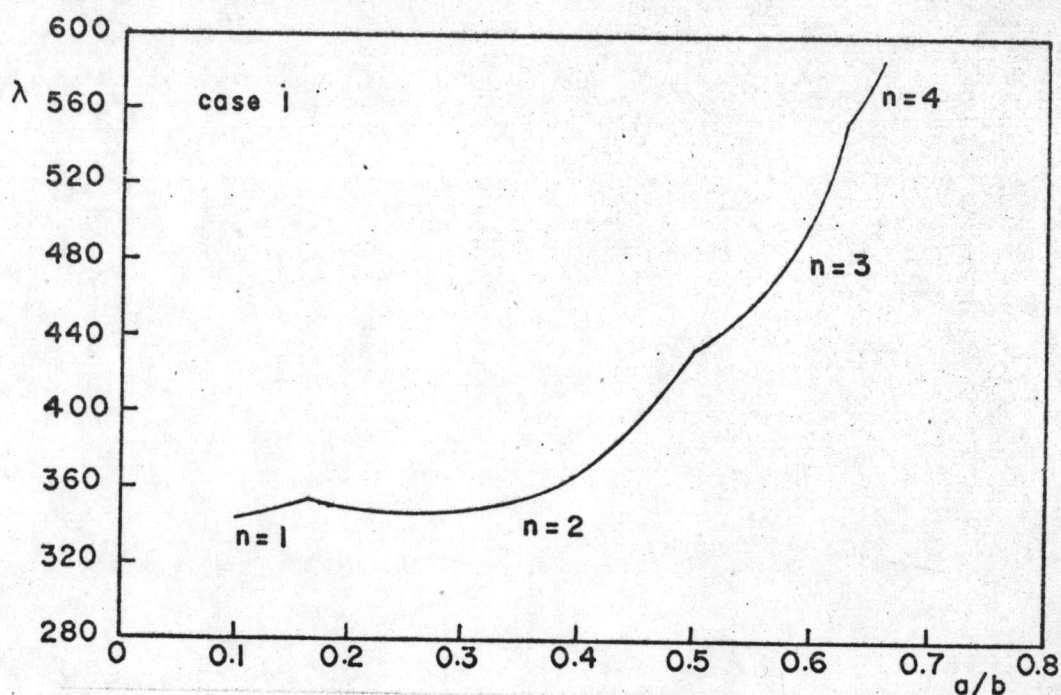


Fig. 3.5 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.0$ and $k = 6.0$

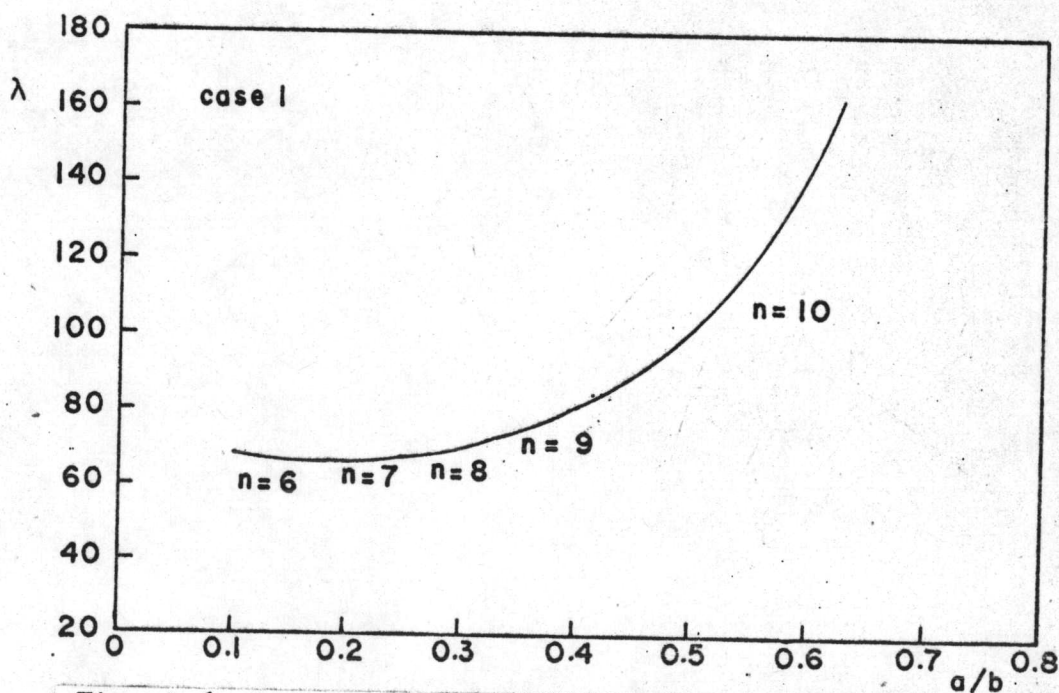


Fig. 3.6 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 0.3$

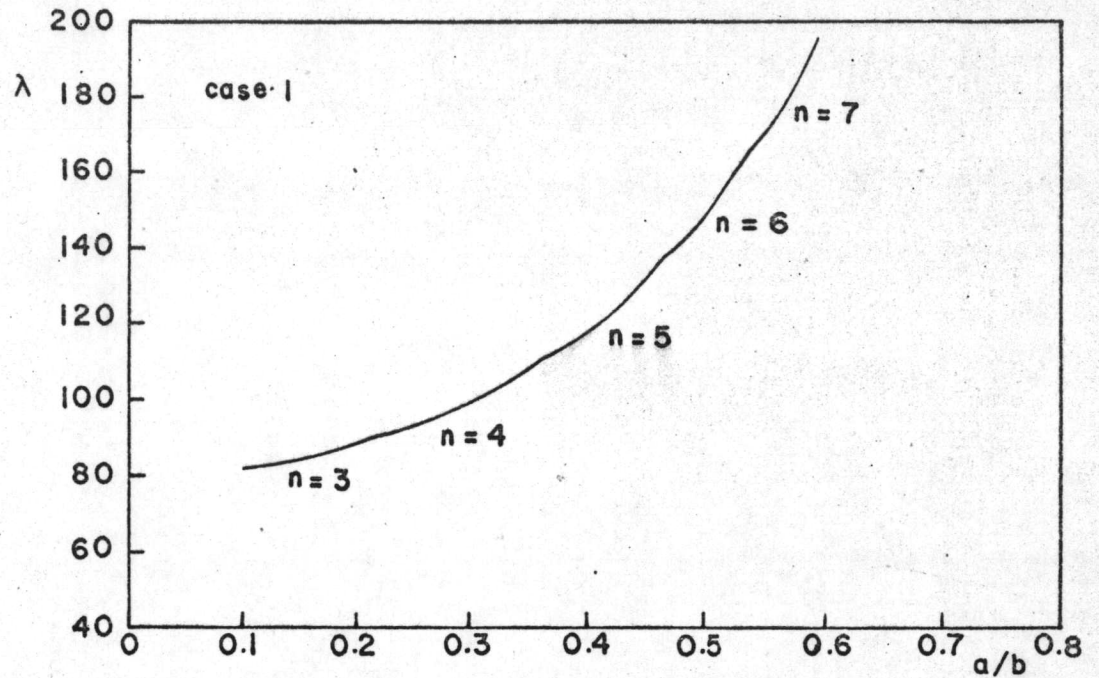


Fig. 3.7 Buckling of polar orthotropic annular plates
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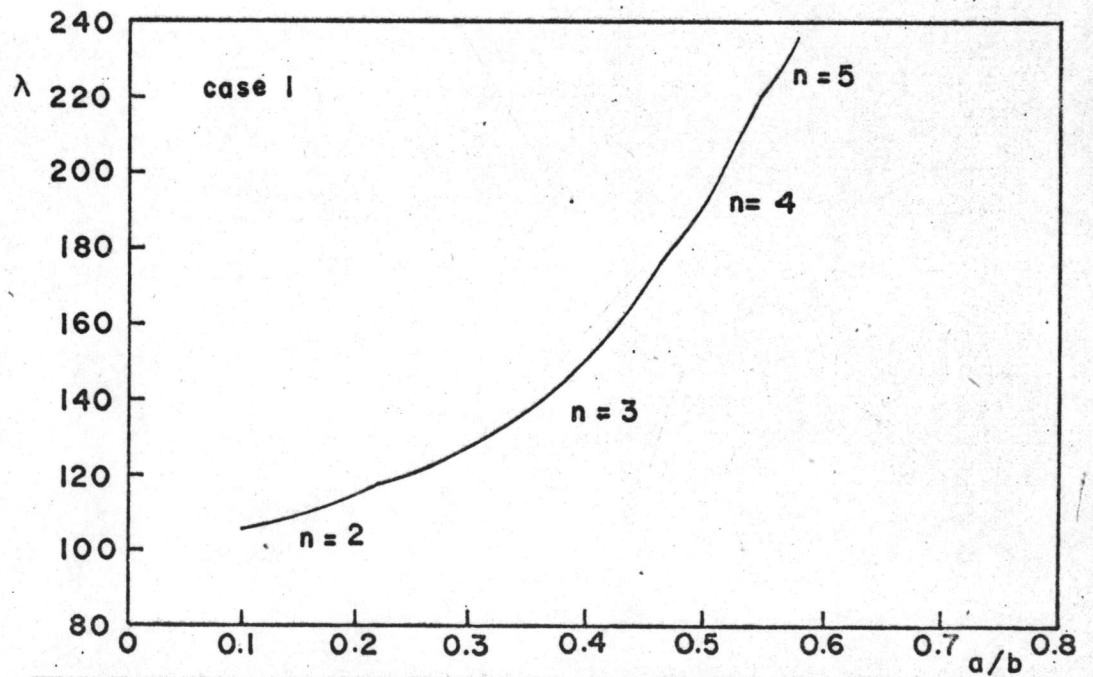


Fig. 3.8 Buckling of polar orthotropic annular plates
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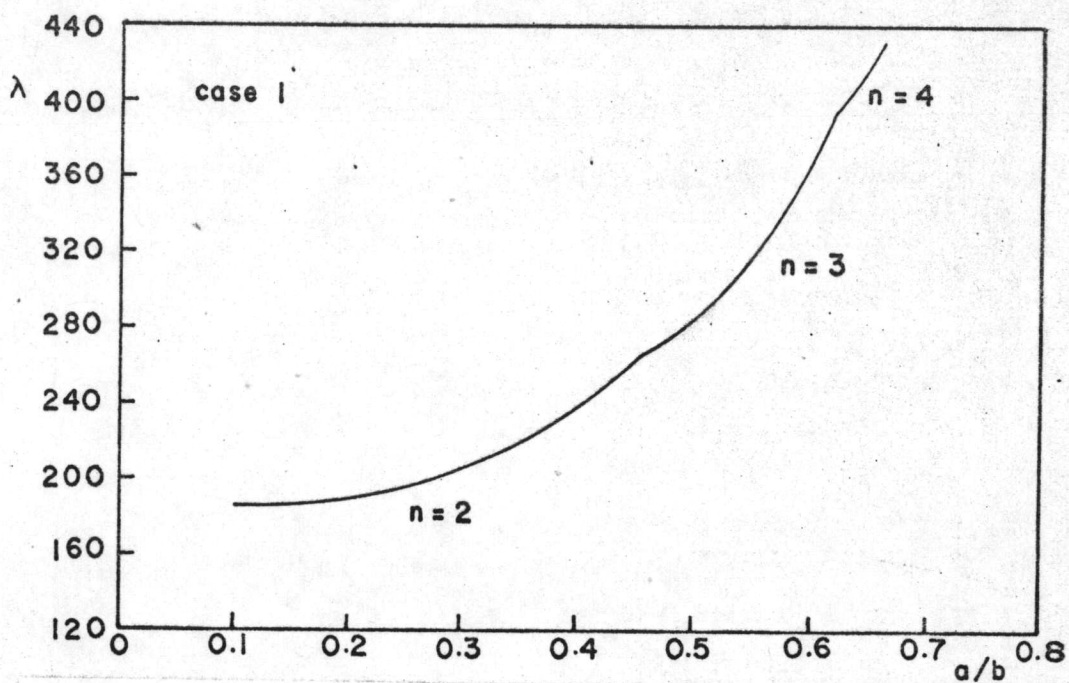


Fig. 3.9 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 4.0$

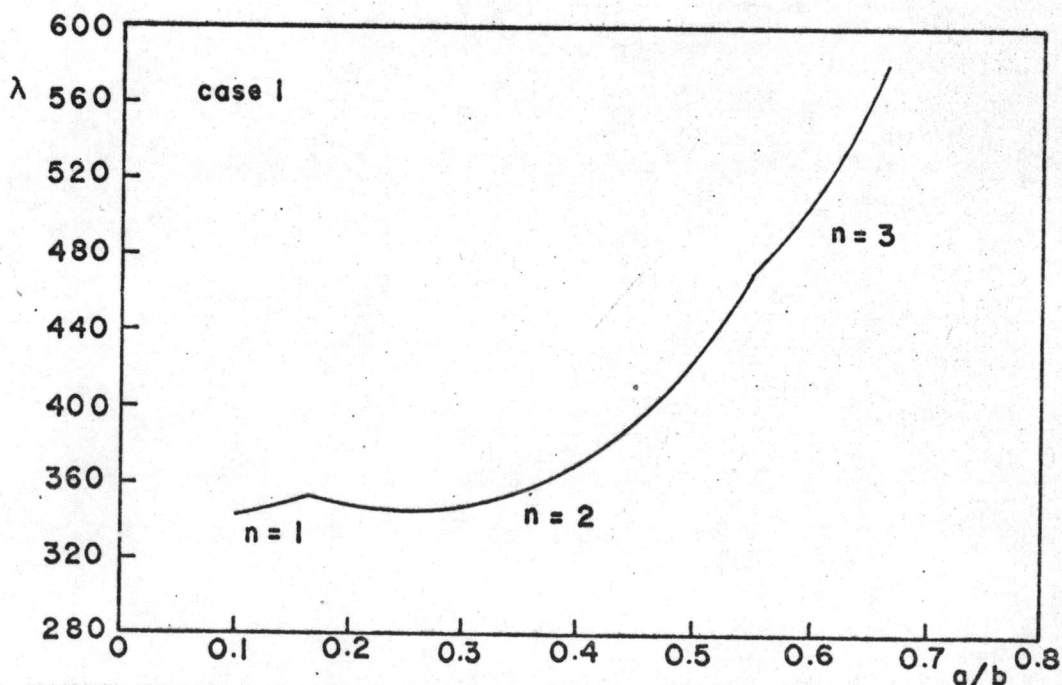


Fig. 3.10 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 6.0$

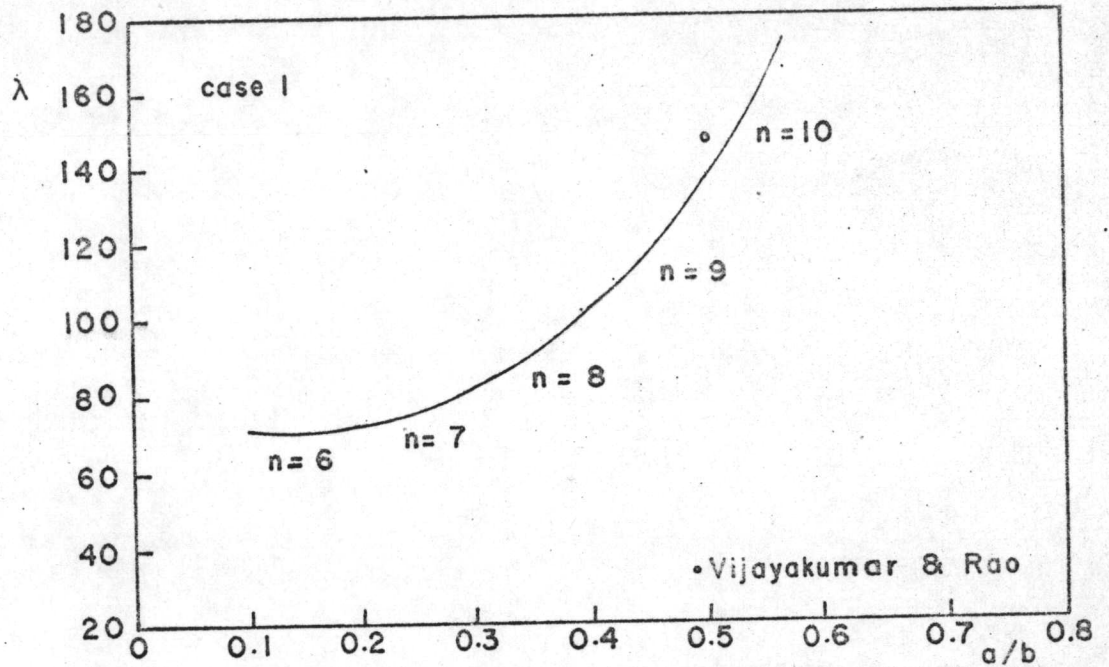


Fig. 3.11 Buckling of polar orthotropic annular plates
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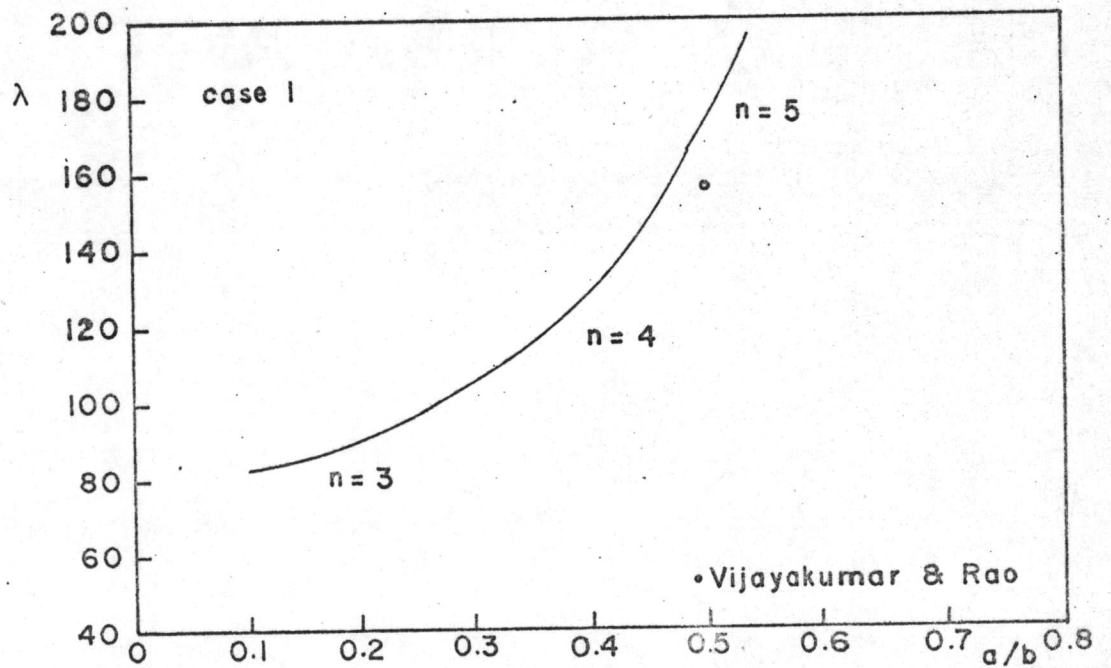


