CHAPTER III

PROBLEM DESCRIPTION

3.1 Introduction

This chapter focuses on the multi-item multi-depot inventory routing problem which is the problem of coordination between inventory control and transportation planning. In this problem, inventory decision and transportation decision are made simultaneously in order to achieve minimum level of total logistic cost. The logistic cost includes inventory holding cost and transportation cost.

The mathematical model is developed where the objective is to minimize total inventory and transportation cost in the multi-item multi-depot system with deterministic demand for finite horizon. The model includes equations that represent the coordination of inventory and transportation decisions. In addition, limited storage capacity at outlets and carrying capacity of vehicle are considered.

The remainder of this chapter is organized as follows. Problem description is provided in Section 3.2. A numerical example will be presented in Section 3.3. The conclusions of this work and some recommendations for the next are presented in Section 3.4.

3.2 Problem Description

This dissertation considers a distribution system consisting of several depots and outlets. All of them are geographically dispersed in a considered area. Depots distribute non-identical items to outlets where face periodically deterministic demand in a finite time horizon. However, the demand is dynamic that means demand in each period is not the same amount. For example demand at an outlet can be 5 units in the first period while 7 units in the second period. Each depot has a fleet of capacitated vehicles which are used to distribute the diversified items to the different places of outlets within considered periods. The number of vehicles is given for any specific depot. Each outlet has limited storage capacity. Illustration of the system is shown in Figure 3.1

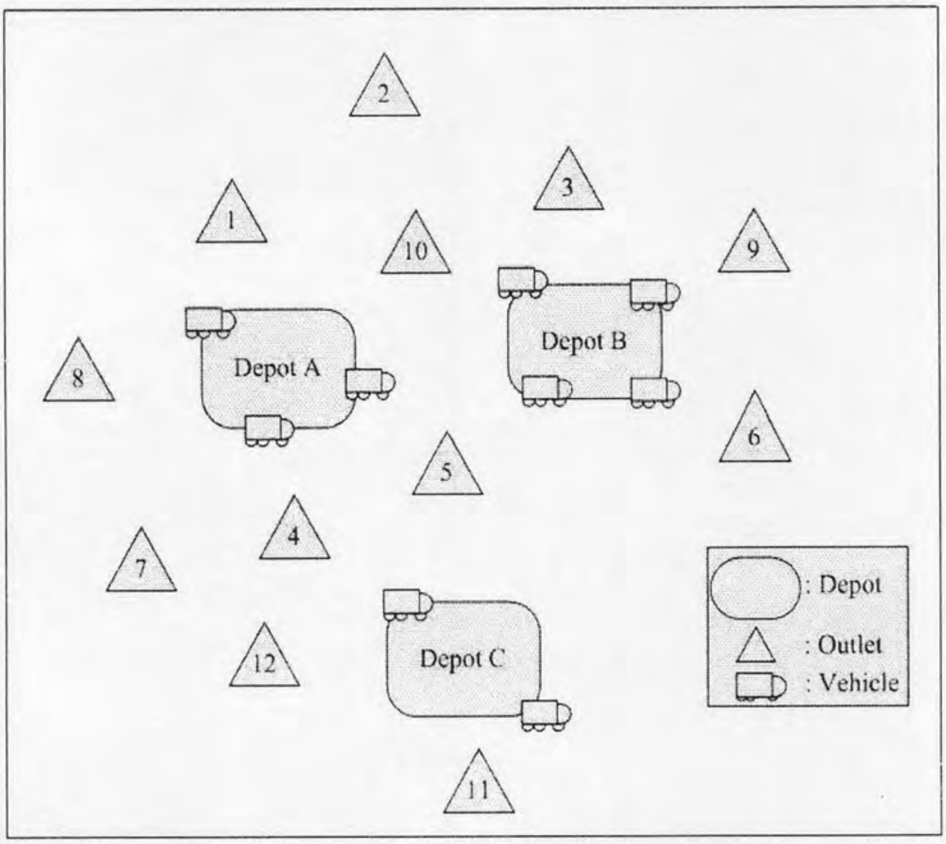

Figure 3.1 Illustration of considered system

Objective of this system is to determine replenishment quantity to satisfy all demands and make deliveries for all replenishment while minimizing total logistic cost. The total logistics cost comprises of inventory holding cost, vehicle fixed-charge cost and vehicle routing cost. The inventory holding cost depends on outlet and item. The inventory holding cost is calculated from inventory holding cost per unit per period to the inventory kept at the outlet and incurred at a constant rate per unit item per period.

