REFERENCES

[1] Booth, G. L.: A note on Γ-nearrings, *Stud. Sci. Math. Hungarica* **23**, 471–475 (1988).

[2] Barnes, W. E.: On the Γ-rings of Nobusawa, *Pacific J. Math.* **18**(3), 411–422 (1966).

[3] Chinram, R.: *Semigroup theory*, Department of Mathematics, Faculty of Science, Prince of Songkla University, Songkla, 2006.

[4] Chinram, R.: A note on quasi-ideals in Γ-semirings, *IMF.* **3**(26), 1253–1259 (2008).

[5] Dutta, T. K. and Sardar, S. K.: On the operator semirings of a Γ-semiring, *SEA Bull. Math.* **26**, 203–213 (2002).

[6] Enderton, H. B.: *Elements of Set Theory*, San Diego: Academic Press, 1977.

[7] Glazek, K.: *A guide to the literature on semirings and their applications in mathematics and information science*, Dordrecht: Kluwer Academic Publishers, 2002.

[8] Hoorn, W. G. V. and Rootselaar, B. V.: Fundamental notions in the theory of seminearrings, *Compositio mathematic.* **18**, 65–78 (1967).

[9] Jun, Y. B., Sapanci, M. and Öztürk, M. A.: Fuzzy ideals in gamma nearrings, *Tr. J. of Math.* **22**, 449–459 (1998).

[10] Kyuno, S.: On prime gamma rings, *Pacific J. Math.* **75**, 185–190 (1978).

[11] Luh, J.: On the theory of simple Γ-rings, *Michigan Math. J.* **16**, 65–75 (1969).

[12] Nobusawa, N.: On a generalization of the ring theory, *Osaka J. Math.* **1**, 81-89 (1964).

[13] Pianskool, S., Sangwirotjanapat, S. and Tipyota, S.: Valuation Γ-semirings and valuation Γ-ideals, *Thai J. Math., Spec. Issue for Annual Meetings in Mathematics 2008*, 93-102 (2008).

[14] Ramamurthi, V. S.: Weakly regular rings, *Canad. Math. Bull.* **16**(3), 317–321 (1973).

[15] Rao, M. K.: Γ-semirings-I, *SEA Bull. Math.* **19**, 49–54 (1995).

[16] Satyanarayana, Bh. and Syam, K. P.: On fuzzy cosets of gamma nearrings, *Turk. J. Math.* **29**, 11–22 (2005).

[17] Shabir, M. and Ahmed, I.: Weakly regular seminearrings, *IEJA.* **2**, 114–126 (2007).

APPENDIX

APPENDIX

We follows substances from Elements of Set Theory [6] written by H.B.Enderton. First, we give necessary concept of set theory for the proof of Example 2.1.4. We know that $\aleph_0$ is the first cardinal number which is countable and $\aleph_1$ is the first cardinal number which is uncountable. Note that $\aleph_0$ and $\aleph_1$ are also ordinals. Furthermore, ON is the class of all ordinals, i.e., ON$=\{\alpha \mid \alpha$ is an ordinal$\}$.

For any ordinals α and β, the *ordinal sum* of α and β, written by $\alpha \oplus \beta$, is defined as follows:

$$\alpha \oplus 0 = \alpha$$

$$\alpha \oplus \gamma^+ = (\alpha \oplus \gamma)^+ \quad \text{where } \gamma \text{ is an ordinal}$$

$$\alpha \oplus \lambda = \bigcup\{\alpha \oplus \xi \mid \xi \in \lambda\} \quad \text{for a limit ordinal } \lambda,$$

and the *ordinal product* of α and β, written by $\alpha \otimes \beta$, is defined as follows:

$$\alpha \otimes 0 = 0$$

$$\alpha \otimes \gamma^+ = (\alpha \otimes \gamma) \oplus \alpha \quad \text{where } \gamma \text{ is an ordinal}$$

$$\alpha \otimes \lambda = \bigcup\{\alpha \otimes \xi \mid \xi \in \lambda\} \quad \text{for a limit ordinal } \lambda.$$

Besides, it can be proved that $0 \oplus \alpha = \alpha$, $0 \otimes \alpha = 0$ and $\alpha \otimes 1 = \alpha = 1 \otimes \alpha$ for any ordinal α.

Furthermore, the following properties hold for any ordinals α, β and γ:

the associative law for addition: $(\alpha \oplus \beta) \oplus \gamma = \alpha \oplus (\beta \oplus \gamma)$,

the associative law for multiplication: $(\alpha \otimes \beta) \otimes \gamma = \alpha \otimes (\beta \otimes \gamma)$,

the left distributive law: $\alpha \otimes (\beta \oplus \gamma) = (\alpha \otimes \beta) \oplus (\alpha \otimes \gamma)$.

Theorem (*Transfinite Induction Principle for* ON)

If C is a subclass of ON *such that for any ordinal α, $\alpha \subseteq C$ implies $\alpha \in C$, then $C =$* ON.

Proposition (*Example 2.1.4*)

(i) $(\aleph_1, \oplus)$ *is a noncommutative semigroup.*

(ii) $\aleph_1$ *is a left* $\aleph_0$*-seminearring under the mapping from* $\aleph_1 \times \aleph_0 \times \aleph_1$ *into* $\aleph_1$ *defined by* $(a, \alpha, b) \mapsto a \otimes \alpha \otimes b$ *for all* $a, b \in \aleph_1$ *and* $\alpha \in \aleph_0$.