Fig. 3.12 Buckling of polar orthotropic annular plates
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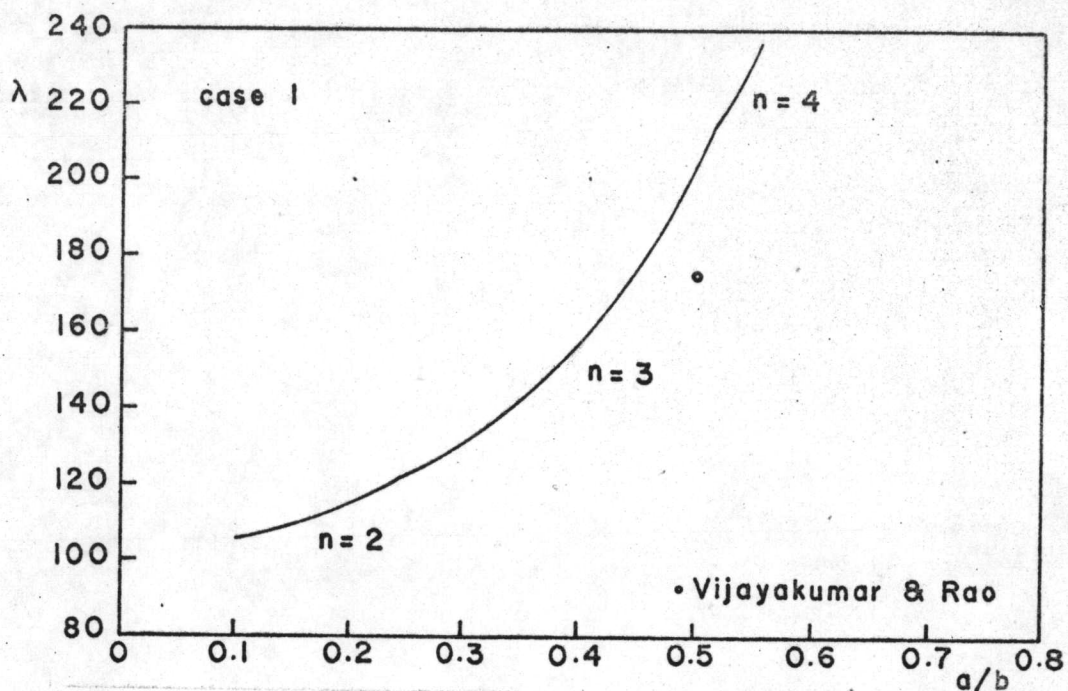


Fig. 3.13 Buckling of polar orthotropic annular plates for $P_i / P_o = 1.0$ and $k = 2.0$

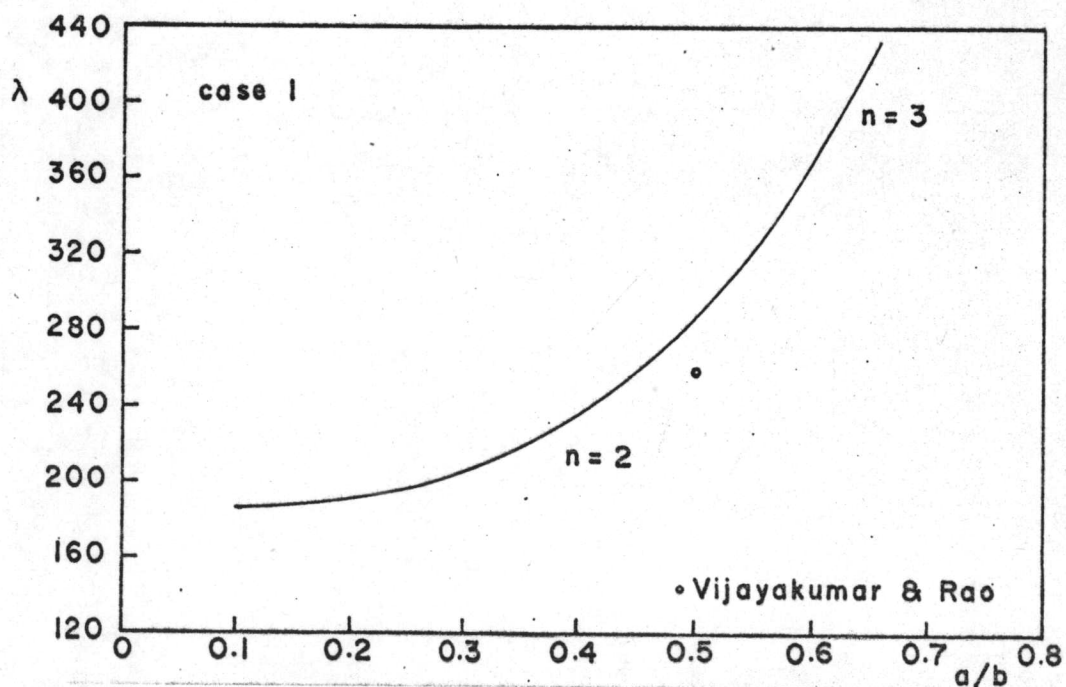


Fig. 3.14 Buckling of polar orthotropic annular plates for $P_i / P_o = 1.0$ and $k = 4.0$

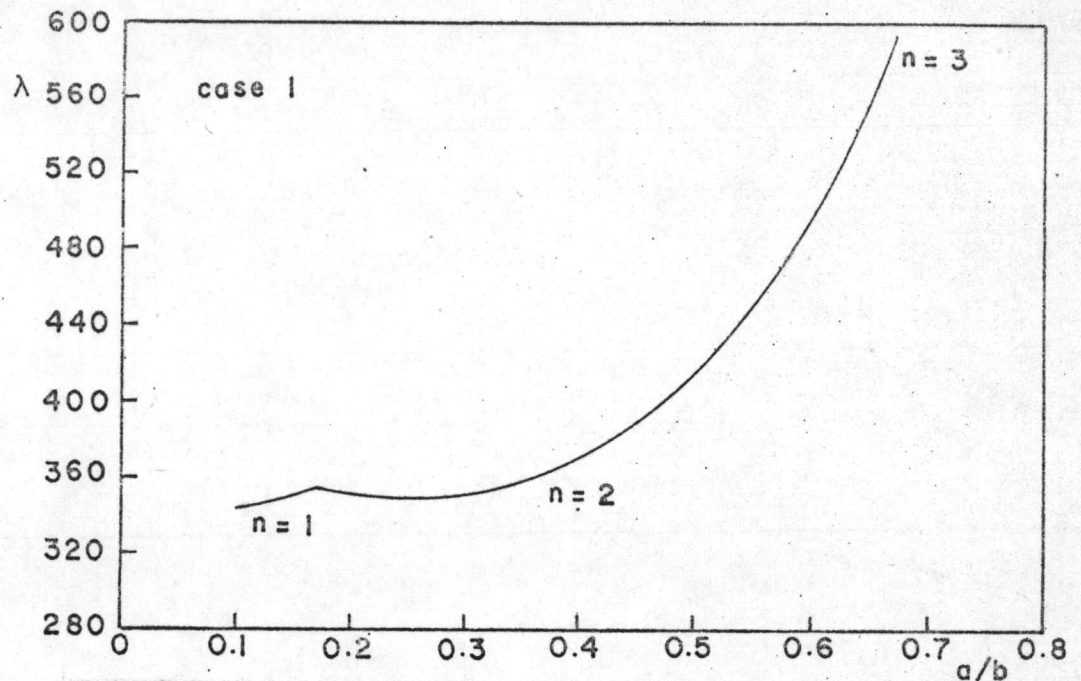


Fig. 3.15 Buckling of polar orthotropic annular plates
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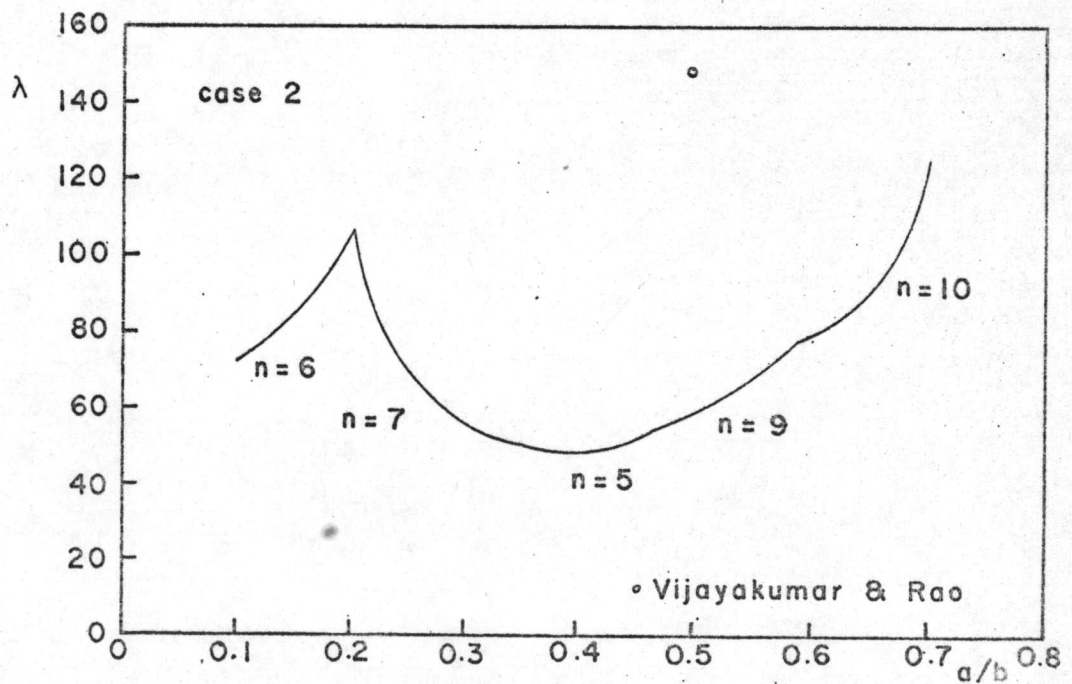


Fig. 3.16 Buckling of polar orthotropic annular plates
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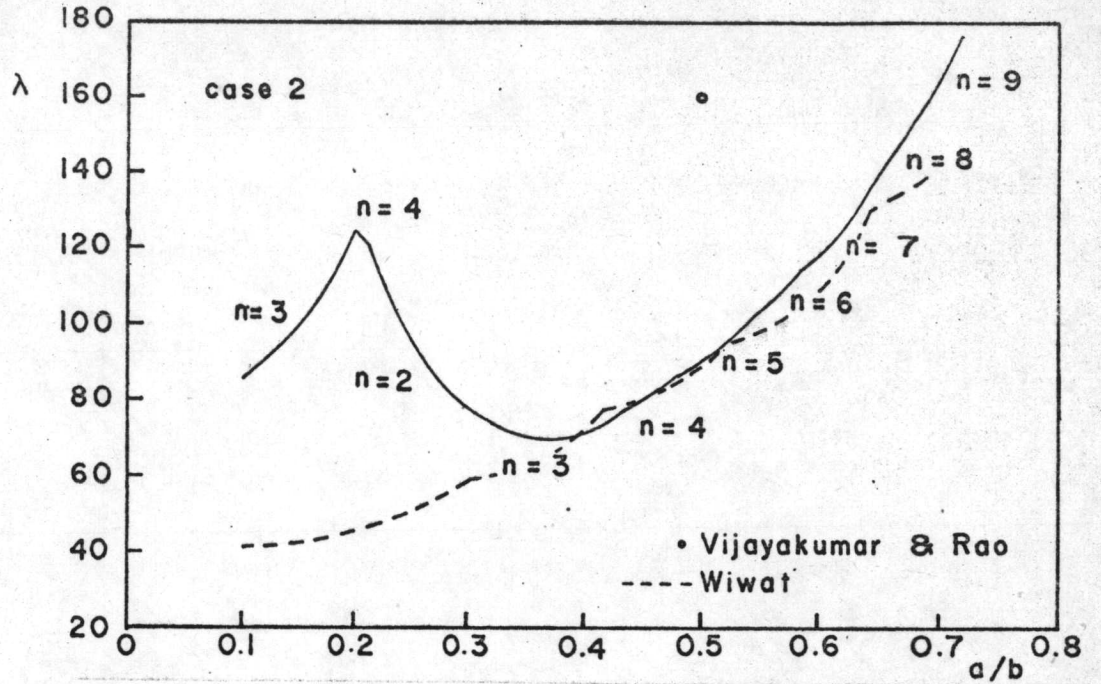