The vehicle fixed charge cost is a constant and identical cost for all vehicles. It is incurred whenever a vehicle is dispatched to deliver a set of replenishment orders assigned to it. Fixed-charge can include driver wages or vehicle rental cost.

The vehicle routing cost is distance dependent costs without consideration of time spent on depot and outlets such as load/unload time and stop over time. An example of this cost is fuel expenses. This cost depends on total route length of all vehicles.

3.2.1 Assumptions

Assumptions for an integrated inventory-transportation system are made based on demand, inventory and transportation policy, and limitation of capacity mainly on vehicle capacity and storage capacity.

3.2.1.1 Demand

The demand of each item in each outlet is assumed to be dynamic and deterministic. Dynamic demand means that its value changes over time and the value of demand is known in advance, so the deterministic demand is considered in this dissertation. Deterministic demand cases in real world system may occur by make to order system or precise forecasting of demand. However, the value of demand may not be constant in each period but it can be different from one time period to another time period.

Moreover, the demand pattern can vary from one outlet to another outlet. This case is realistic due to each outlet is located in different location which has different buying power and demand.

3.2.1.2 Policy

- The customer satisfaction is a main concern of this system so stock outs situation is prohibited in this system. This policy relates to competitive environment of business which causes it to maintain a good customer satisfaction level by serving their demand with out any stock out.

- All demand must be met during the considered time horizon. This assumption can be realistic by sufficient supply of external suppliers which are not considered in this dissertation.

- The inventory is kept at outlets' locations. Depots operate as transshipment point and do not hold any inventory so inventory is kept at outlets only.

- Transshipment between depots is not considered in this research, because it may cause additional cost for transshipment and more decisions.

- Each replenishment quantity of an item in an outlet can belong to one and only one depot for certain period, but can be reassigned to another depot in other periods.

- For delivery service, all region considered in the system can be served within a period by a set of homogeneous vehicles without any traffic problem. Each item (but not all items in a single delivery) at outlets is fully served according to the schedules (split delivery is not allowed).

3.2.1.3 Capacity

There are two types of capacity which are studied in this dissertation. The first one is inventory storage capacity. Each outlet has a limited stocking capacity for keeping all items. This capacity is outlet specific. The last type of capacity is vehicle carrying capacity which is limited and identical for all vehicles.

There are a limited certain number of vehicle of each depot and available to serve all outlets during planning horizon.

3.2.2 Model Formulation

All notations used in this dissertation are defined and presented as follows.

Decision Variables

I_i^t	Inventory level of Item-Outlet i at period t.
o_i^{tk}	1 if Order Item-Outlet i at period t cover demand from period t to k-1, and 0 otherwise
a_{ij}^{tv}	1 if there is an arc for delivery from Node i *to* Node j in period t by vehicle v, and 0 otherwise
f_{ij}^{tv}	Flow of Item from Node i *to* Node j at period t by vehicle v.

Set

DP={1..d}	Depot Set
I={1..s}	Item Set
O={1..o}	Outlet Set
IO=I$\times$O	Item-Outlet Set(consider each item in each outlet as a demand node)
N=DP$\cup$IO	Node Set :consists of Item-Outlet and Depot
V={1..v}	Vehicle Set
P={1..p}	Period Set

Parameter

H_i	Holding cost of Item-Outlet i.
F_v	Fix-charge cost per period of using vehicle v.
SC_o	Storage Capacity at Outlet o.
CC^v	Carrying Capacity of Vehicle v.
SI_i^o	1 if Item-Outlet i is related to Outlet o, and 0 otherwise
S_d^v	1 if vehicle v is stationed at Depot d, and 0 otherwise

C_{ab}	Distance between Node *a* and *b*.
B_i	Beginning Inventory level of Item-Outlet *i*.
D_i^t	Demand of Item-Outlet *i* at period *t*.
$CumD_i^{tk}$	Demand of Item-Outlet *i* at from period *t* to *k*-1.