Proof. (i) We initially show that $(\aleph_1, \oplus)$ is a semigroup. It suffices to show that $a \oplus b \in \aleph_1$ for all ordinals $a, b \in \aleph_1$ because of the associative law for addition. The Transfinite Induction Principle for ON is applied here.

Let $a \in \aleph_1$ and $C_a = \{b \in \mathrm{ON} \mid \text{ if } b < \aleph_1, \text{ then } a \oplus b < \aleph_1\}$ and $\delta \in \mathrm{ON}$ be such that $\delta \subseteq C_a$. If we can show that $\delta \in C_a$, then $C_a = \mathrm{ON}$. This means that for each $a, b \in \aleph_1$, $a \oplus b \in \aleph_1$ so that $(\aleph_1, \oplus)$ is a semigroup.

In fact, there are three kinds of ordinal numbers, 0 which is the least ordinal, successor ordinals and limit ordinals. Note that a is countable since $a \in \aleph_1$. We prove that $\delta \in C_a$ in three cases.

Case I: $\delta = 0$. It is clear that $0 \in C_a$ since $a \oplus 0 = a < \aleph_1$.

Case II: δ is a successor ordinal. Then $\delta = \gamma^+$ for some ordinal γ. Assume that $\delta < \aleph_1$. Since $\gamma < \gamma^+ < \aleph_1$, this implies that γ is countable. Then $a \oplus \delta = a \oplus \gamma^+ = (a \oplus \gamma)^+$ is also countable. Hence $a \oplus \delta < \aleph_1$.

Case III: δ is a limit ordinal. Assume that $\delta < \aleph_1$. If $\xi \in \delta$, then ξ is countable so that $a \oplus \xi$ is countable since a is countable. Hence $a \oplus \delta = \cup\{a \oplus \xi \mid \xi \in \delta\} < \aleph_1$.

From all cases, we can conclude that $\delta \in C_a$ as claimed.

In set theory, ω is the set of all natural numbers, i.e. $\omega = \mathbb{N} \cup \{0\}$, and ω is a limit ordinal. Moreover, $1 \oplus \omega = \omega \neq \omega \cup \{\omega\} = \omega^+ = \omega \oplus 1$. Consequently, the semigroup $(\aleph_1, \oplus)$ is noncommutative.

As a result, $(\aleph_1, \oplus)$ is a noncommutative semigroup.

(ii) We show that $\aleph_1$ is a left $\aleph_0$-seminearring. It is competent to show that $a \otimes \alpha \otimes b \in \aleph_1$ for any ordinals $a, b \in \aleph_1$ and $\alpha \in \aleph_0$ because the facts $(a\otimes\alpha\otimes b)\otimes\beta\otimes c = a\otimes\alpha\otimes(b\otimes\beta\otimes c)$ and $a\otimes\alpha\otimes(b\oplus c) = (a\otimes\alpha\otimes b)\oplus(a\otimes\alpha\otimes c)$ for any ordinals $a, b, c \in \aleph_1$ and $\alpha, \beta \in \aleph_0$ follow from the associative law for multiplication and the left distributive law. Similarly, we apply the Transfinite Induction Principle for ON again.

Let $a \in \aleph_1$, $\alpha \in \aleph_0$ and $C_{a,\alpha} = \{b \in \text{ON} \mid \text{ if } b < \aleph_1, \text{ then } a\otimes\alpha\otimes b < \aleph_1\}$ and $\delta \in \text{ON}$ be such that $\delta \subseteq C_{a,\alpha}$. Then a is countable and α is finite. Similarly, we separate the proof into three cases.

Case I: $\delta = 0$. Then $0 \in C_{a,\alpha}$ since $a \otimes \alpha \otimes 0 = 0 < \aleph_1$.

Case II: δ is a successor ordinal. Then $\delta = \gamma^+$ for some ordinal γ. Assume that $\delta < \aleph_1$. Since $\gamma < \gamma^+ < \aleph_1$, this implies that γ is countable. Then $a \otimes \alpha \otimes \delta = a \otimes \alpha \otimes \gamma^+ = (a \otimes \alpha) \otimes \gamma^+ = \big((a \otimes \alpha) \otimes \gamma\big) \oplus (a \otimes \alpha)$ is also countable. Thus $a \otimes \alpha \otimes \delta < \aleph_1$.

Case III: δ is a limit ordinal. Assume that $\delta < \aleph_1$. If $\xi \in \delta$, then ξ is countable so that $a \otimes \alpha \otimes \xi$ is countable because a is countable and α is finite. Thus $a \otimes \alpha \otimes \delta = \cup\{a \otimes \alpha \otimes \xi \mid \xi \in \delta\} < \aleph_1$.

We can conclude from all cases that $\delta \in C_{a,\alpha}$. Then $C_{a,\alpha} = \text{ON}$. This implies that $a \otimes \alpha \otimes b \in \aleph_1$ for any ordinals $a, b \in \aleph_1$ and $\alpha \in \aleph_0$.

As a result, $\aleph_1$ is a left $\aleph_0$-seminearring.

□

Furthermore, in an $\aleph_0$-seminearring $\aleph_1$, the right distributivity is not true since $(1 \oplus 1) \otimes \omega = 2 \otimes \omega = \omega \neq \omega \oplus \omega = (1 \otimes \omega) \oplus (1 \otimes \omega)$. Hence $\aleph_1$ is not an $\aleph_0$-semiring. Besides, $(\aleph_1, \oplus)$ is not a group since each ordinal does not have its inverse except 0. Thus $\aleph_1$ is not an $\aleph_0$-nearring. Therefore $\aleph_1$ is not an $\aleph_0$-ring.

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