CHAPTER I

BOSE-EINSTEIN STATISTICS AND BOSE CONDENSATION

1.1 BOSE-EINSTEIN STATISTICS AND QUANTUM CORRECTION[1]

Bose-Einstein statistics state that each energy level of many boson systems can be occupied by any number of particles, the dominant characteristic of this system being statistical attraction between particles. However, because bosons would like to have the same quantum numbers, there is a vital and fascinating distinction between bosons and fermion. Thus, this research investigates an idea of quantum correction and how the system produces its statistical potential, automatically.

By introducing a function $f(\bar{r})$

$$f(\bar{r}) = \frac{\int d\bar{p} Exp\left[-\beta \frac{p^2}{2m} + i\frac{\bar{p}}{\hbar}\cdot\bar{r}\right]}{\int d\bar{p} Exp\left[-\beta \frac{p^2}{2m}\right]} = Exp\left(\frac{-\pi r^2}{\lambda^2}\right) \qquad (1.1)$$

where, $\lambda = \sqrt{2\pi\hbar^2/mkT}$

it can be seen that $f(\bar{r})$ is the expectation value of $Exp\left(i\frac{\bar{p}}{\hbar}\cdot\bar{r}\right)$ of the free particle system. From the expression of partition function,

$$Q = Tr\left[e^{-\beta H}\right] = Tr\left[e^{-\beta K}\right] \tag{1.2}$$

is obtained, where, K is a kinetic energy operator and a system is non-interacting system.

$$Q = Tr\left[e^{-\beta K}\right] = \frac{(2\pi)^{3N}}{N!\hbar^{3N}}\int d\bar{p}\int d\bar{r}e^{-\beta(p_1^2+\ldots+p_N^2)/2m} \times \sum_P (\pm)^P \left[f(\bar{r}_1 - \mathrm{P}\bar{r}_1)\ldots f(\bar{r}_N - \mathrm{P}\bar{r}_N)\right] \tag{1.3}$$

where P is an permutation operator and the plus sign for boson and the minus sign for fermion.

For very high temperatures the integrand may be approximated as follows. The sum $\sum_P$ contains $N!$ terms, while the term corresponding to the unit permutation is $P = 1$ is $\left[f(0)\right]^N = 1$. Because the term corresponding to the unit permutation which only interchanges $\bar{r}_i$ and $\bar{r}_j$ is $\left[f\left(\bar{r}_i - \bar{r}_j\right)\right]^2$, then

$$\sum (\pm)^P \left[f\left(\bar{r}_1 - \mathrm{P}\bar{r}_1\right)\ldots f\left(\bar{r}_N - \mathrm{P}\bar{r}_N\right)\right] = 1 \pm \sum_{i \neq j} f_{ij}^2 + \sum_{i,j,k} f_{ij} f_{jk} f_{ki} \pm \ldots \tag{1.4}$$

where, $f_{ij} \equiv f\left(\bar{r}_i - \bar{r}_j\right)$ - the plus sign applying to bosons and the minus sign applying to fermions. According to $f(\bar{r})$ in the equation (1.1), f_{ij} vanishes if $\left|\bar{r}_i - \bar{r}_j\right| >> \lambda$. In another way, this may approximat the right hand side of equation (1.4) by $1 \pm \sum_{i \neq j} f_{ij}^2$.

Further, this can be expressed as:

$$1 \pm \sum_{i \neq j} f_{ij}^2 \approx \prod_{i<j} \left(1 \pm f_{ij}^2\right)$$
$$= Exp\left(- \beta \sum_{i<j} \tilde{\upsilon}_{ij} \right) \tag{1.5}$$

where,

$$\tilde{\upsilon}_{ij} \equiv -kT \, Log\left(1 \pm f_{ij}^2\right)$$
$$\equiv -kT \, Log\left(1 \pm Exp\left(- \frac{2\pi \left| \bar{r}_i - \bar{r}_j \right|}{\lambda^2} \right) \right) \tag{1.6}$$

using the plus sign for bosons and the minus sign for fermions. Therefore, an improvement over equation (1.3) is the formula

$$Q = Tr\left[e^{-\beta K}\right] = \frac{(2\pi)^{3N}}{N! \hbar^{3N}} \int d\bar{p} \int d\bar{r} e^{-\beta \left(\sum_i \frac{p_i^2}{2m} + \sum_{i<j} \tilde{\upsilon}_{ij} \right)} \tag{1.7}$$

This shows that a free particle system, i.e. ideal gas, has the same effect as that of endowing the particles with inter-particle potential [1] $\tilde{\upsilon}(r)$, as the potential $\tilde{\upsilon}(r)$ is attractive for bosons and repulsive to fermions, as illustrate in Figure 1.1. This first quantum correction to the partition function of the free particle system is sometimes called statistical attraction between bosons and statistical repulsion between fermions.