The model of multi-item multi-depot inventory routing problem (MMIRP) can be formulated as follows:

Objective function

$$\text{Minimize} \quad \sum_{i \in IR}\sum_{t \in P} H_i I_i^t + \sum_{i \in N}\sum_{j \in N}\sum_{t \in P}\sum_{v \in V} C_{ij} a_{ij}^{tv} + \sum_{w \in W}\sum_{i \in N}\sum_{t \in P}\sum_{v \in V} F_v a_{wi}^{tv} \qquad (0)$$

Constraints

$$I_i^t = I_i^{(t-1)} + CumD_i^{tk} o_i^{tk} - D_i^t \qquad \forall i \in IO, t \in P \qquad (1)$$

$$\sum_{j \in P} o_i^{jt} = \sum_{k \in P \cup \{p+1\}} o_i^{tk} \qquad \forall i \in IO, t \in P\text{-}\{1\} \qquad (2)$$

$$\sum_{j \in P} o_i^{jt} = 1 \qquad \forall i \in IO, t = \{p+1\} \qquad (3)$$

$$\sum_{k \in P \cup \{p+1\}} o_i^{tk} = 1 \qquad \forall i \in IO, t = 1 \qquad (4)$$

$$o_i^{tk} = 0 \qquad \forall i \in IO, t,k \in P \cup \{p+1\}, t >= k \qquad (5)$$

$$\sum_{v \in V}\sum_{i \in N\text{-}\{j\}} a_{ij}^{tv} = \sum_{k \in P \cup \{p+1\}, k>t} o_j^{tk} \qquad \forall j \in IO, t \in P \qquad (6)$$

$$\sum_{v \in V}\sum_{j \in N\text{-}\{i\}} a_{ij}^{tv} = \sum_{k \in P \cup \{p+1\}, k>t} o_j^{tk} \qquad \forall i \in IO, t \in P \qquad (7)$$

$$\sum_{i \in N\text{-}\{j\}} a_{ij}^{tv} = \sum_{h \in N\text{-}\{j\}} a_{jh}^{tv} \qquad \forall j \in IO, t \in P, v \in V \qquad (8)$$

$$f_{ij}^{tv} \le CC^v a_{ij}^{tv} \qquad \forall i,j \in N, t \in P, v \in V \qquad (9)$$

$$\sum_{i \in N\text{-}\{j\}} f_{ij}^{tv} \ge \sum_{h \in N\text{-}\{j\}} f_{jh}^{tv} \qquad \forall j \in IO, t \in P, v \in V \qquad (10)$$

$$\sum_{v \in V}\sum_{i \in N\text{-}\{j\}} f_{ij}^{tv} = \sum_{v \in V}\sum_{h \in N\text{-}\{j\}} f_{jh}^{tv} - CumD_j^{tk} o_j^{tk} \qquad \forall j \in IO, t \in P \qquad (11)$$

$$\sum_{i \in IO} I_i^t SI_i^o \le SC_o \qquad \forall o \in O, t \in P \qquad (12)$$

$$\sum_{i \in IO} a_{di}^{tv} \le S_d^v \qquad \forall t \in P, v \in V, d \in DP \qquad (13)$$

$$\sum_{i \in IO} a_{id}^{tv} \le S_d^v \qquad \forall t \in P, v \in V, d \in DP \qquad (14)$$

Description of Constraints

(0) Objective Function: Inventory holding, Vehicle routing, and Vehicle fixed-charge cost.

(1) Inventory balance.

(2) Continuity of Replenishment Period.

(3) There must be exact one link between the first period to an exact period.

(4) There must be exact one link from an exact period to period $p+1$.

(5) There is no link between later periods to former periods.

(6)-(7) Each outlet is exactly visited by a vehicle only equal to order variable

(8) Route Continuity.