Fig. 3.17 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.0$ and $k = 1.0$

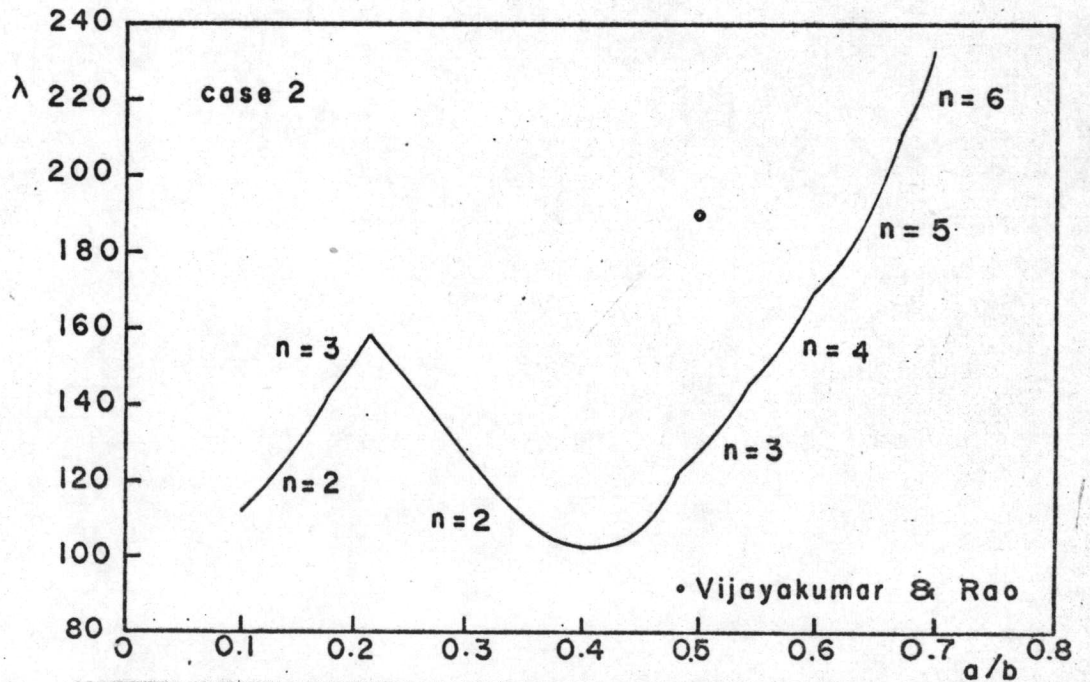


Fig. 3.18 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.0$ and $k = 2.0$

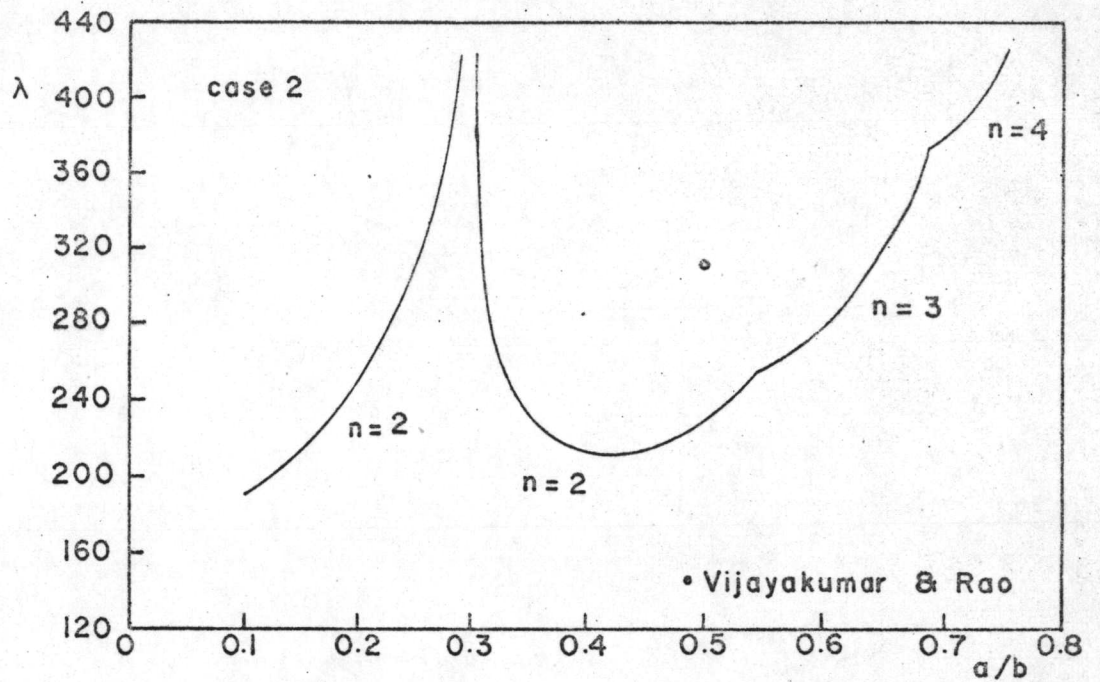


Fig. 3.19 Buckling of polar orthotropic annular plates
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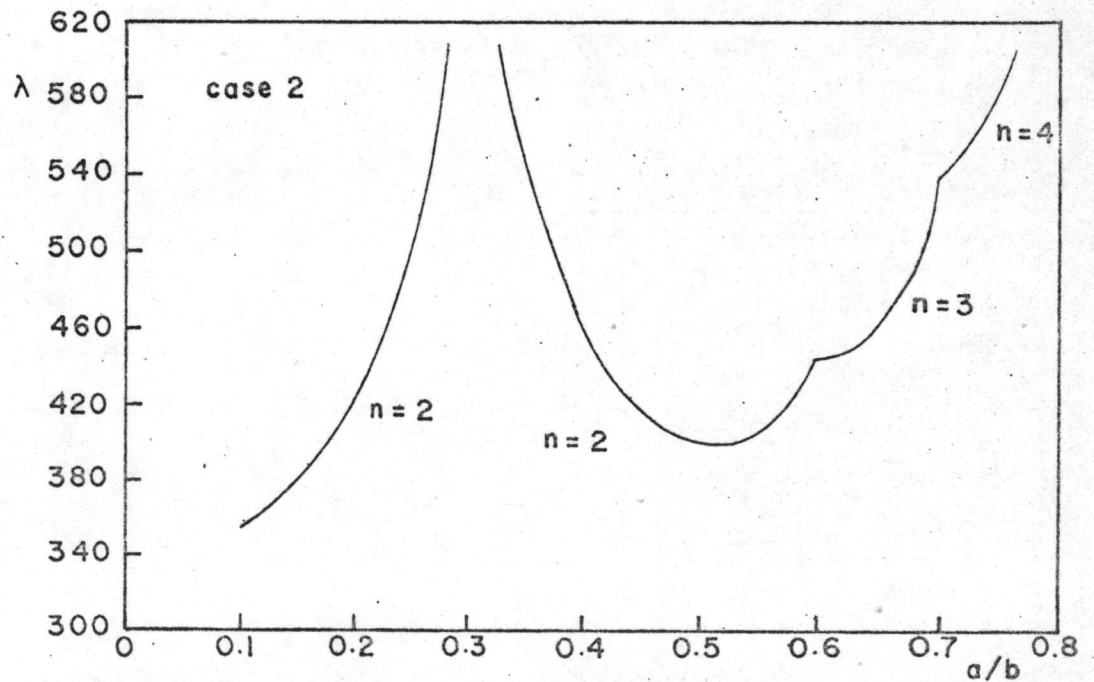


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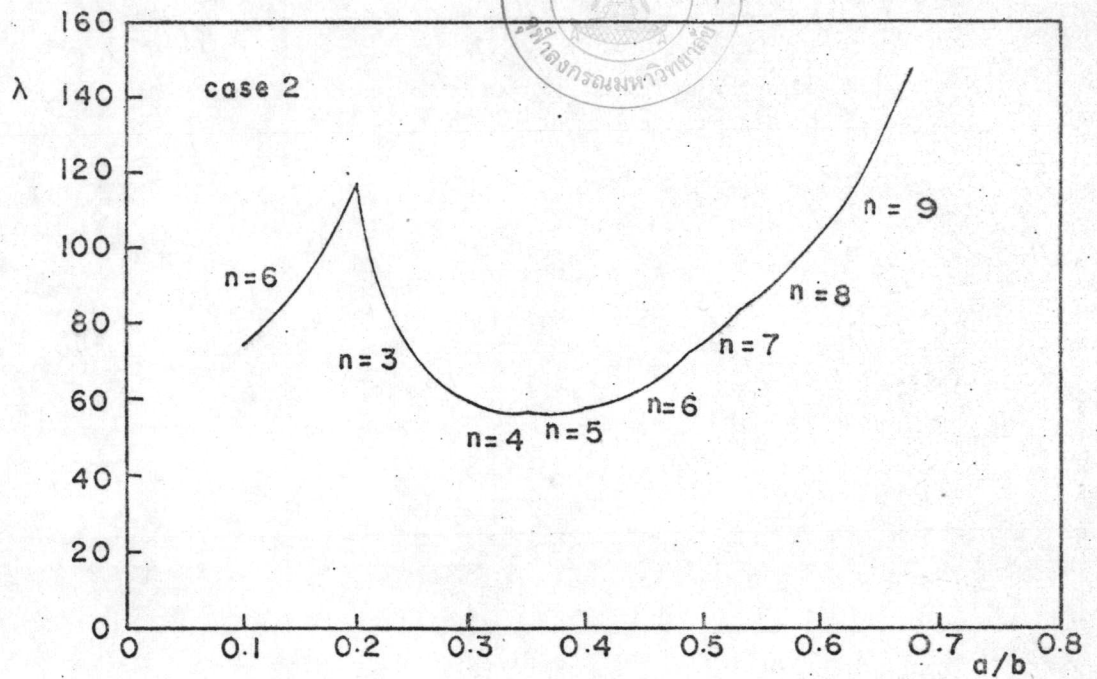


Fig. 3.21 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 0.3$

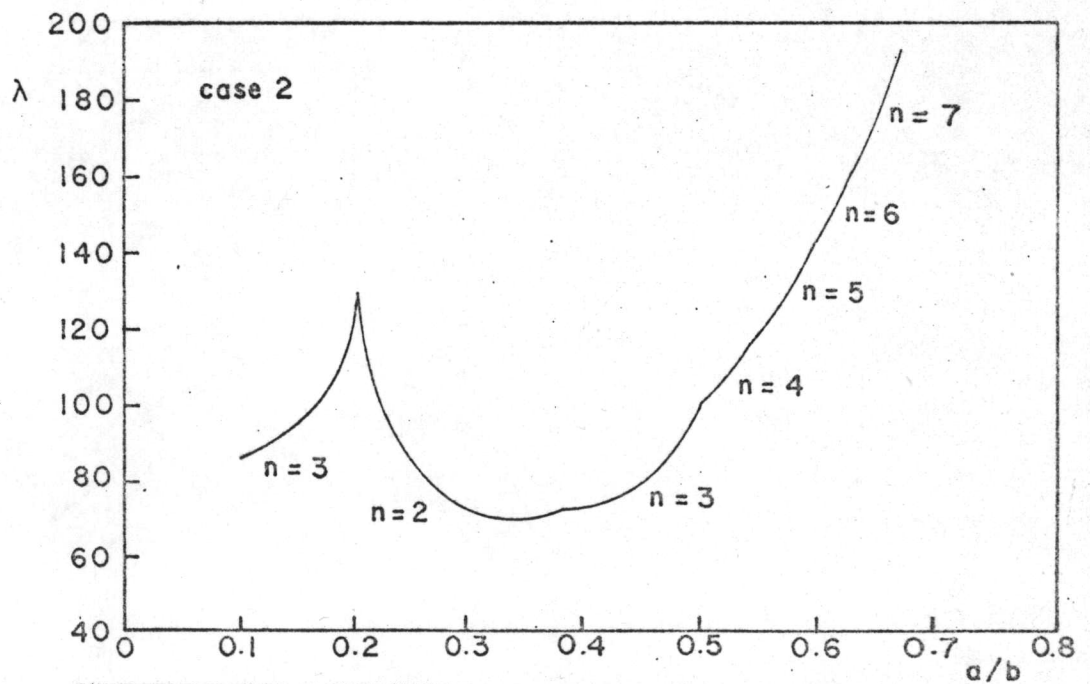


Fig. 3.22' Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 1.0$

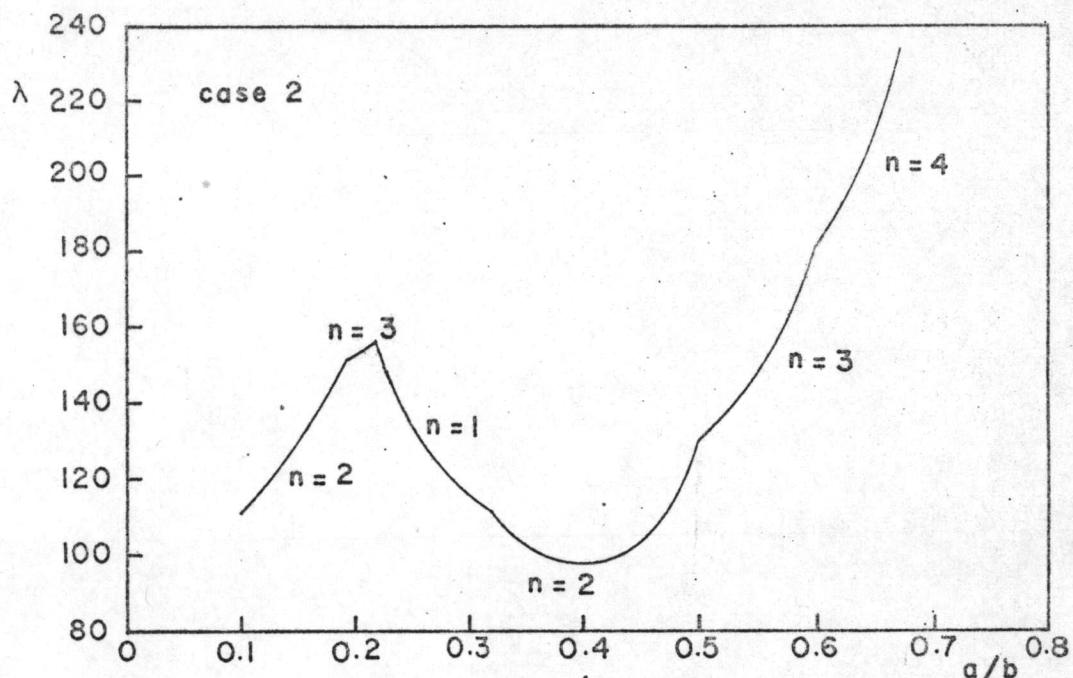


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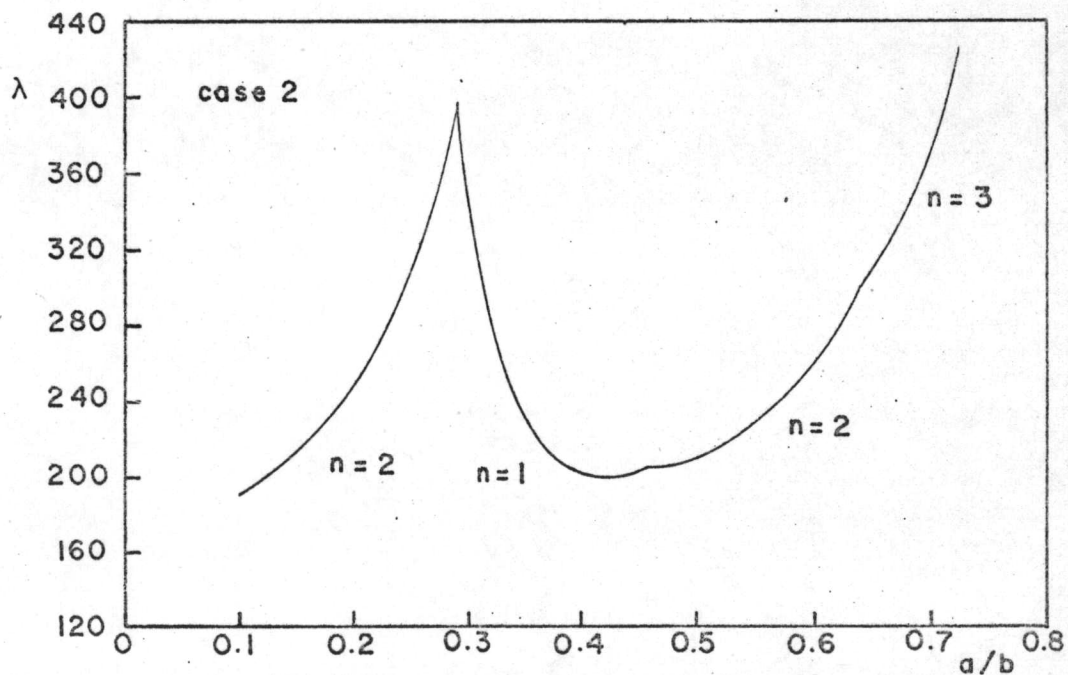


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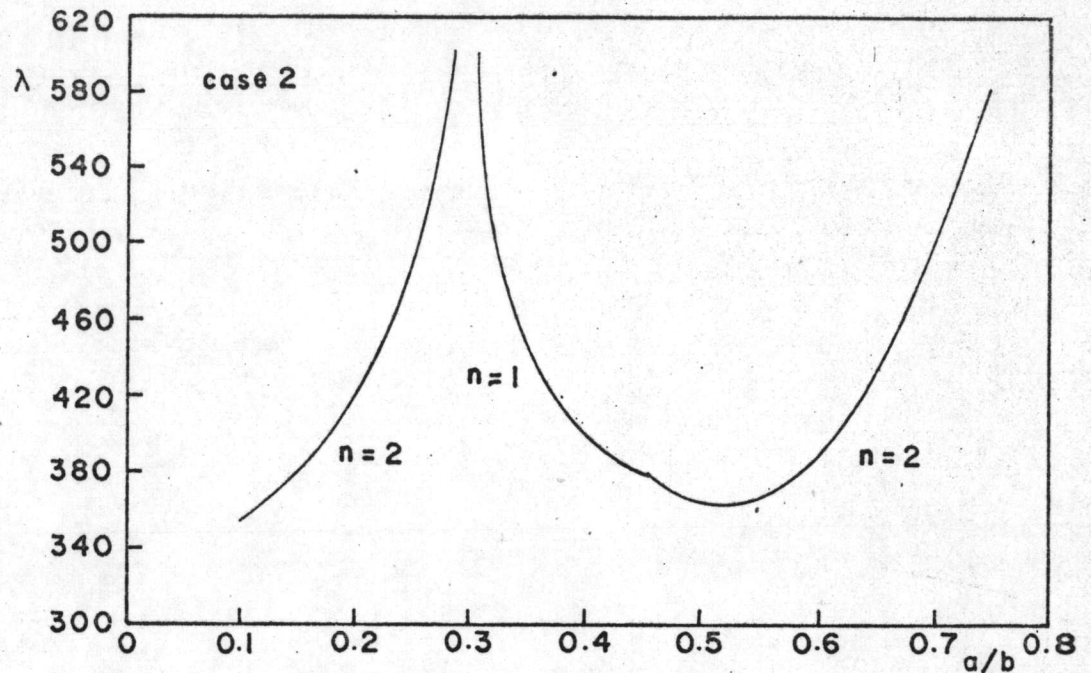


