CHAPTER I

INTRODUCTION

Vibrations in a solid body implies periodic motion in which the space-time history of a particle in the solid repeats itself every period. The number of repetitions in unit time is called the frequency of the vibration system.

In general vibrations are divided into two large classes namely longitudinal and transversal. We will discuss only transverse vibrations. The motivation arises from musical instruments several of which utilize transverse vibrations (such as Xylophone, Drums and Violin etc.) to obtain melodious sounds. Usually the cross-sections of the vibrating parts are not uniform. This thesis will deal with a method of calculating mode shapes and frequencies of a non-uniform bar as a step to a better understanding of the musical instruments.

We start with the differential equation of motion of a bar vibrating transversely. It is known[1] that

$$\frac{\partial^2}{\partial x^2}\left(QSK^2\frac{\partial^2}{\partial x^2}\psi\right) = -\rho S\frac{\partial^2\psi}{\partial t^2},$$

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[1] PHILIP M. MORSE, Vibration and Sound (New York: McGraw-Hill Book Company, Inc., 1948), p.164.

where $\psi(x,t)$ is the displacement of the center line of a bar away from equilibrium (we assume this displacement to be in the xy - plane).

Q is a constant, called Young's modulus and its values are given in Table 1[2]; S is the area of cross - section:

ρ is the density of the material of the bar ;

K is the radius of gyration of the cross-section and normally its value equals the square root of the moment of inertia divided by the area of cross-section. The moments of inertia for some of the simpler cross section shapes are shown in Table 2. The axis AA applies in the present study.

The differential equation corresponds to the equation of motion of the uniform bar if ρ, S, K^2 and Q are independent of x, If any of them are not, one may assume some form for ψ, say $y(x)\,e^{i\omega t}$, where y(x) defines the shape of the normal mode of vibration under consideration, i.e., the normal function as is limited by terminal conditions to be one of an infinite series, which equals $2\pi F$, where F is the frequency of vibration.

Next we divide the non-uniform bar into sections and approximate the differential equation by a set of difference

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[2] Ibid p.152.

equations. From the set of difference equations and the given physical parameters we construct an approximate solution by using numerical methods. By varying the initial conditions we seek solutions that satisfy the boundary conditions. We can thus obtain the mode shape and the frequencies of a non-uniform bar. The calculation was programmed for an electronic computer by Dr. R.H.B. Exell.

Table 1

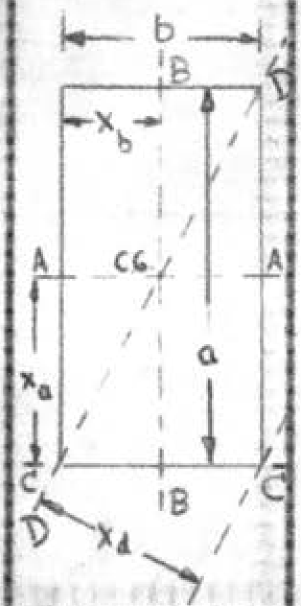
Elastic Constants of Materials

Material	Q	ρ
Iron, cast	9×10^{11}	7.1
wrought	19×10^{11}	7.6
Iron cobalt (70 % Fe)	21×10^{11}	8.0
Nickel	21×10^{11}	8.7
Steel, annealed	19×10^{11}	7.7
invar	14×10^{11}	8.0

Values of Young's modulus Q in dynes per square centimeter, and of density ρ, in grams per cubic centimeter, for various materials.

Table 2

Properties of a Simpler cross section

Section	x	J	k
	$x_a = \frac{a}{2}$	$J_{AA} = \frac{a^3 b}{12}$	$K_{AA} = \frac{a}{\sqrt{12}} = 0.289\ a$
	$x_b = \frac{b}{2}$	$J_{BB} = \frac{ab^3}{12}$	$K_{BB} = \frac{b}{\sqrt{12}} = 0.289\ b$
	$x_d = \frac{ab}{\sqrt{a^2+b^2}}$	$J_{CC} = \frac{a^3 b}{3}$	$K_{CC} = \frac{a}{\sqrt{3}} = 0.577\ a$
		$J_{DD} = \frac{a^3 b^3}{6(a^2+b^2)}$	$K_{DD} = \frac{ab}{\sqrt{6(a^2+b^2)}}$
		$J_p = \frac{a^3 b + ab^3}{12}$	

J = Moment of inertia.

J_p = Polar moment of inertia, refers to an axis through the center of gravity.

x = Distance of center of gravity.

K = Radius of gyration.