(9) The amount transported between two locations will always be zero whenever there is no vehicle moving between these locations, and the amount transported is less than or equal to the vehicle's capacity.

(10) Sub-tours Elimination.

(11) Relationship between flow variable and replenishment quantity.

(12) Inventory level at outlet can not exceed Storage Capacity.

(13)-(14) All arcs between depot d and any outlet must be delivered by vehicles which station at depot d only.

3.3 A Numerical Example

In this section, a simple problem is illustrated. This example consists of two depots and five outlets. Each depot has a vehicle for delivery items to outlets. All vehicles have a capacity limit at one hundred units that means the total amount of all items must not exceed one hundred units. There are two items in each outlet. Decisions are made for three periods. Figure 3.2 illustrates the graphical location of all depots and outlets in the example problem. Table 3.1 shows the distance between all locations. Demand of all outlets is given in Table 3.2. Table 3.3 gives the information about the holding cost of the outlets. Beginning inventory of all outlets are assumed to be zero. The example problem is solved by improving initial solution, which is the solution obtained by the Lot-For-Lot policy. The initial solution from Lot-for-Lot policy is the solution with minimum holding cost and maximum transportation cost. The Lot-For-Lot will set the replenishment quantity for all item-outlets follow their demand so there is no inventory kept in outlet store and result in zero holding cost. However all item-outlets must be replenished in every period so the transportation is maximal. The initial solution can be improved by reschedule replenishment period of item-outlets which result in reduction on transportation cost. Nevertheless the rescheduling of replenishment period of item-outlets causes the increase on holding cost, the trade off between the two costs must be considered to obtain the solution. With the concept of balancing the holding and transportation cost the solution of this example are provided in Table 3.4 and illustrations of delivery routes are shown in Figure 3.3 to 3.5.

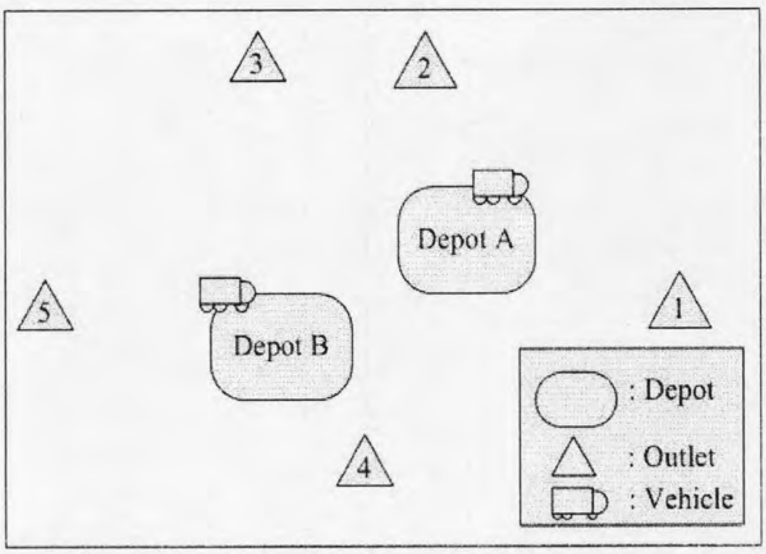

Figure 3.2 Illustration of the graphical location of all depots and outlets in the example problem.

Table 3.1 Distance matrix between all locations

		Depot		Outlet				
		A	**B**	**1**	**2**	**3**	**4**	**5**
Depot	**A**	0	3	7	10	9	9	11
	B	3	0	9	12	7	7	13
Outlet	**1**	9	7	0	13	16	11	16
	2	12	10	13	0	15	19	4
	3	7	9	16	15	0	11	13
	4	7	9	11	19	11	0	20
	5	13	11	16	4	13	20	0

Table 3.2 Demand of all outlets

		Period	1	2	3
		[tem			
Outlet	1	X	20	19	20
		Y	19	20	19
	2	X	20	21	20
		Y	17	20	21
	3	X	21	22	20
		Y	24	20	15
	4	X	21	18	25
		Y	20	20	21
	5	X	16	20	23
		Y	22	22	17

