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นางสาวอภิญญา ติรสุวรรณวาสี

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EDGE-ODD GRACEFUL LABELINGS OF PRISM-LIKE GRAPHS OF CYCLES

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A Thesis Submitted in Partial Fulfillment of the Requirements
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กราฟอย่างง่าย G ที่มี q เส้นเชื่อม เรียกว่า กราฟด้านคี่อย่างสวยงาม เมื่อมีฟังก์ชันหนึ่งต่อหนึ่ง f จากเส้นเชื่อมของกราฟไปทั่วถึงเซต $\{1, 3, 5, \dots, 2q-1\}$ และจุดยอดแต่ละจุดกำกับด้วยจำนวนที่เป็นผลรวมของค่าของฟังก์ชัน f บนเส้นเชื่อมทุกเส้นที่ตกกระทบกับจุดนั้นมอดุโล $2q$ โดยจำนวนที่กำกับจุดเหล่านั้นแตกต่างกันทั้งหมด ในวิทยานิพนธ์ฉบับนี้ได้ให้บทนิยามกราฟคล้ายปริซึมบางชนิด แล้วพิสูจน์ว่ากราฟเหล่านี้เป็นกราฟด้านคี่อย่างสวยงาม

ภาควิชา คณิตศาสตร์และวิทยาการคอมพิวเตอร์ ลายมือชื่อนิสิต ....

ลายมือชื่อ อ.ที่ปรึกษาหลัก ....

สาขาวิชา คณิตศาสตร์ประยุกต์และวิทยาการคณนา ลายมือชื่อ อ.ที่ปรึกษาร่วม ....

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A simple graph G with q edges is called an edge-odd graceful graph, if there is a bijection f from the edge set of the graph to $\{1,3,5,\dots,2q-1\}$ such that, when each vertex is assigned the sum of all values of the edges incident to it modulo $2q$, the resulting vertex labels are distinct. In this thesis, we define new prism-like graphs and prove that they are edge-odd graceful graphs.

Department : Mathematics and Computer Science Student's Signature ....

Advisor's Signature ....

Field of Study : Applied Mathematics and Computational Science Co-advisor's Signature ....

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CONTENTS

LIST OF FIGURES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Let G be a simple undirected graph with q edges. We let $V(G)$ and $E(G)$ denote the vertex set and the edge set of G, respectively. A Graph labeling is a function from either $V(G)$, $E(G)$ or $V(G) \cup E(G)$ to a set of integers. There are several types of graph labelings such as a graceful labeling, a harmonious labeling, and a magic-type labeling. Labeled graphs can be used as mathematical models for several situations. For example, in coding theory, we can use graph labeling to design missile guidance codes, good radar type codes and convolution codes with optimal autocorrelation properties. In local area networks between buildings, it might be useful to use the similar idea as in graph labeling to assign each user terminal a node label subject to the constraint that all connecting edges (communication links) receive distinct labels. Besides that, it can be applied widely in ambiguities in X-ray crystallography, communication network labeling, finite additive number theory and ruler problems, circuit layout, etc. [1].

In 1967, Rosa [2] gave the definition of a graceful labeling of G which is an injection f from $V(G)$ to the set $\{0, 1, 2, \ldots, q\}$ such that each edge xy is assigned label $|f(x) - f(y)|$ and the resulting edge labels are distinct. In 1991, Gnanojothi [2] introduced an odd-graceful concept for a graph. Later, in 2009, Solairaju and Chithra [6] reversed the concepts of those two previous vertex labelings. The new type of labeling is called an edge-odd graceful labeling. A graph G admits an *edge-odd graceful labeling* if there exists a bijection f from $E(G)$ to the set $\{1, 3, 5, \ldots, 2q - 1\}$ such that the induced mapping f^+ from $V(G)$ to the set $\{0, 1, 2, \ldots, 2q - 1\}$ given by

$$f^+(x) = \sum_{xy \in E(G)} f(xy) (\text{mod } 2q)$$

where the sum is taken over all vertices y adjacent to x and the vertex labels are distinct. A graph that admits an edge-odd graceful labeling is called an *edge-odd graceful graph*. Solairaju and Chithra [6] showed edge-odd graceful labelings of graphs related to paths. In 2013, Singhun [5] showed edge-odd graceful labeling of graphs related to cycles, $SF(n,m)$ for $n \geq 3$ and wheel graphs W_{n+1} for n is even. Recently, Wongpradit [7], showed edge-odd graceful labeling of graphs related to prisms, $\mathrm{Prism}(C_n)$ for $n \geq 3$ and shaft graphs, $\mathrm{Shaft}(n,1)$ for n odd integer and $n \geq 3$. Thus, we extend the idea of Wongpradit [7] to prism-liked graphs and try it in such a way that it becomes edge-odd graceful graph.