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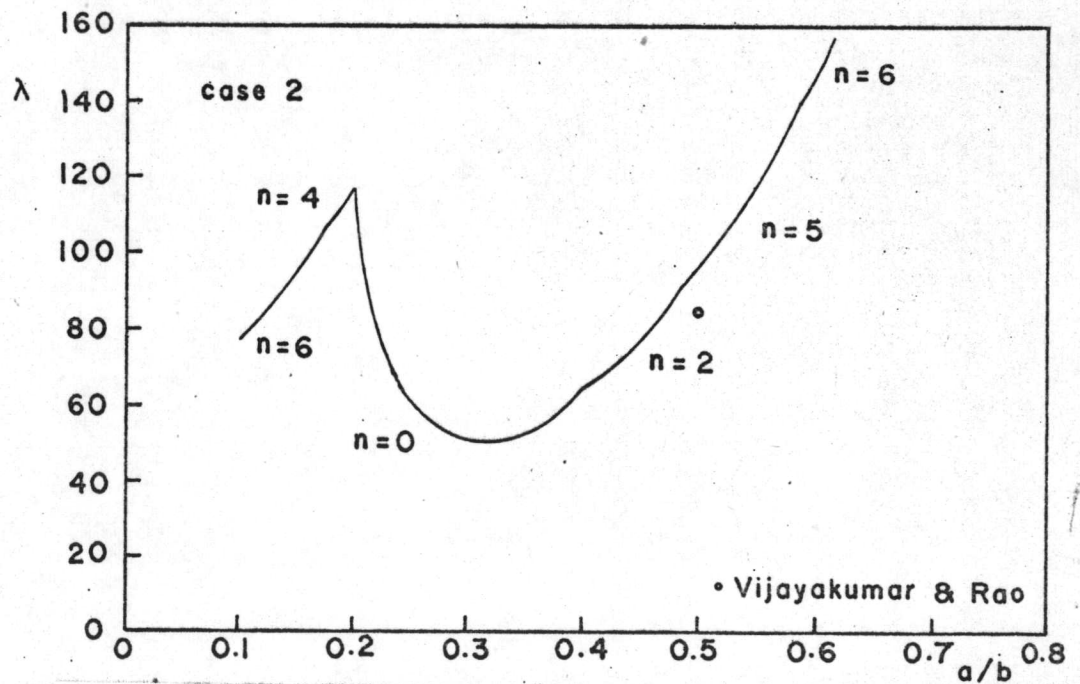


Fig. 3.26 Buckling of polar orthotropic annular plates
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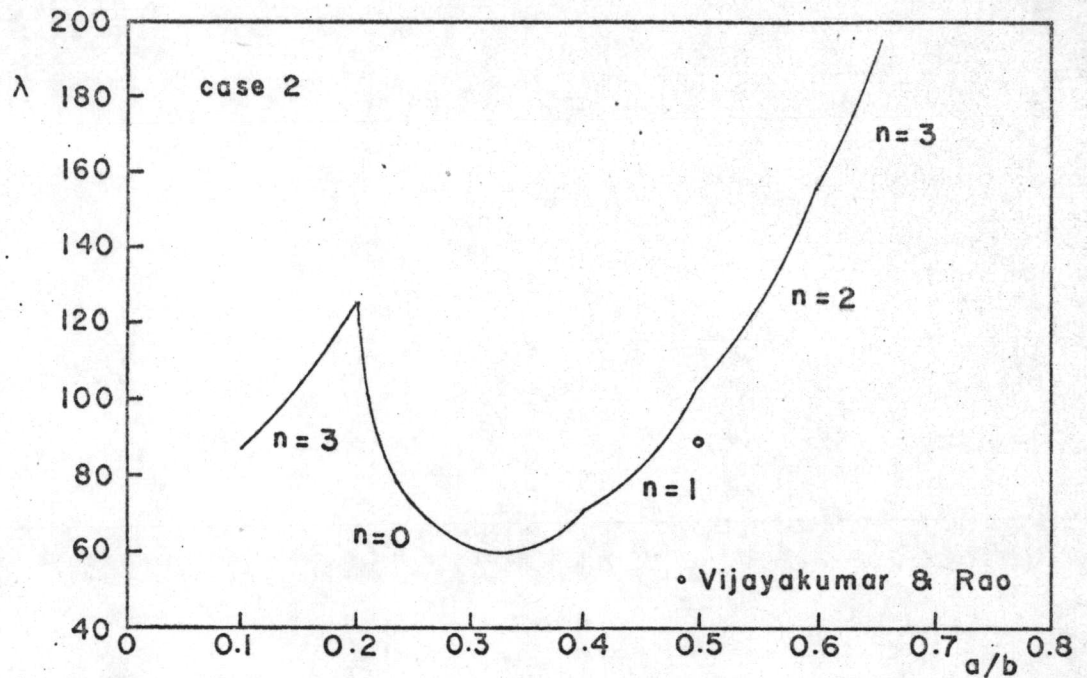


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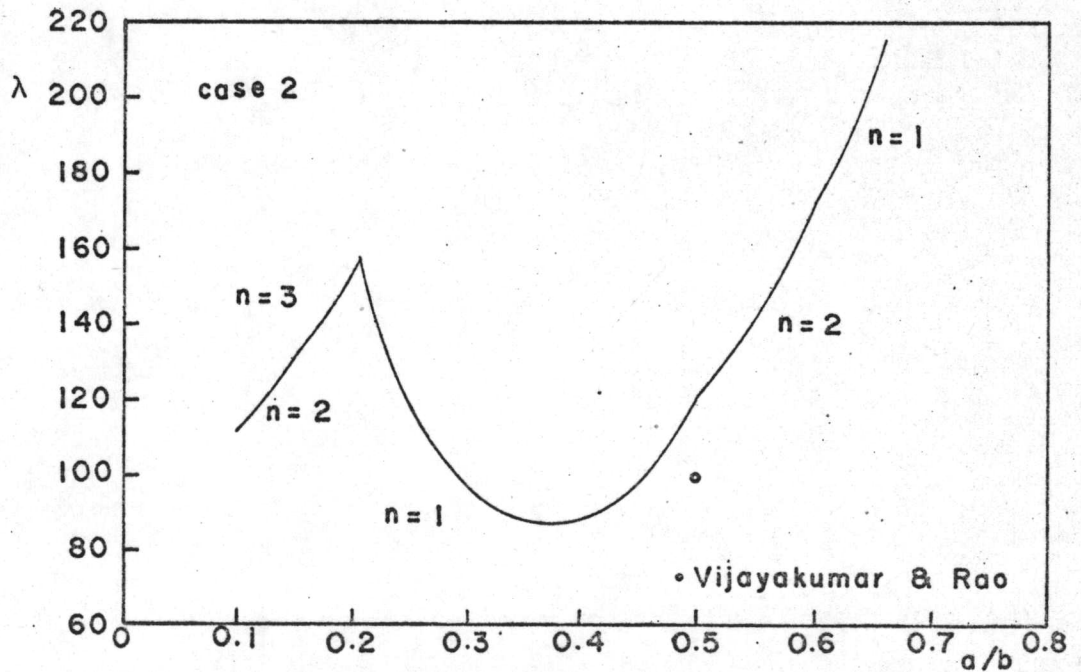


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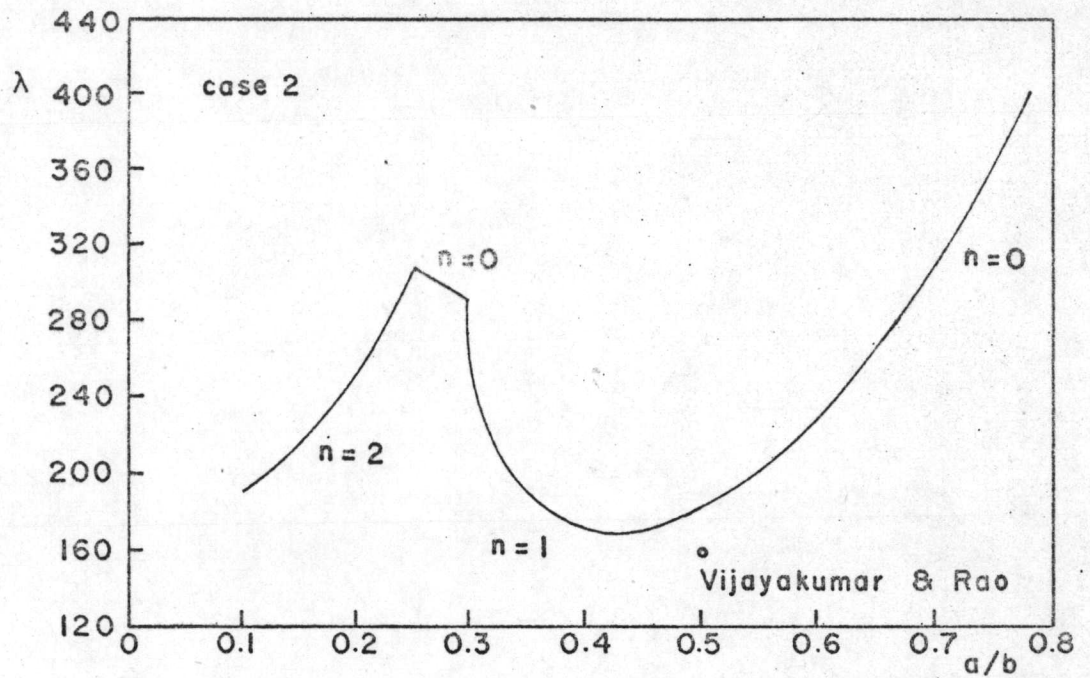


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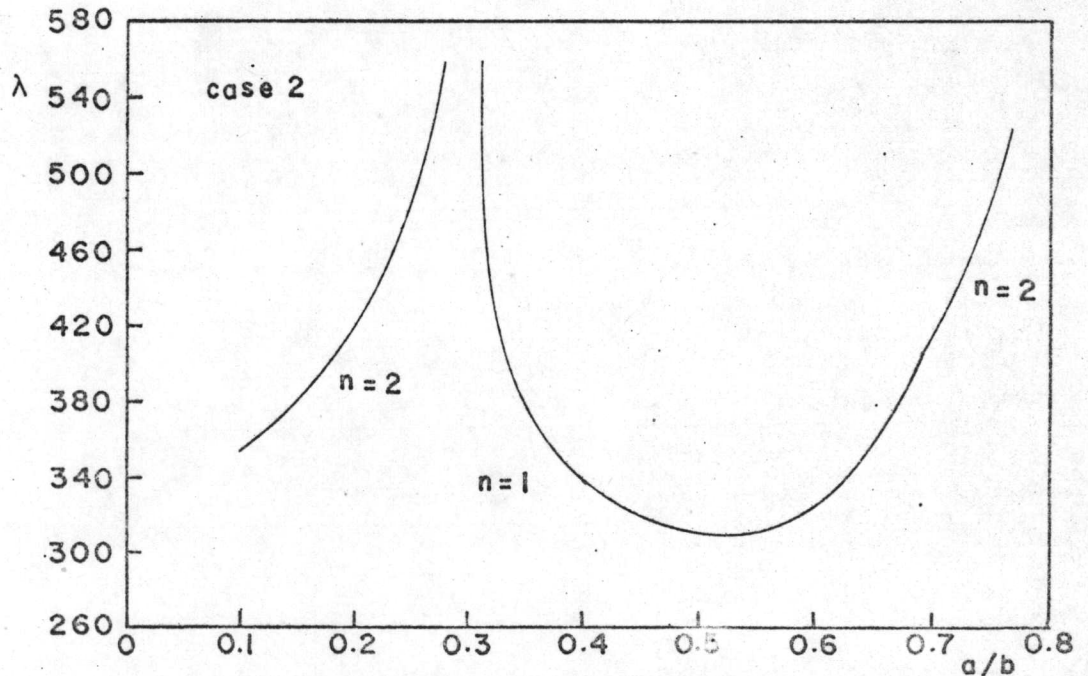


Fig. 3.30 Buckling of polar orthotropic annular plates
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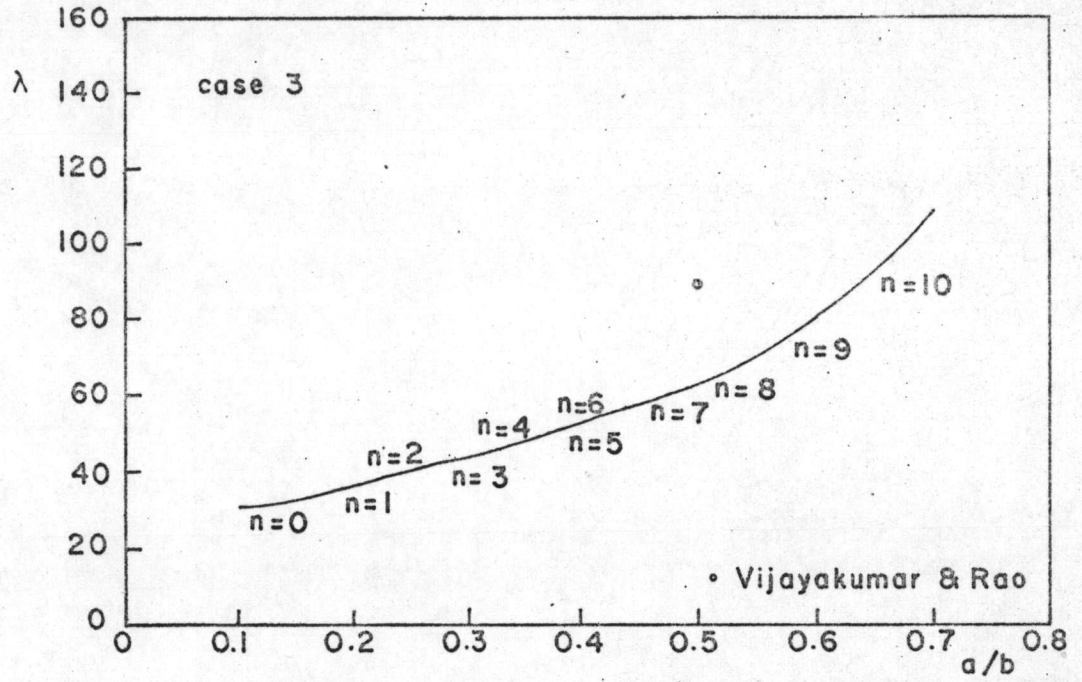


Fig. 3.31 Buckling of polar orthotropic annular plates for $P_i / P_o = 0.0$ and $k = 0.3$

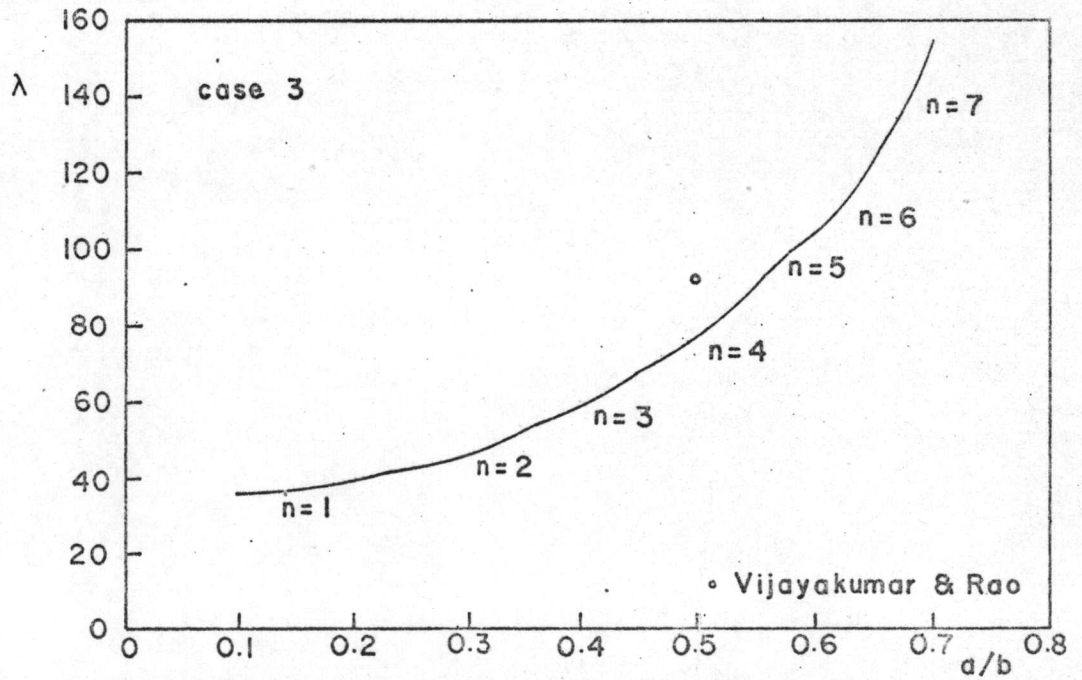


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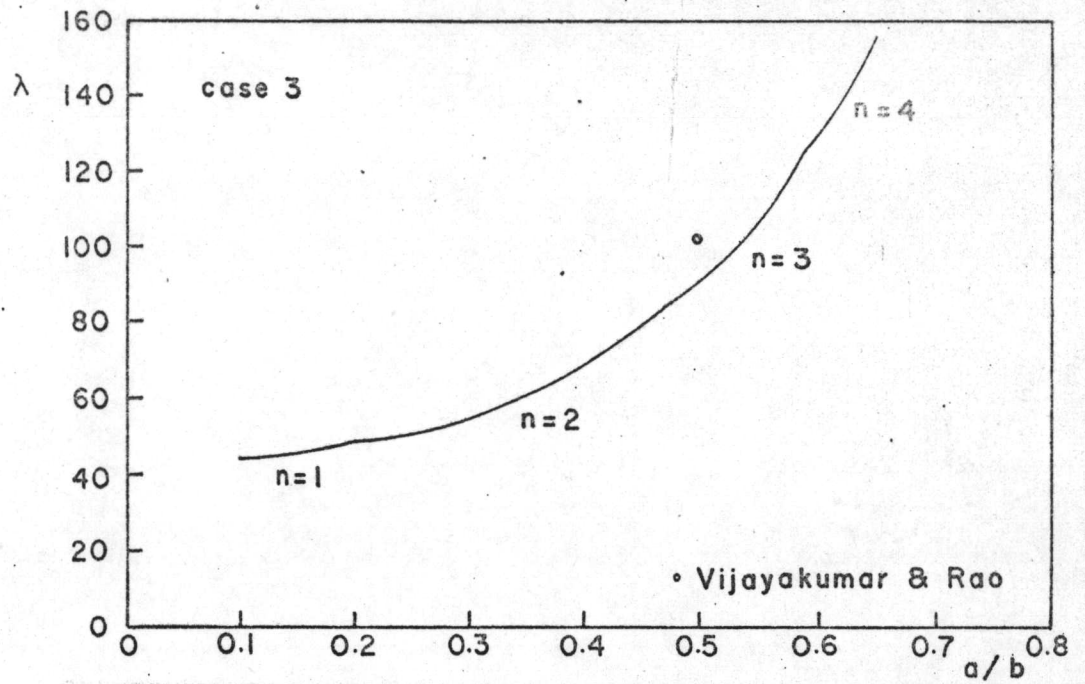