Table 3.3 Holding cost of all outlets

		Item	Holding Cost
Outlet	1	X	0.10
		Y	0.15
	2	X	0.25
		Y	0.30
	3	X	0.07
		Y	0.10
	4	X	0.35
		Y	0.40
	5	X	0.14
		Y	0.17

Table 3.4 Replenishment quantities and Routes for example problem

		Period / Item	1	2	3
Outlet	1	X	20	19	20
		Y	19	20	19
	2	X	20	21	20
		Y	17	20	21
	3	X	63	0	0
		Y	59	0	0
	4	X	21	43	0
		Y	20	41	0
	5	X	16	43	0
		Y	22	39	0
Route		Route 1	A-1-4-3(X)-A	A-1-4-A	A-2-1-A
		Route 2	B-2-5-3(Y)-B	B-2-5-B	

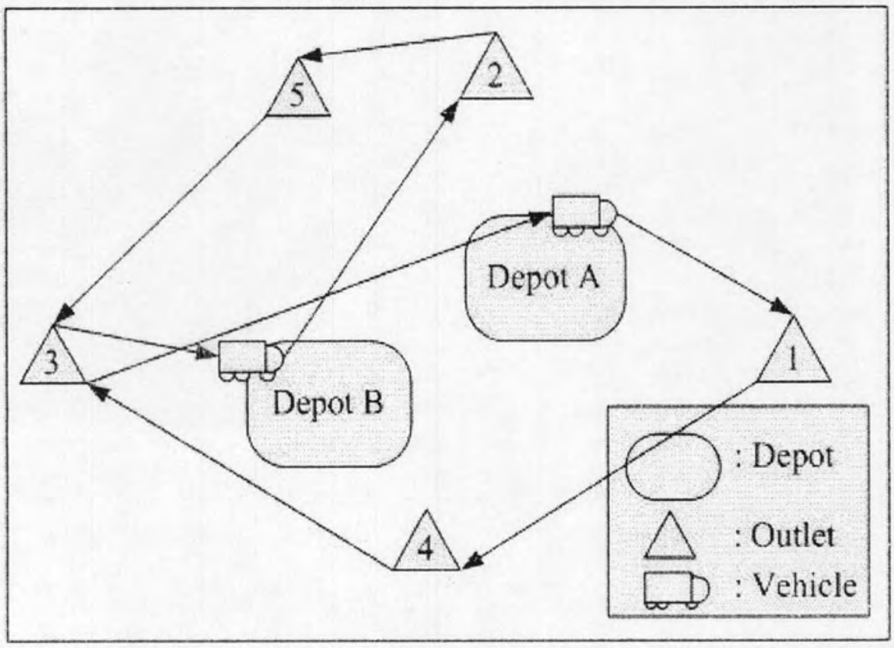

Figure 3.3 Illustration of delivery routes in the first period for example problem