In Chapter 2, we give some preliminaries as a background knowledge as well as the definition of a prism-liked graph, namely $\mathrm{Prism}_k(C_n)$, for $n \geq 3$ and $k \geq 3$. In Chapter 3, we construct an algorithm for edge-labeling of $\mathrm{Prism}_3(C_n)$ and prove that $\mathrm{Prism}_3(C_n)$ is edge-odd graceful for $n \geq 3$. In Chapter 4, we construct four algorithms for edge-labeling of $\mathrm{Prism}_k(C_3)$ and prove that $\mathrm{Prism}_k(C_3)$ is edge-odd graceful for $k \geq 3$. Finally, conclusion and discussion are given in the last Chapter.

CHAPTER II

PRELIMINARIES AND LITERATURE REVIEWS

The following are some definitions that we use throughout this thesis as well as a list of known results that motivates us to consider this type of problems.

Definition 2.1 [4] A *simple graph* $G = (V, E)$ consists of V, a nonempty set of vertices, and E, a set of unordered pairs of distinct elements of V called edges.

Example 2.1 A simple graph G with $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ and $E(G) = \{u_1u_2, u_2u_3, u_3u_4, u_1u_4, u_1u_5, u_2u_6, u_3u_7, u_4u_8\}$

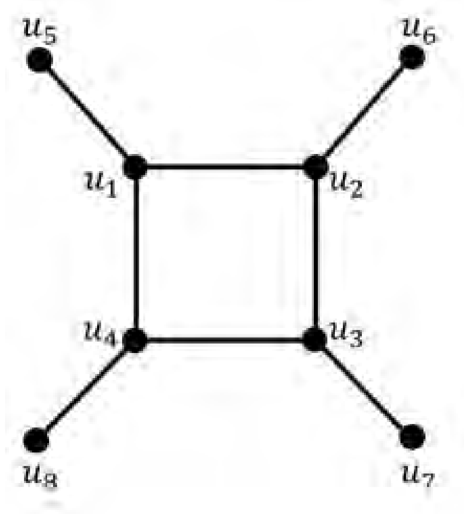

Figure 2.1. A simple graph G

Definition 2.2 [4] A *path graph*, P_n, is a simple graph whose n-vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list.

Figure 2.2. The path P_4

Definition 2.3 [4] A *cycle* C_n, is a graph with an equal number of n vertices and n edges whose vertices can be placed around a circle so that two vertices are adjacent if and only if they appear consecutively along the cycle. In this thesis, we usually write a cycle C_n as $u_1u_2u_3 \cdots u_n$ and we name the vertices in the counterclockwise direction.

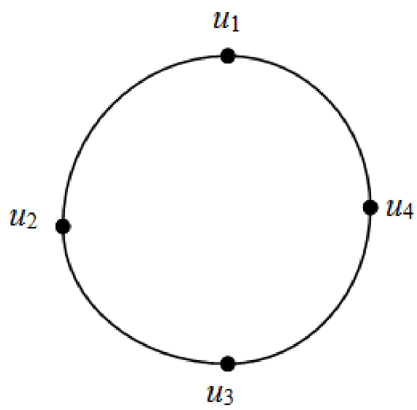

Figure 2.3. The cycle C_4

Definition 2.4 [4] The *Cartesian product* of G and H, written $G\square H$, is the graph with vertex set $V(G) \times V(H)$ specified by putting (u, v) adjacent to (u', v') if and only if (1) $u = u'$ and $vv' \in E(H)$, or (2) $v = v'$ and $uu' \in E(G)$.