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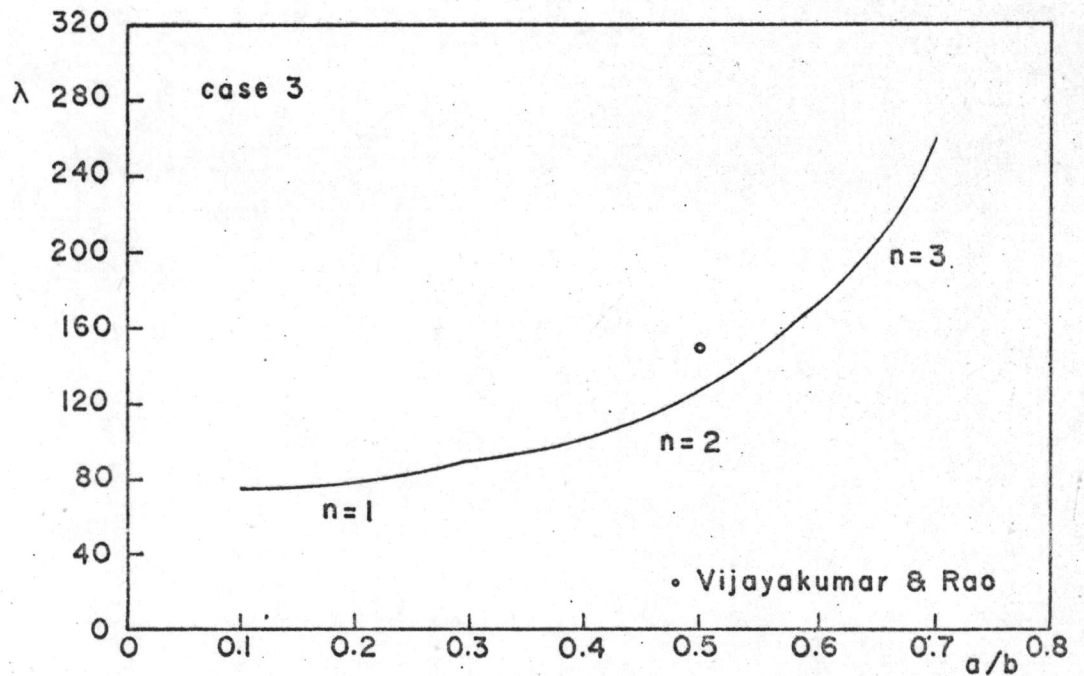


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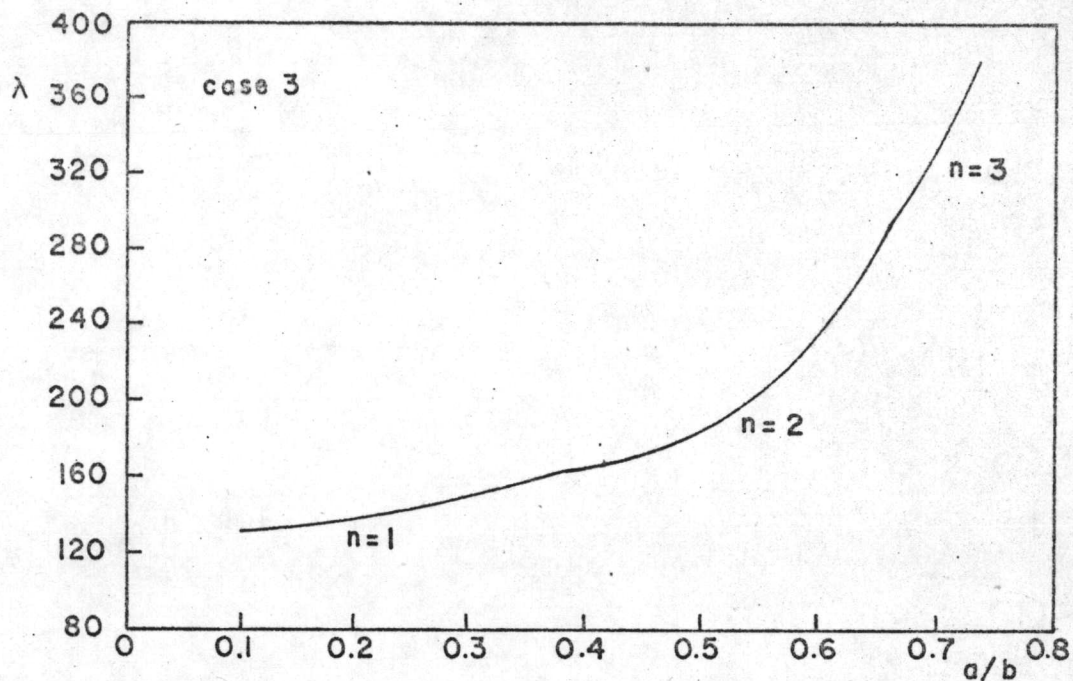


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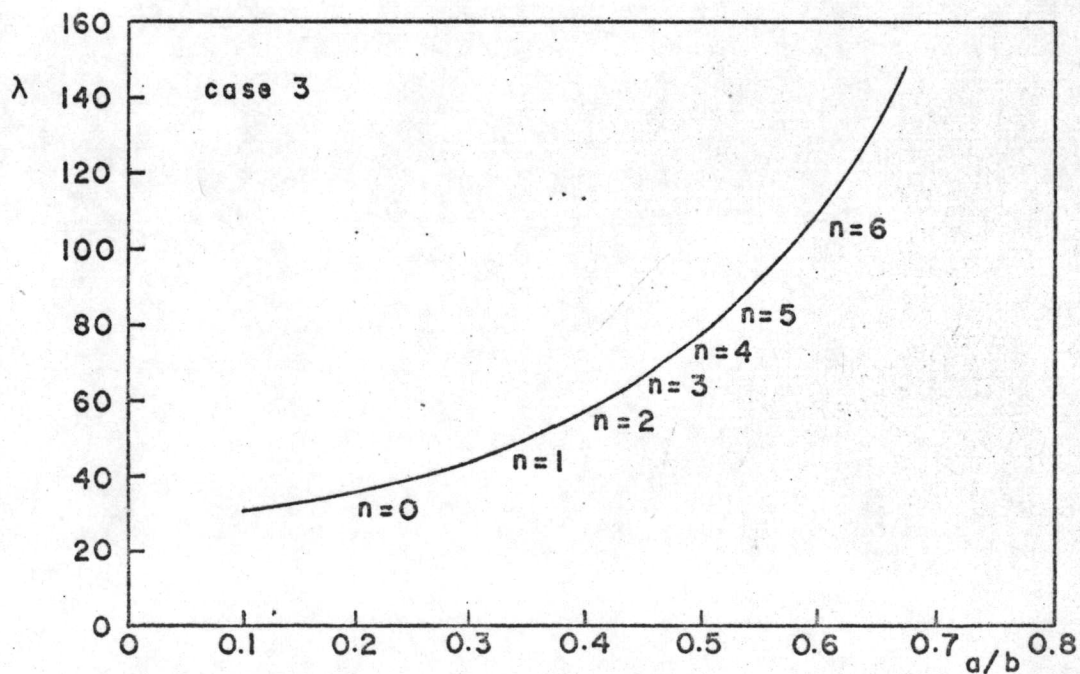


Fig. 3.36 Buckling of polar orthotropic annular plates
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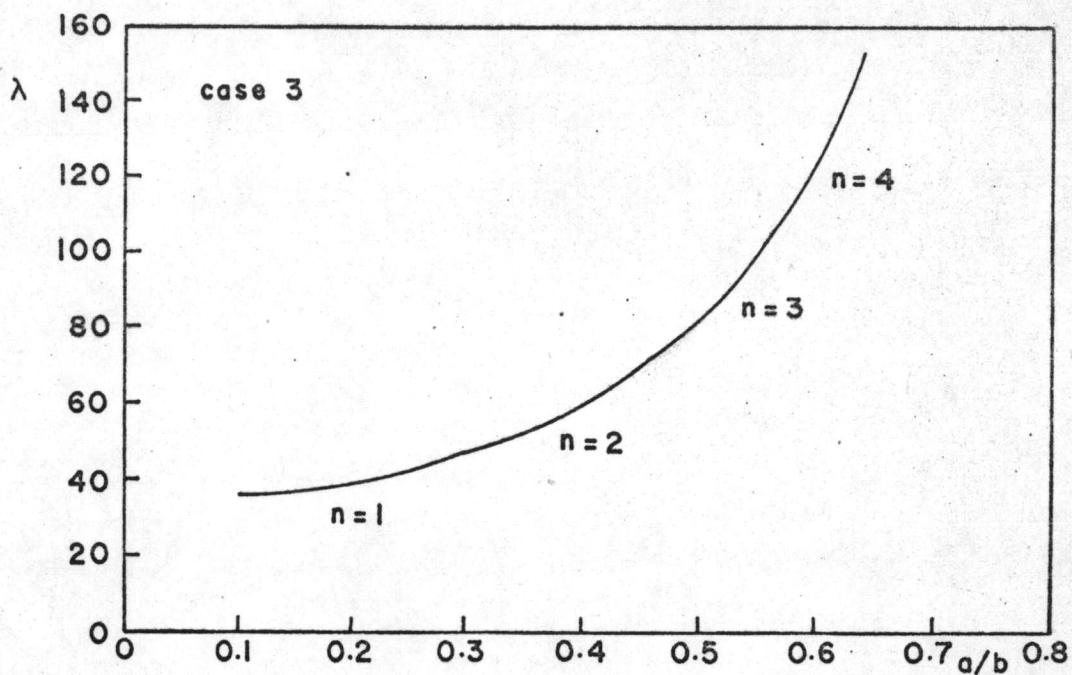


Fig. 3.37 Buckling of polar orthotropic annular plates for $P_i / P_o = 0.5$ and $k = 1.0$

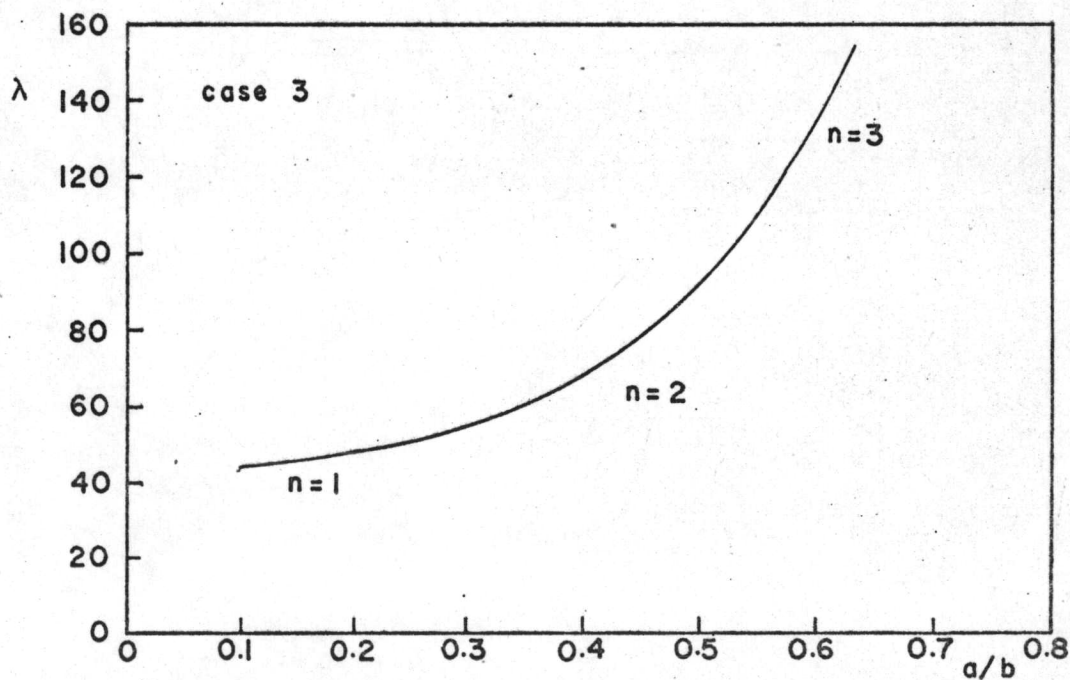


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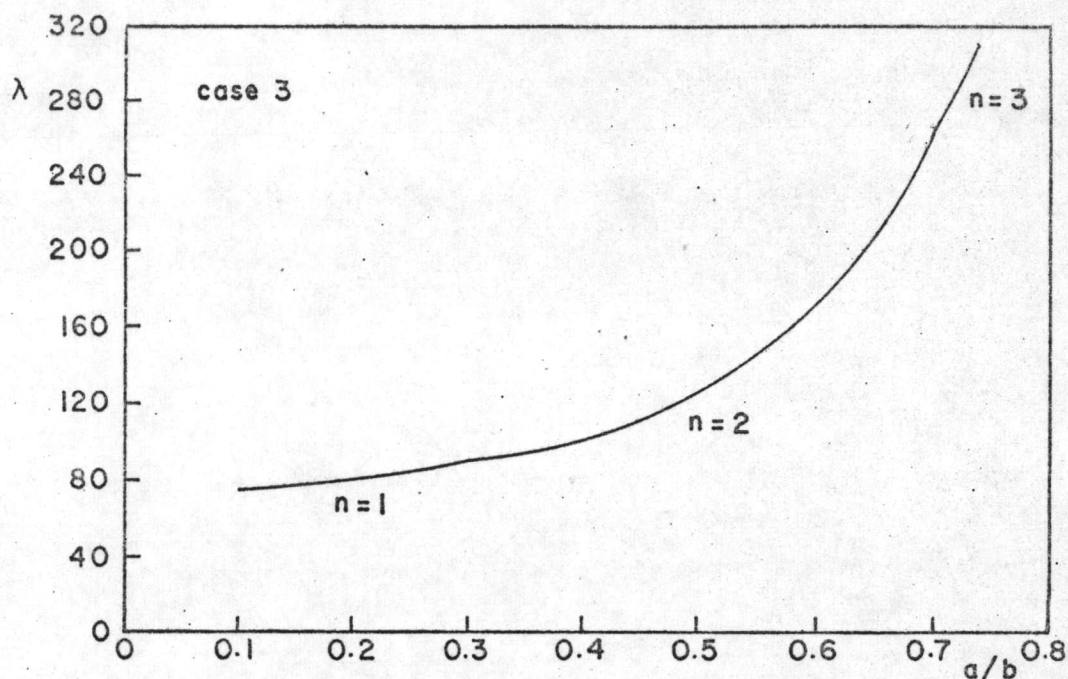