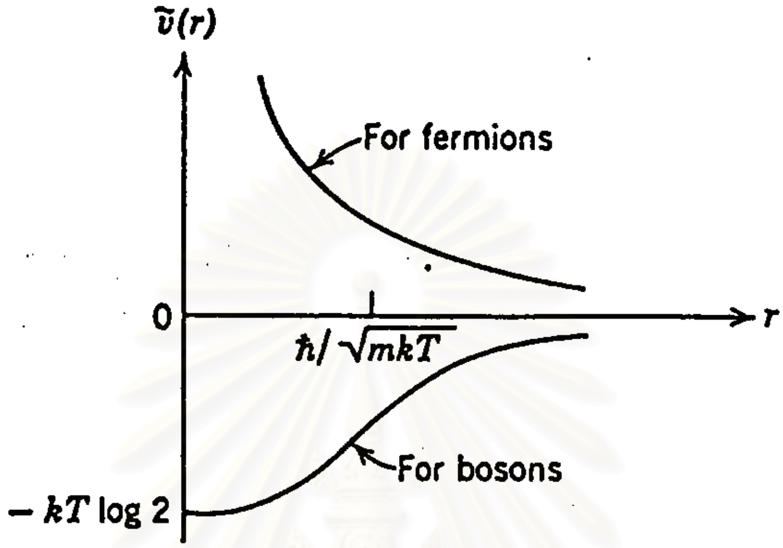

Figure 1.1 The inter-potential of bosons and fermions system[1].

1.2 BOSE CONDENSATION

In the previous section the statistical attraction between bosons in a system of non-interacting particle was reviewed. This fascinating attraction leads to a review of Bose condensation as follows.

Onnes (1911) [2] condensed ^4He into a liquid state at 5.22 K and over the next 13 years observed the sudden change in properties of ^4He at 2.2 K. In 1924, using Bose statistics [3] for any particles with an integer spin quantum number, Einstein [4] predicted a new state of matter, a system which has all particles at the lowest energy level. This system can be interpreted as the single particle state system. Fröhlich (1937) [5] tried to connect and form conclusions about both ideas, suggesting that the λ transition at T_λ, is a special case of order-disorder transition. London (1938) [6, 7] tried to interpret the Fröhlich (1937) model starting with his idea about the new theory of Einstein, the Bose-Einstein condensation state. He also suggested that Bose condensation is a special case of order-disorder transition and focused most of his interest on a discontinuous point on a graph of heat capacity, C_v, and temperature, T, of the ideal boson gas. Finally, he concluded that the superfluidity in liquid ^4He is a manifestation of Bose condensation. Tisza (1938) [8] also conjectured that the superfluid component in his phenomenological two-fluid model could be interpreted as the fraction of the ^4He Bose condensed atoms. However, the condensate fraction and the superfluid fraction are not the sane in a Bose condensed liquid like superfluid ^4He . At zero temperature, the superfluid fraction is 100% while the condensate fraction is only about 9%.

Landau (1941) [9, 10] classical paper strongly argues against any connection between a Bose condensate and the two-fluid model. However, the modern view vindicates Landau and London in that the microscopic basis of Landau's quasiparticle picture and the two-fluid model lies in the existence of a Bose condensate.

The Landau theory of superfluidity is based on the low-lying excited state of a Bose liquid, showing that the low temperature thermodynamic and transport properties of superfluid ^4He could be understood in terms of a weakly interacting gas of Bose "quasiparticles", phonon and roton.

The microscopic basis of Landau's picture was developed by Feyman [11, 12, 13, 14] in the period 1953-1957. Feynman dealt directly with the excited states of Bose liquid, as oppose to Bogoliubov (1947) [15] who dealt directly with the excited states of boson gas.

Bogoliubov [15], in one of the first treatments of a broken symmetry, gives the derivation of the phonon spectrum in a weakly interesting dilute Bose gas.

From the above information about 9% of the condensate fraction of the condensate fraction in liquid ^4He is decreasing to zero at T_λ. This fact stimulated some physicists to study the high momentum scattering of liquid ^4He [16] over the next 40 years. Finally, the results of the above study give more information about the two-branch structures of the sharp peak: the narrow *(n)* and wide *(w)* branches discussed in the maxon-roton regime. The first of them *(n)* could exist at any high temperature *T*.

1.3 SCOPE OF THIS RESEARCH

We argue with the idea of Landau [9, 10] of the exciatation picture and follows with the Bogoliubov [15] idea of how to define the excitation. The functional integration, formulated by Feynman and including some mathematical technique for calculating from Popov [17] and Yarunin [18, 19, 20], were used. It is also suggested that the microscopic theory of the two-branches excitation in the weakly interacting dilute Bose gas and an ansatz interaction between Bose gas be incorporated.