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APPENDIX

NEVILLE'S PROCESS OF INTERPOLATION BY ITERATION

Let $f_a, f_b, f_c, \ldots$ denote the values of a function coorresponding to the arguments a, b, c, ... and let $[ab], [abc] \ldots$ be the divided differences. Now f(x; a, b, c,) denotes the interpolation polynomial with values of f given at a, b, c, with similar meaning for the other symbols, thus

$$f(x;\ a,b) = f_a + (x-a)\,[ab]$$

$$f(x;\ a,b,c) = f_a + (x-a)\,[ab] + (x-a)(x-b)\,[abc]$$

$$f(x;\ a,b,c,d) = f_a + (x-a)\,[ab] + (x-a)(x-b)\,[abc] + (x-a)(x-b)(x-c)\,|abcd|$$

$$\text{or } f\ (x;\ a,b,c,d) = \begin{vmatrix} f(x;a,b,c) & c-x \\ f(x;a,b,c) & d-x \end{vmatrix} \div (d-c)$$

The calculation can be arranged as in the following scheme :

Argument	Function	1	2	3	Parts
a	f_a				a-x
b	f_b	f(x;a,b)			b-x
c	f_c	f(x;a,c)	f(x;a,b,c)		c-x
d	f_d	f(x;a,d)	f(x;a,b,d)	f(x;a,b,c,d)	d-x

Given the numerical value of x, each number in the i^{th} column is the value of an interpolation polynomial which equals f(x) at i+1 points, and

$$f(x) = f(x;a,b,c,d) + (x-a)(x-b)(x-c)(x-d)\,\frac{f^{(iv)}}{4!}(\zeta)$$ [1]

where ζ lies in the smallest interval containing a,b,c,d,x. With the same notation, a somewhat different technique has been developed by Neville[2] as follows:

Argument	Parts	Function	1	2	3
a	x-a	f_a			
			f(x;a,b)		
b	x-b	f_b		f(x;a,b,c)	
			f(x;b,c)		f(x;a,b,c,d)
c	x-c	f_c		f(x;b,c,d)	
			f(x;c,d)		
d	x-d	f_d			

Where $f(x;a,b,c) = \begin{vmatrix} x-a & f(x;\ a,b) \\ x-c & f(x;\ b,c) \end{vmatrix} \div (c-a)$

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[1] L.M. MILNE - THOMSON , The Calculus of Finite Differences. (London: Macmillan and co., Limited, 1951), P.77.

[2] Ibid (1) P. 81.

Inverse Interpolation

If a variable y is given in tabular form as a single-valued function of x and a value of x is required for which y takes on a prescribed value the problem is that of inverse interpolation[3].

1. Inverse Interpolation by Divided Differences

Argument	Function	1	2
y_1	x_1		
		$[y_1\ y_2]$	
y_2	x_2		$[y_1\ y_2\ y_3]$
		$[y_2\ y_3]$	
y_3	x_3		
.	.		
.	.		
.	.		
.	.		
.	.		

where $[y_1 y_2] = (x_1 - x_1) \,/\, y_1 - y_2$, etc.

$$x = x_1 + (y-y_1)[y_1y_2] + (y-y_1)(y-y_2)[y_1y_2y_3] + \cdots + (y-y_1)(y-y_2)(y-y_3)\cdots(y-y_n)[y_1y_2y_3\cdots y_n]$$

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3 F.B. HILDEBRAND, Introduction to Numerical Analysis (New York: McGraw-Hill Book Company, Inc., 1956), P.51.

2. Inverse Interpotation by Iterated Linear Interpolation.

The Neville's interpolation by iteration and Aitken's process are very well adapted to inverse interpolation when different interval spacings have to be taken into account. We can interchange argument and function and use the same method as before to obtain the required result by direct interpolation.

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