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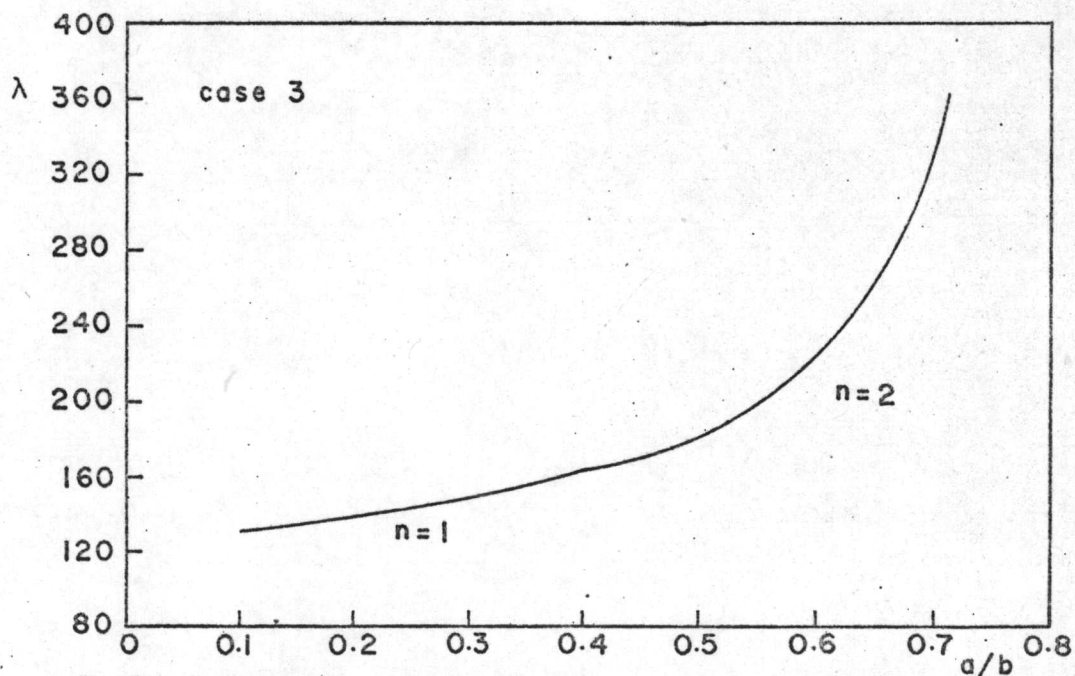


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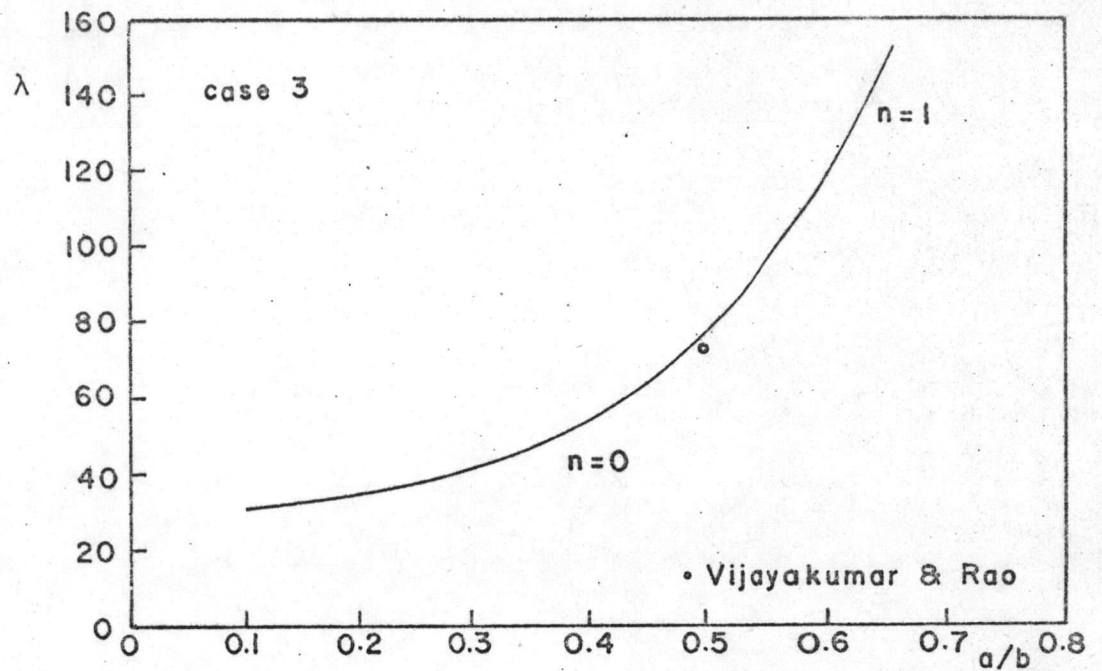


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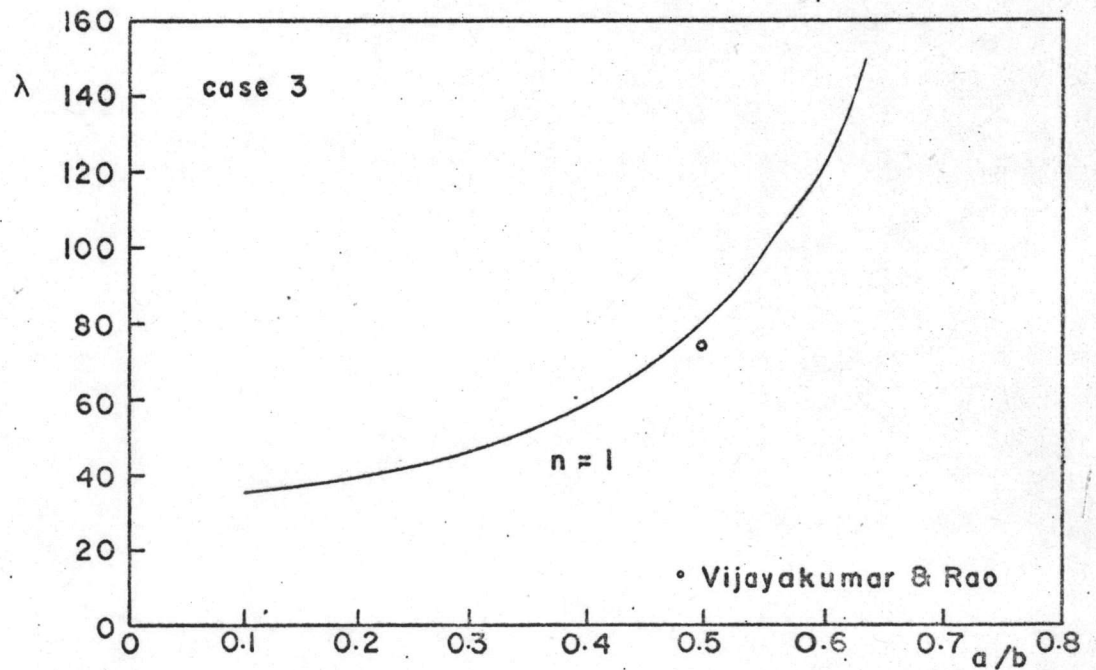


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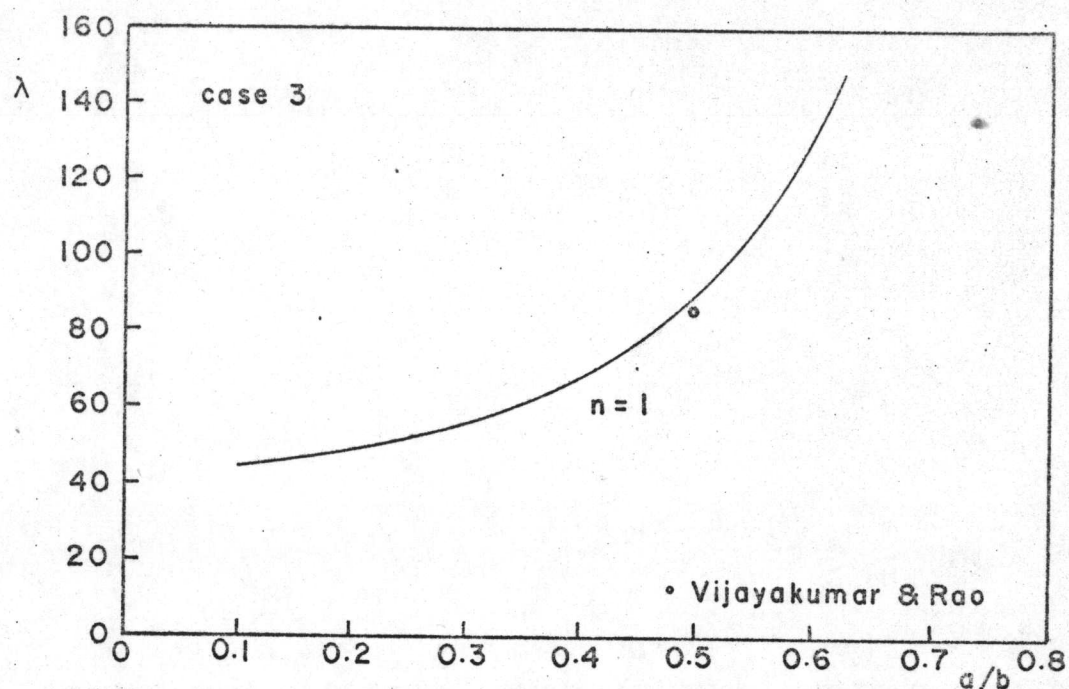


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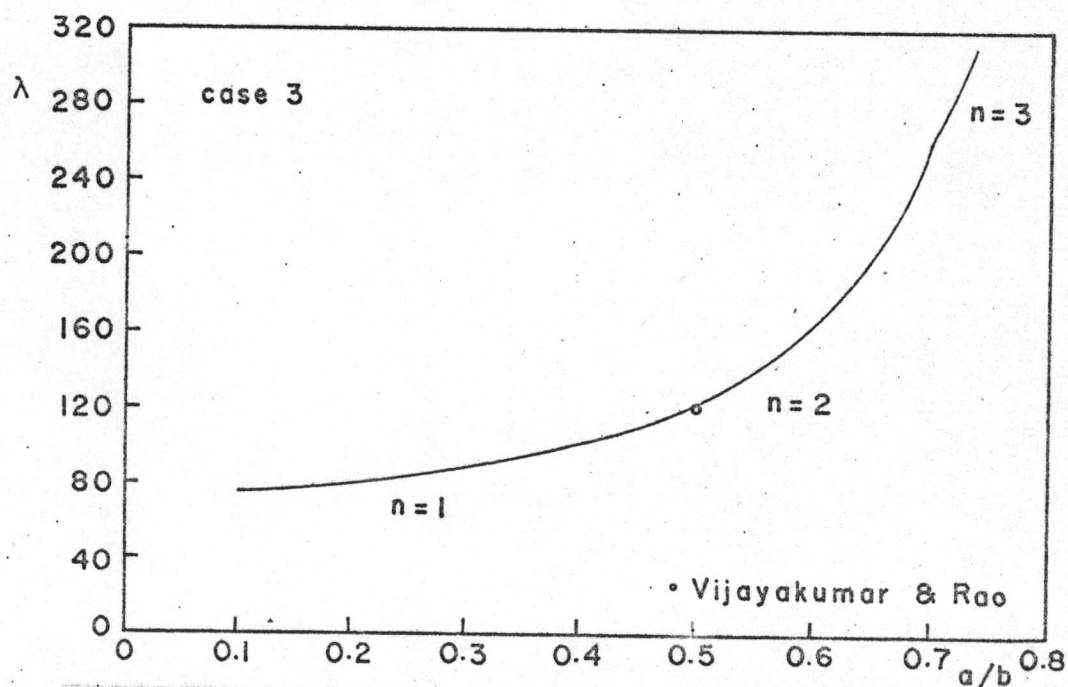


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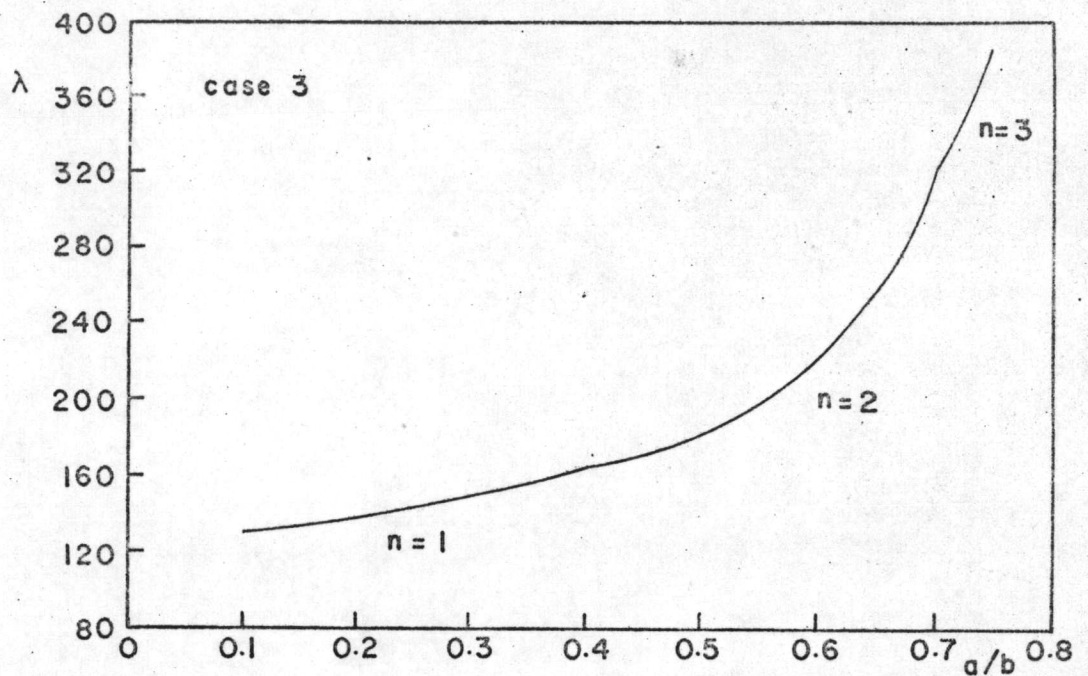


Fig. 3.45 Buckling of polar orthotropic annular plates
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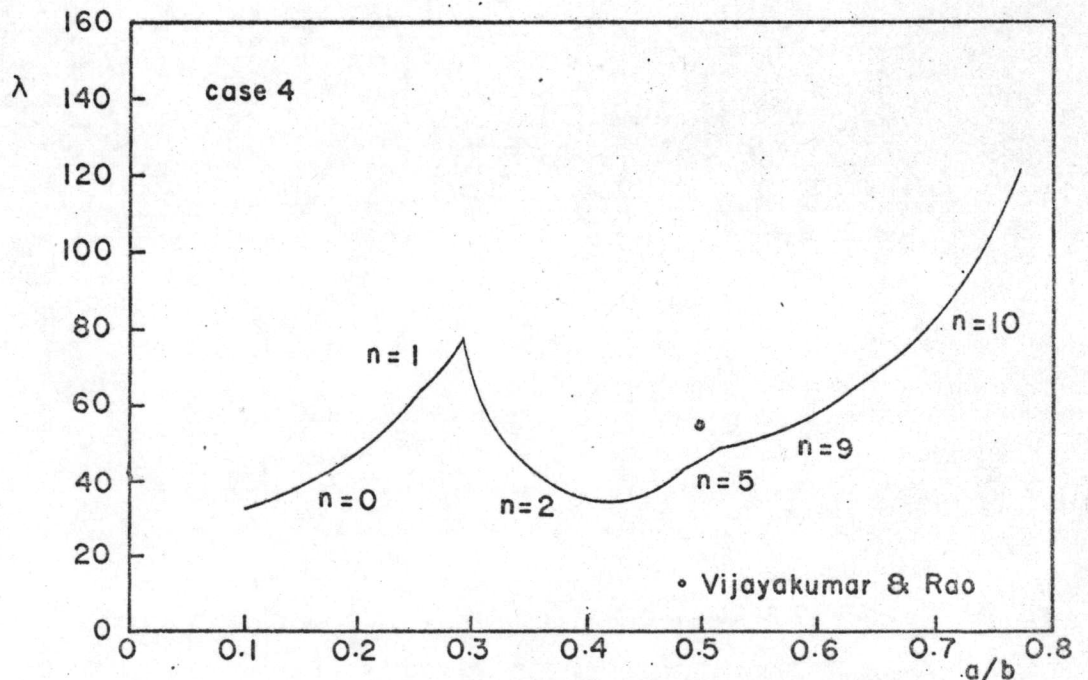