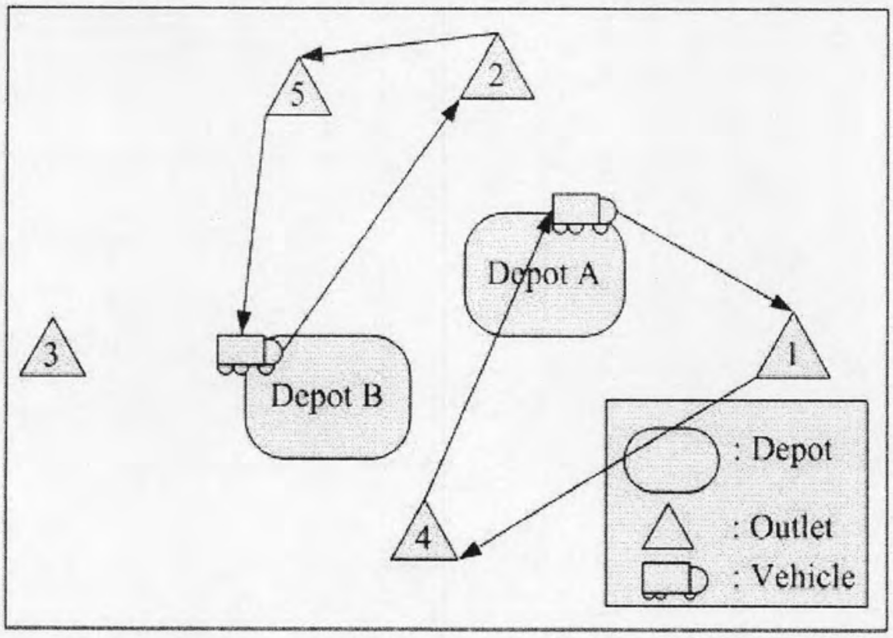

Figure 3.4 Illustration of delivery routes in the second period for example problem

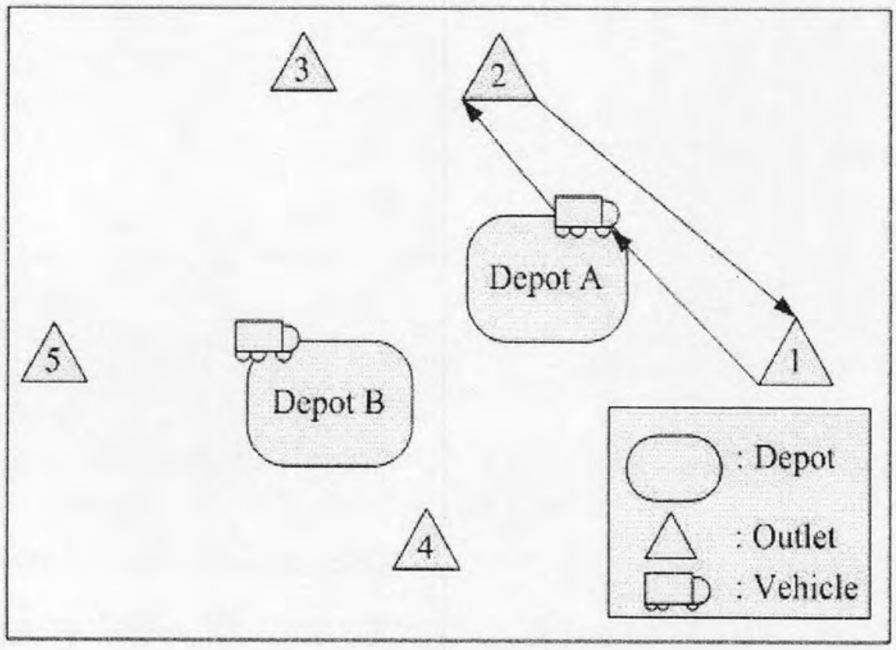

Figure 3.5 Illustration of delivery routes in the last period for example problem

3.4 Conclusion

This chapter gives a clear definition of the multi-item multi-depot inventory routing problem. This chapter begins with the problem description section that describes the characteristics of problem, the system which is studied in this dissertation. In addition, this chapter follows with assumption of the problem for more understanding in the problem in Section 3.2.1. In that section, a detail of characteristic of demand, inventory and delivery policies, and capacity is presented. These assumptions are derived from the cases in industry so that this presented problem can represent the real-world situation.

Furthermore, this chapter provides a mathematical model to minimize total inventory routing cost for the multi-item multi-depot inventory routing problem in section 3.2.2. It starts with the notation of the model which contains parameter of the system, vehicle, depot, outlet, and item-outlet. The mathematical model begins with an objective function which composes of three main costs: inventory holding, vehicle routing, and vehicle fix-charge cost. After that the constraints and their description are presented.

However, due to the complexity of the problem the exact solution technique is not suitable for medium to large-sized problem. The computational time of exact solution technique can be very high and impractical for using in the real situation. This problem can be considered as NP-Hard with the degree at least as the vehicle routing problem. For the vehicle routing problem, almost all of this type of problem is usually solved by heuristic. To deal with this highly complex problem, it is important to develop an efficient heuristic algorithm to solve this problem within reasonable time. Hence, in the next chapter, the development of some heuristics to obtain a good solution in acceptable time will be proposed for a simplified problem.