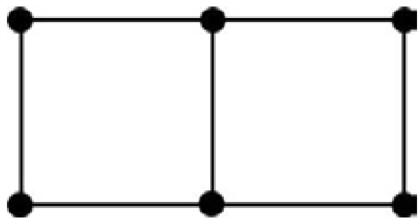

Figure 2.4. $P_2\square P_3$

Definition 2.5 [7] Let $n \geq 3$ and C_n be an n-cycle $u_1u_2u_3 \cdots u_n$. Let $C_n' = u_1'u_2'u_3' \cdots u_n'$ be a copy of C_n. Define $\text{Prism}(C_n)$, called the *prism of* C_n, by joining each corresponding vertices u_i of C_n to u_i' of C_n'. That is, the edges of $\text{Prism}(C_n)$

consists of $u_{i-1}u_i \in E(C_n)$, $u'_{i-1}u'_i \in E(C'_n)$ and $u_iu'_i$ bridges between C_n and C'_n. Thus,

$$E(\text{Prism}(C_n)) = E(C_n) \cup E(C'_n) \cup \{u_iu'_i \mid i \in \{1,2,3,\ldots,n\}\}.$$

Remark 2.1 The $\text{Prism}(C_n)$ can be viewed as $C_n \square P_2$, a Cartesian product of a cycle C_n and a path P_2.

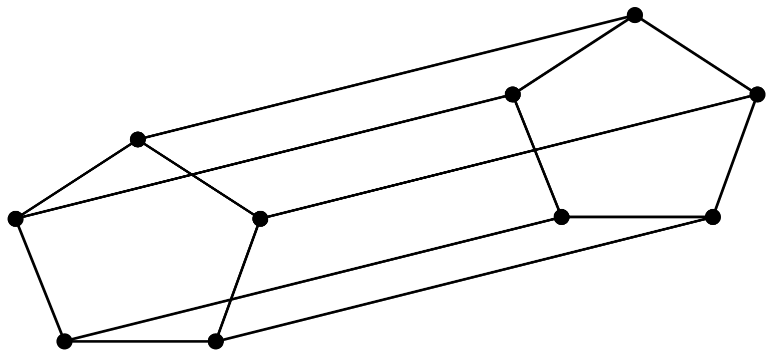

Figure 2.5. $\text{Prism}(C_5)$

Definition 2.5 leads us to define a prism-like graph as in the following definition and this class of graph is the main graph of our study.

Definition 2.6 Let $n \geq 3$ and C_n be an n-cycle $u_1u_2u_3 \cdots u_n$. For $j \in \{1,2,3,\ldots,k\}$, let $C_n^j = u_1^j u_2^j u_3^j \cdots u_n^j$ be the jth copy of C_n. For $k \geq 2$, define $\text{Prism}_k(C_n)$, called the *prism k of* C_n, by joining each corresponding vertices u_i^j of C_n^j to u_i^{j+1} of C_n^{j+1} for $i \in \{1,2,3,\ldots,n\}$ and $j \in \{1,2,3,\ldots,k-1\}$. That is, the edges of $\text{Prism}_k(C_n)$ consists of $u_{i-1}^j u_i^j \in E(C_n^j)$ and $u_i^j u_i^{j+1}$ bridges between corresponding points of C_n^j and C_n^{j+1}. Thus,

$$E(\text{Prism}_k(C_n))$$
$$= (\cup_{j=1}^{k} E(C_n^j)) \cup \{u_i^j u_i^{j+1} \mid i \in \{1,2,3,\ldots,n\}, j \in \{1,2,3,\ldots,k-1\}\}.$$

Remark 2.2 For $k = 2$, the $\mathrm{Prism}_k(C_n)$ is the $\mathrm{Prism}(C_n)$ as defined in definition 2.5 and for $k \geq 2$, the $\mathrm{Prism}_k(C_n)$ can be viewed as $C_n \square P_k$, a Cartesian product of a cycle C_n and a path P_k.