Fig. 3.46 Buckling of polar orthotropic annular plates
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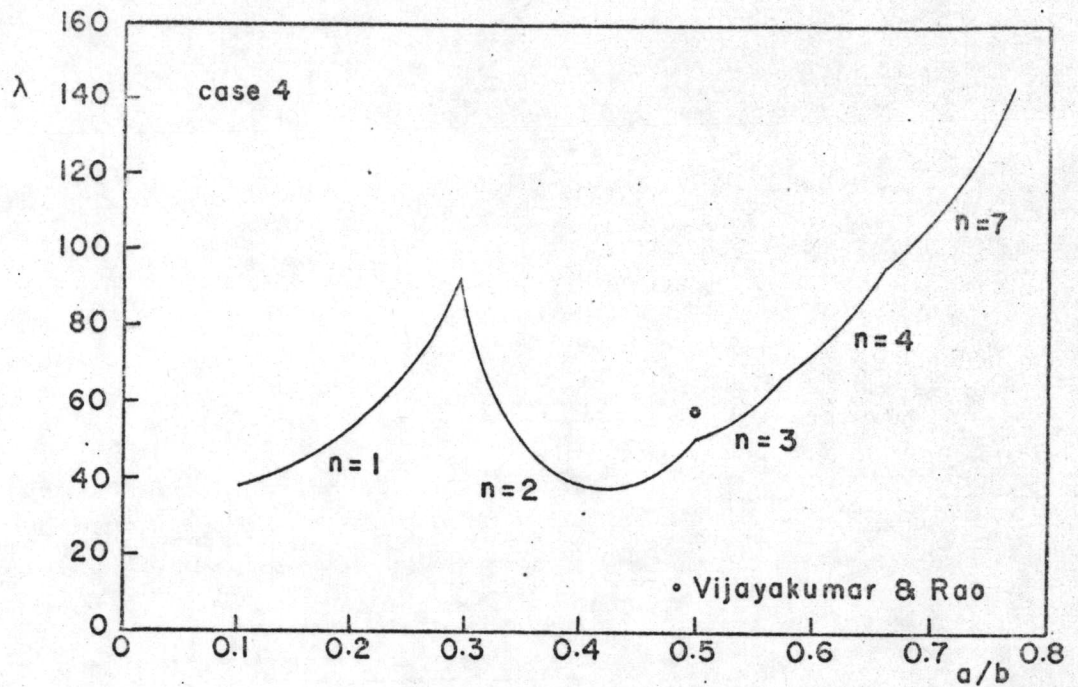


Fig. 3.47 Buckling of polar orthotropic annular plates
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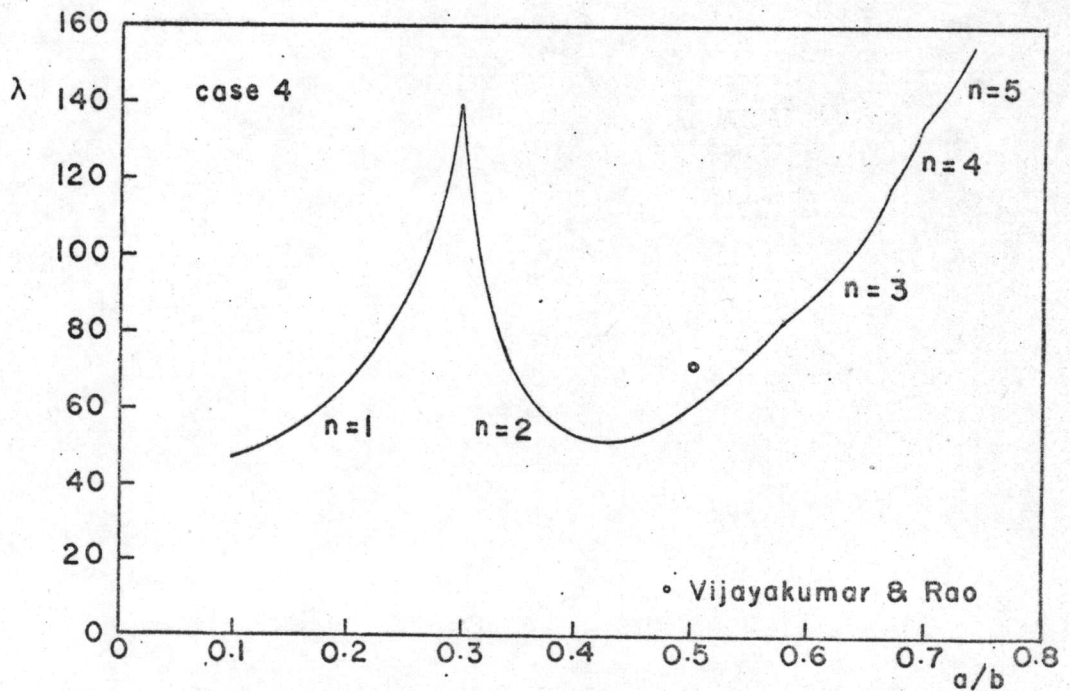


Fig. 3.48 Buckling of polar orthotropic annular plates
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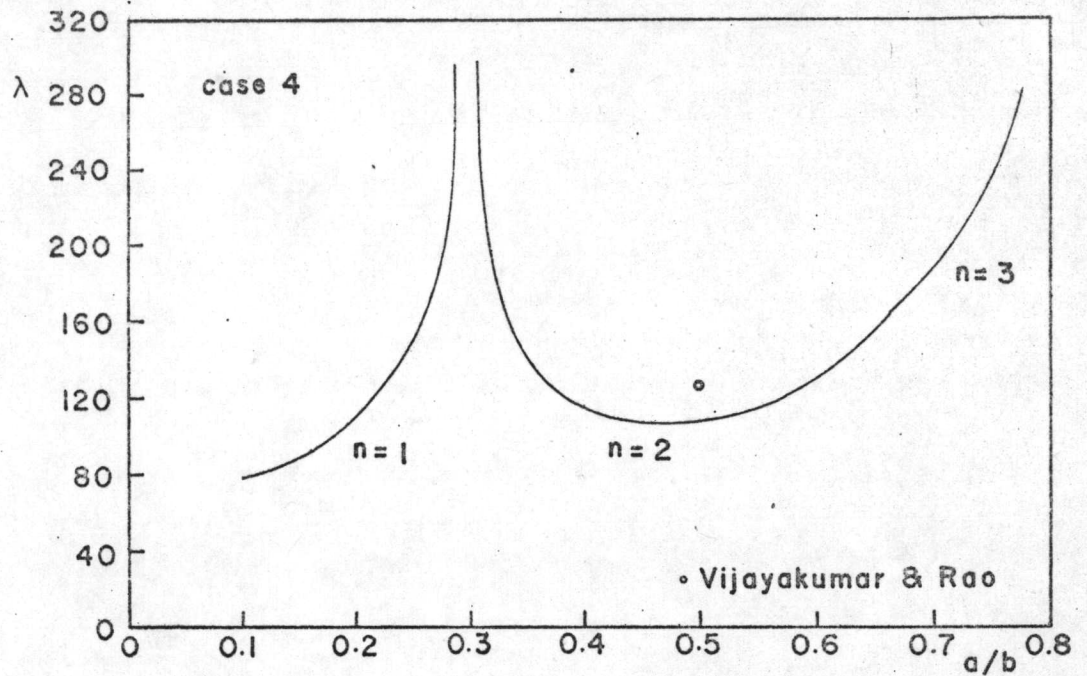


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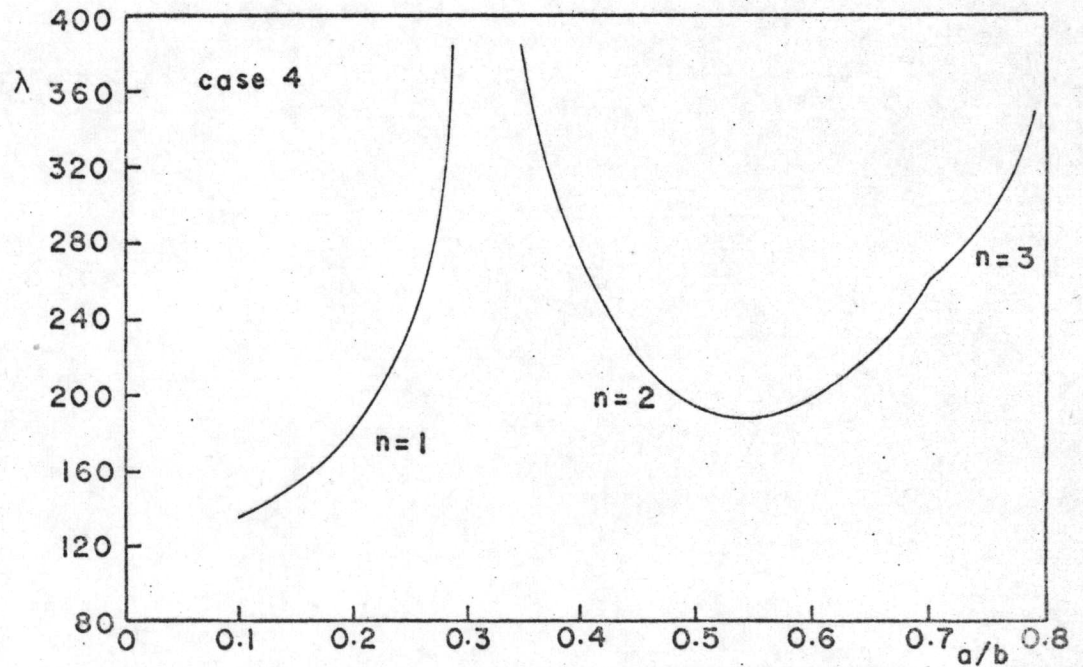


Fig. 3.50 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.0$ and $k = 6.0$

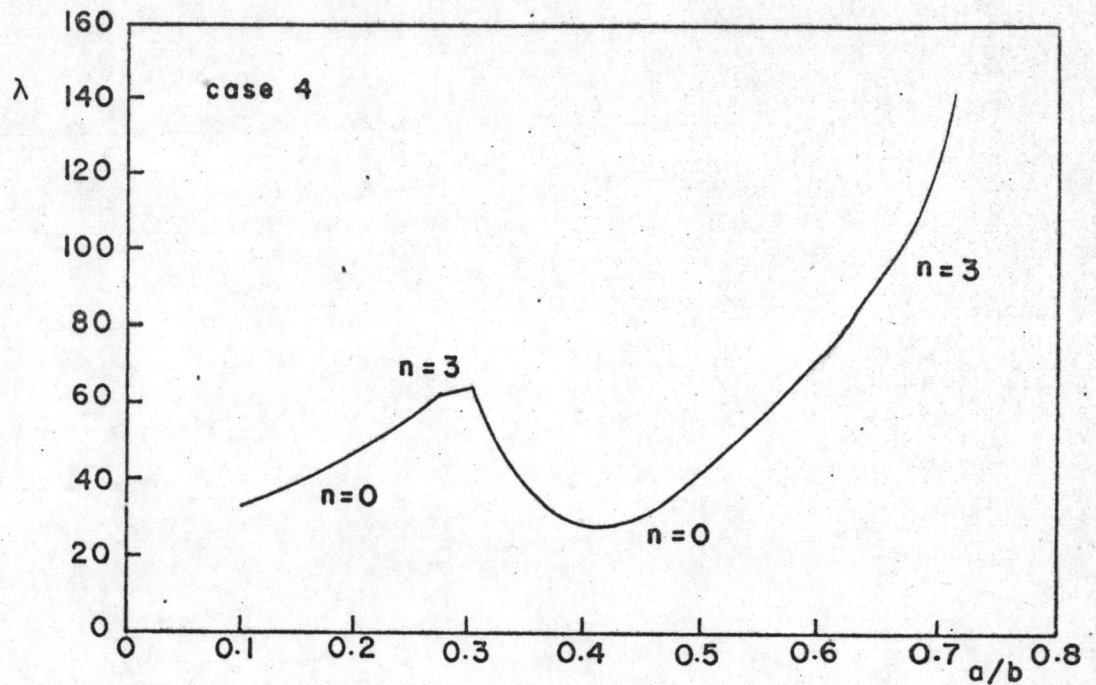


Fig. 3.51 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 0.3$

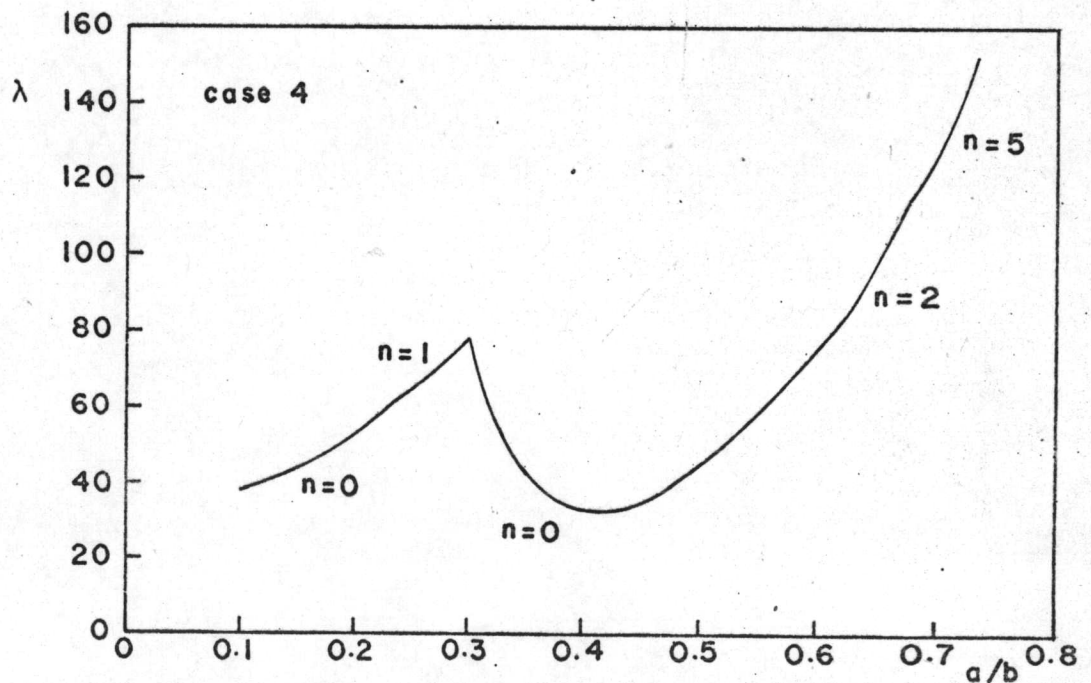


Fig. 3.52 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 1.0$

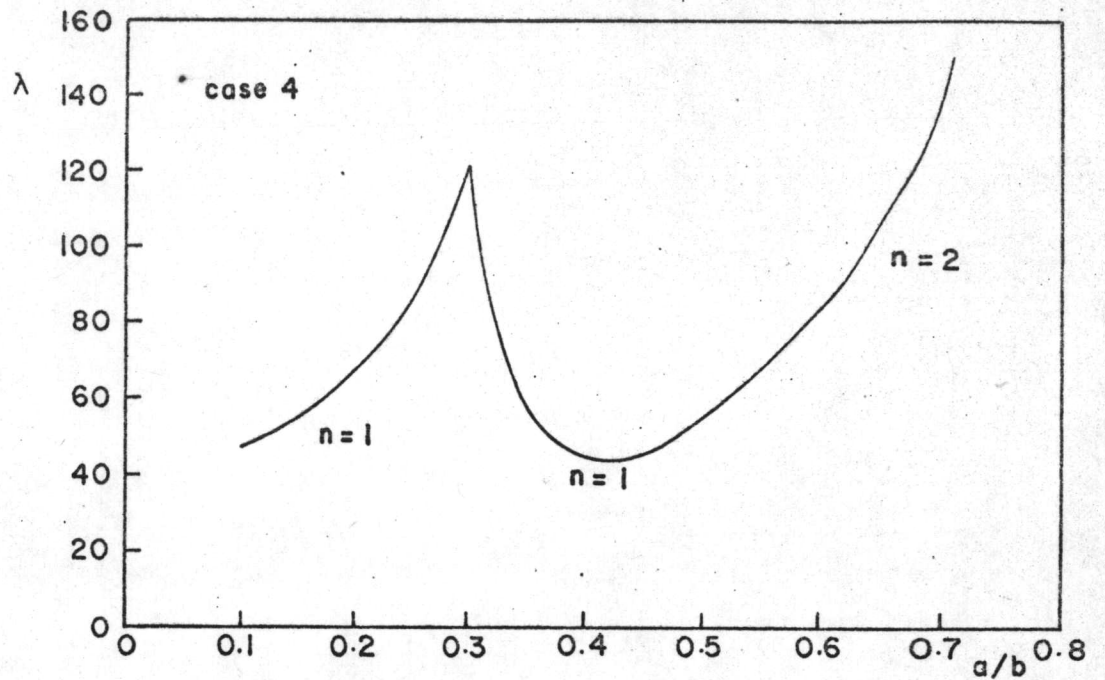


Fig. 3.53 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 2.0$

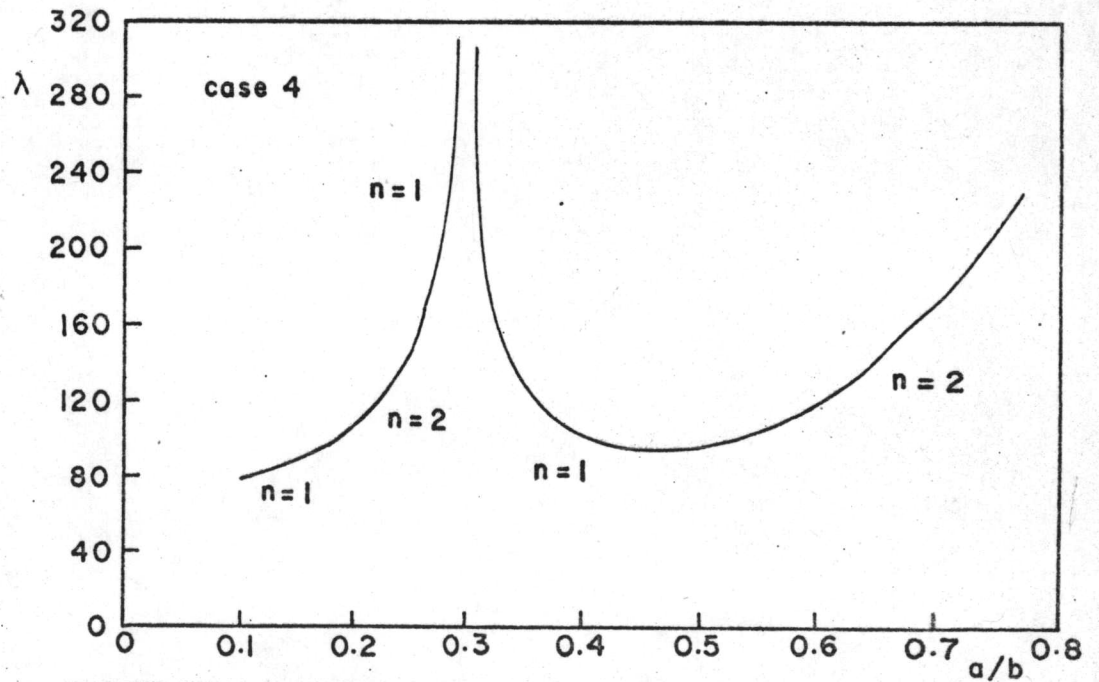


Fig. 3.54 Buckling of polar orthotropic annular plates for $P_i/P_o = 0.5$ and $k = 4.0$