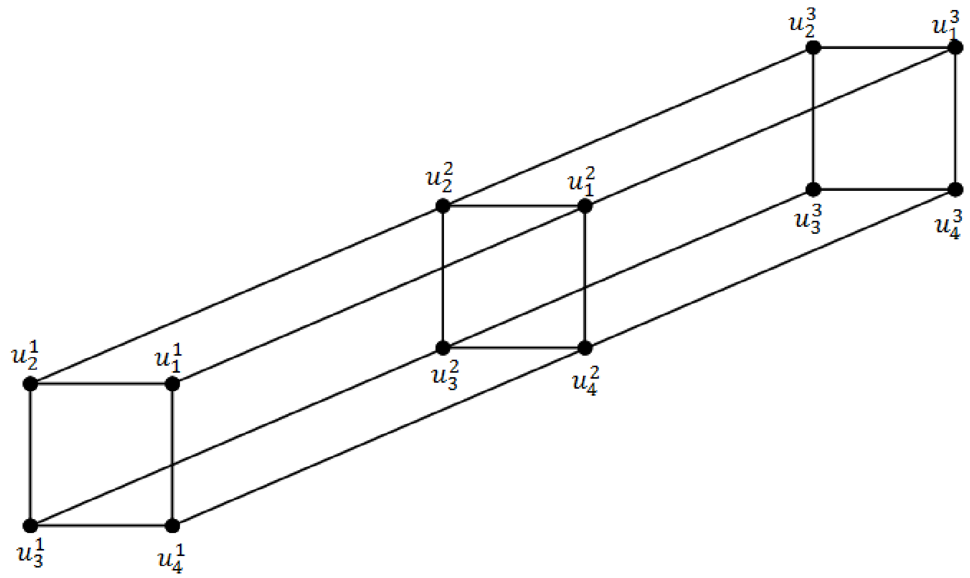

Figure 2.6. $\mathrm{Prism}_3(C_4)$

Definition 2.7 Let G be a simple graph. An *edge-odd graceful labeling* of G is a bijection f from $E(G)$ onto the set $\{1, 3, 5, \ldots, 2q - 1\}$ so that the induced mapping f^+ from $V(G)$ to the set $\{0, 1, 2, \ldots, 2q - 1\}$ given by $f^+(x) = \sum f(xy) \pmod{2q}$, where the sum is taken over all vertices y adjacent to x and the edge labels and vertex labels are distinct. A graph that admitted an edge-odd graceful labeling is called an *edge-odd graceful graph*.

The following shows some known results from literature that we collect as examples of edge-odd graceful graphs.

Theorem 2.1.[6] P_n^+ is an edge-odd graceful graph for all $n \geq 2$.

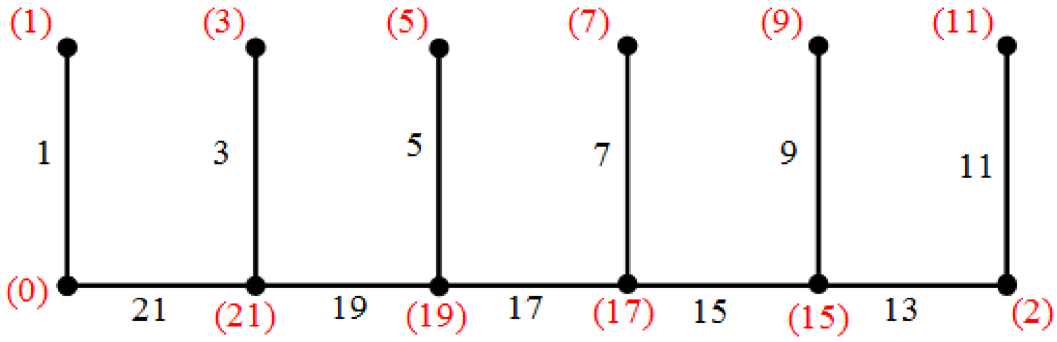

Figure 2.7. An edge-odd graceful labeling of P_6^+

Theorem 2.2.[6] $B_{n,n}$ is an edge-odd graceful graph if n is odd.