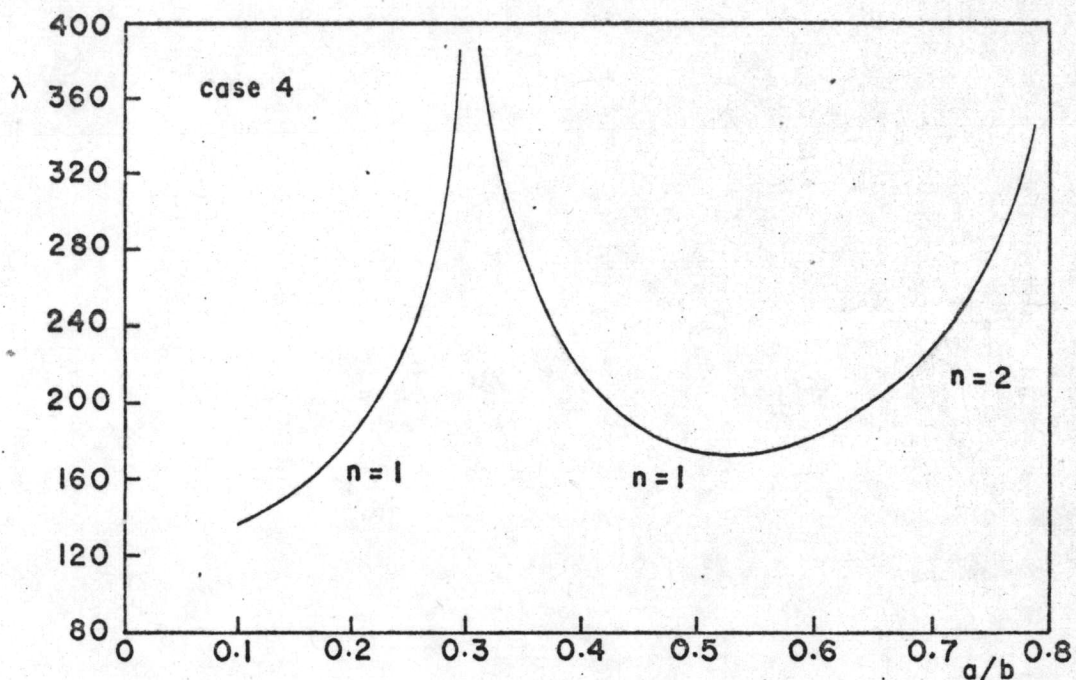


Fig. 3.55 Buckling of polar orthotropic annular plates
for $P_i/P_o = 0.5$ and $k = 6.0$

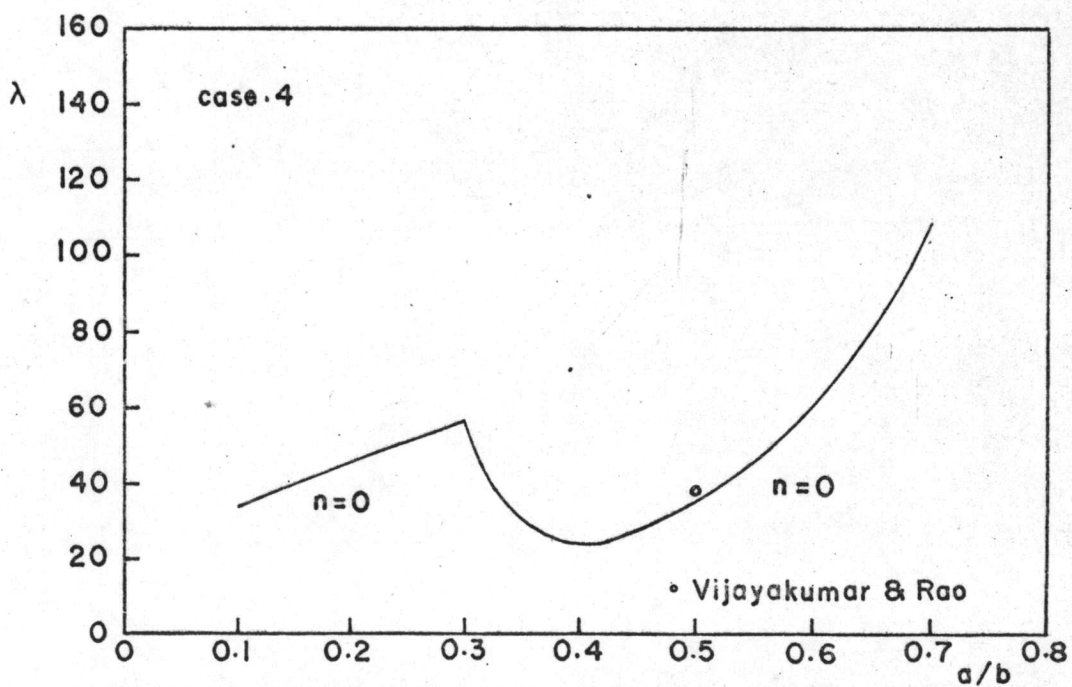


Fig. 3.56 Buckling of polar orthotropic annular plates
for $P_i/P_o = 1.0$ and $k = 0.3$

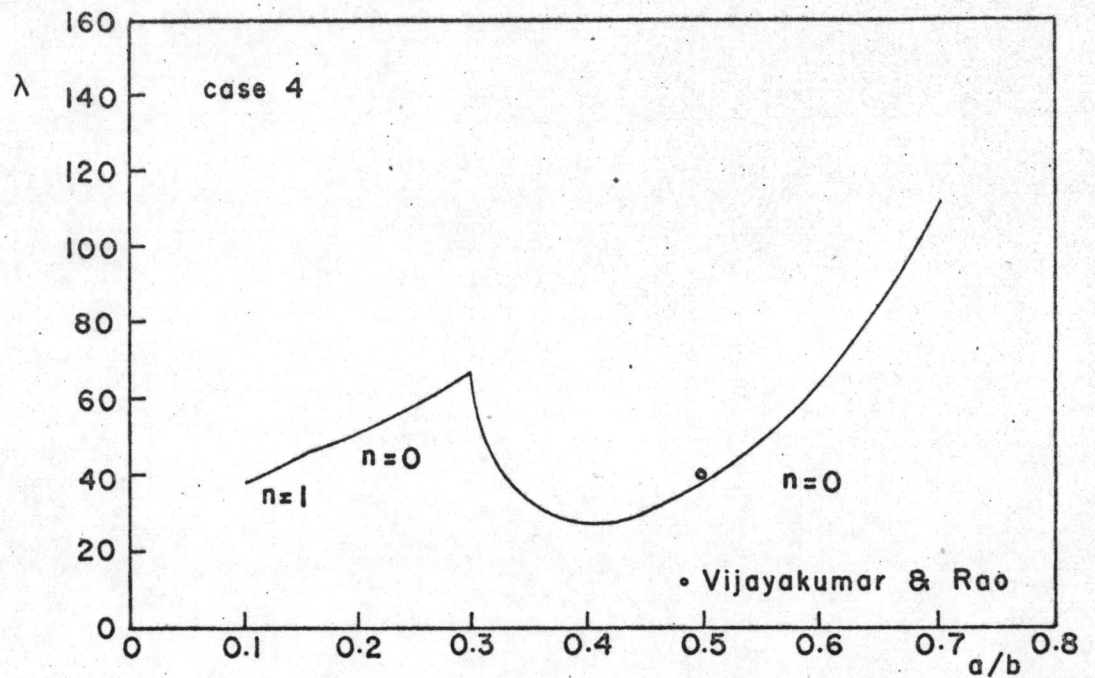


Fig. 3.57 Buckling of polar orthotropic annular plates
for $P_i/P_o = 1.0$ and $k = 1.0$

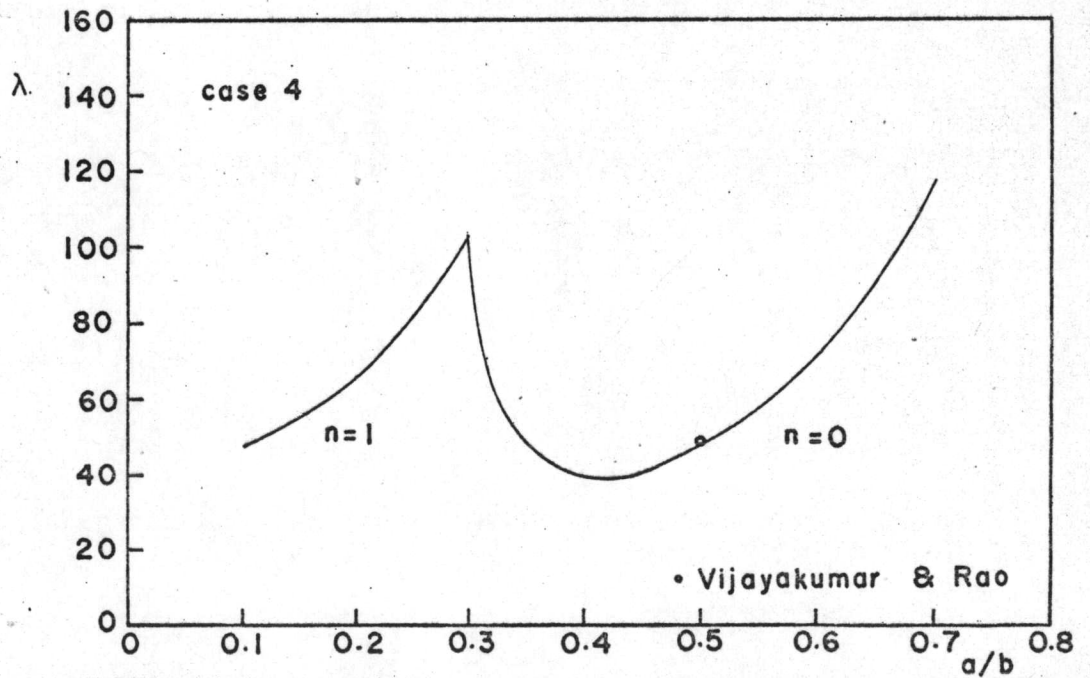


Fig. 3.58 Buckling of polar orthotropic annular plates
for $P_i/P_o = 1.0$ and $k = 2.0$

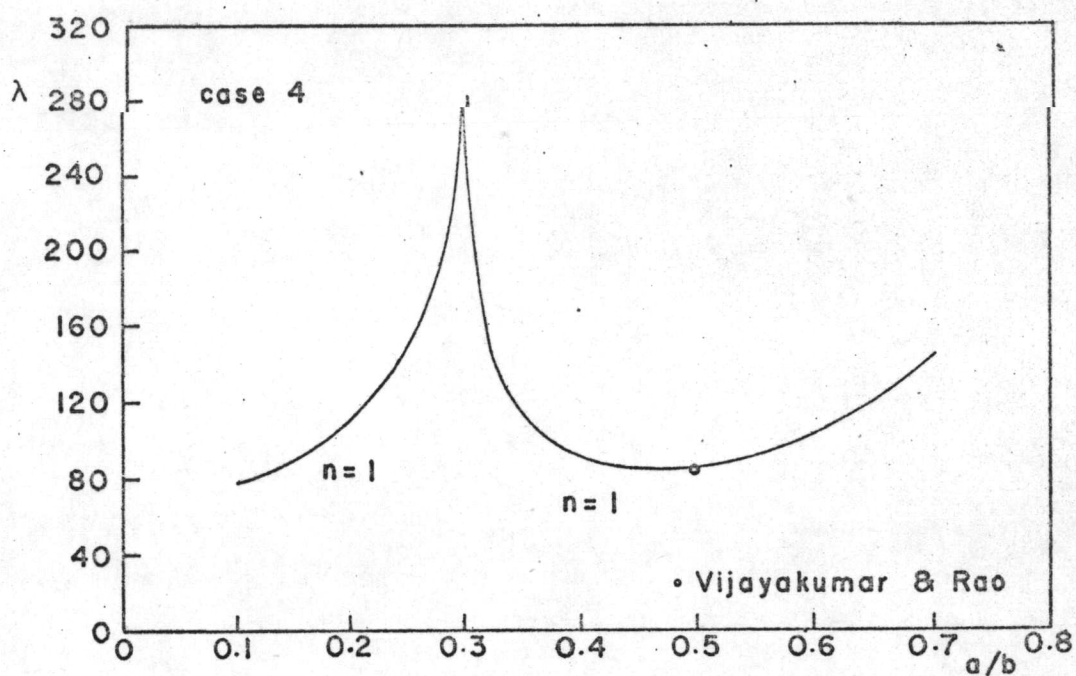


Fig. 3.59 Buckling of polar orthotropic annular plates
for $P_i/P_o = 1.0$ and $k = 4.0$

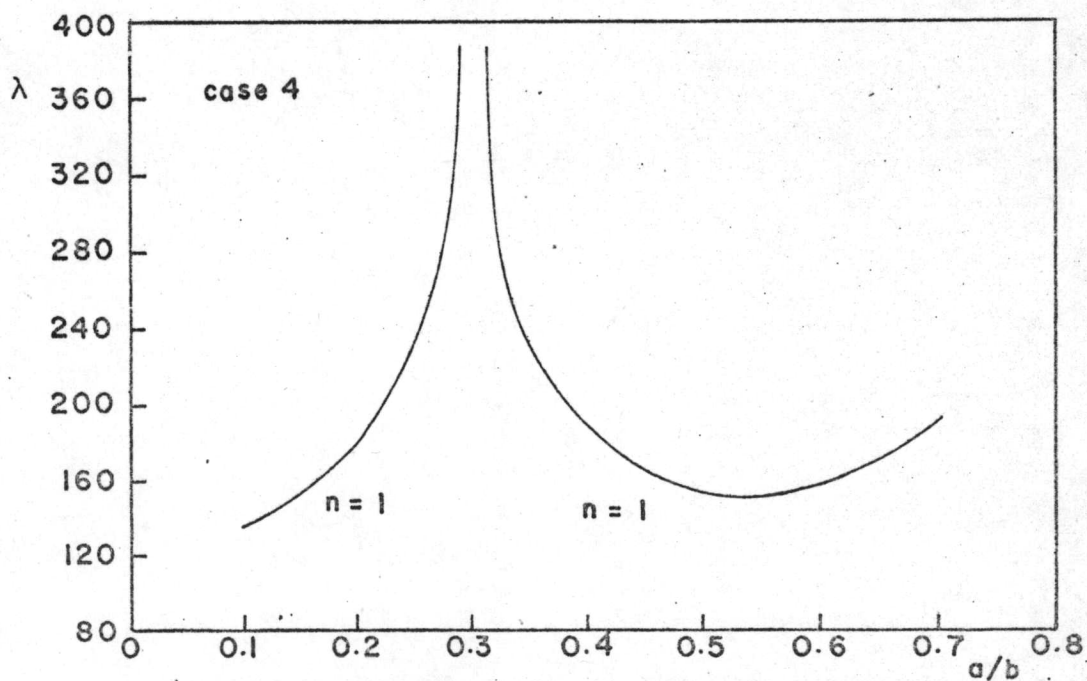


Fig. 3.60 Buckling of polar orthotropic annular plates
for $P_i/P_o = 1.0$ and $k = 6.0$