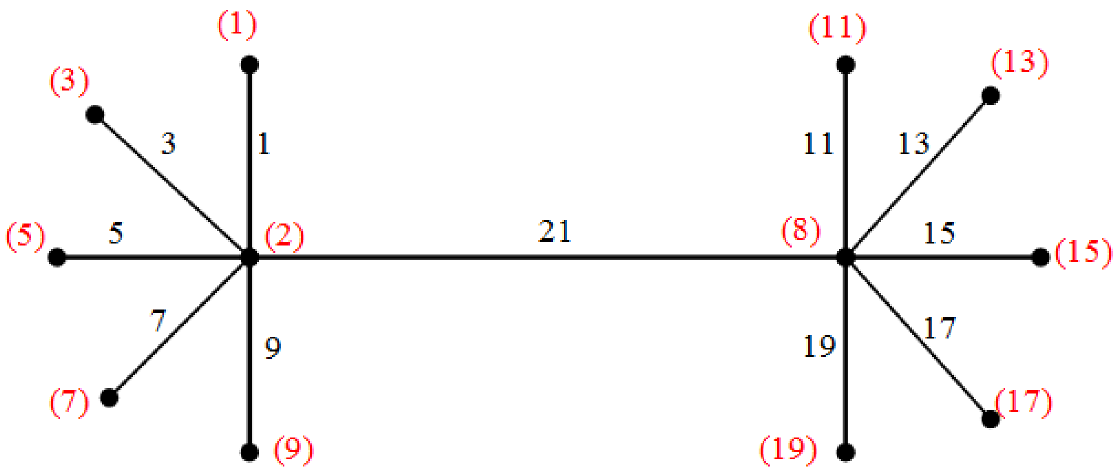

Figure 2.8. An edge-odd graceful labeling of $B_{5,5}$

Theorem 2.3.[6] $\langle K_{1,n}:2\rangle$ is an edge-odd graceful graph if n is odd.

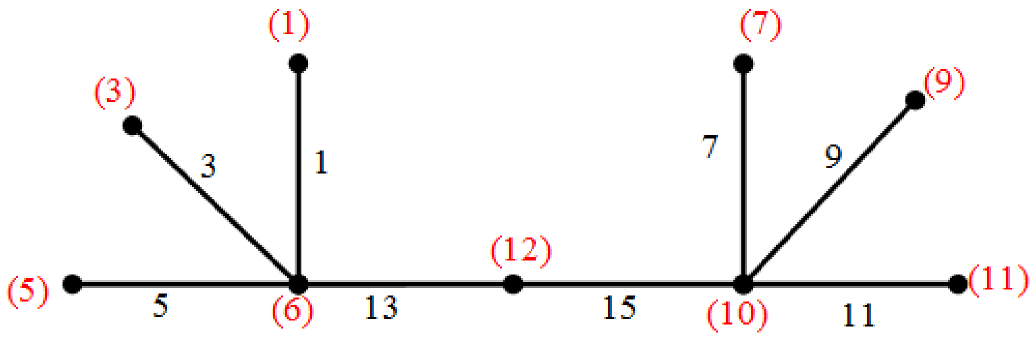

Figure 2.9. An edge-odd graceful labeling of $\langle K_{1,3}:2\rangle$

Theorem 2.4.[6] $K_{1,n,n}$ is an edge-odd graceful graph if n is even.

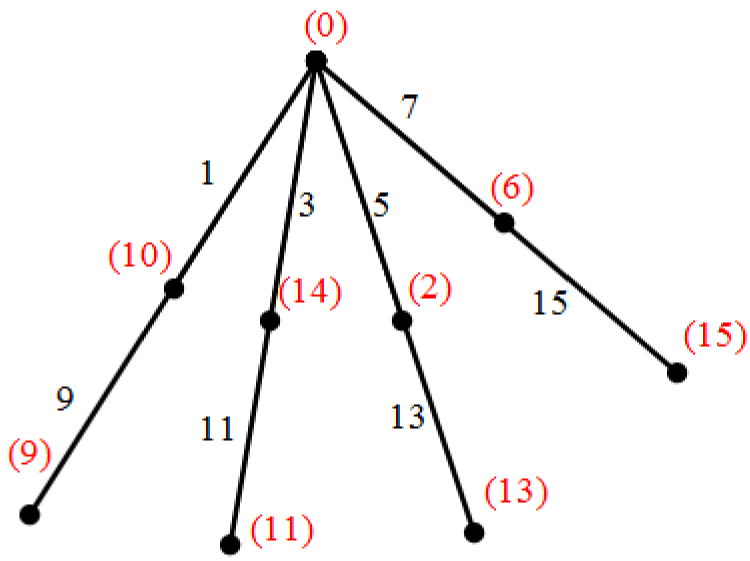

Figure 2.10. An edge-odd graceful labeling of $K_{1,4,4}$

Theorem 2.5.[5] $SF(n,1)$ is an edge-odd graceful graph for all n.

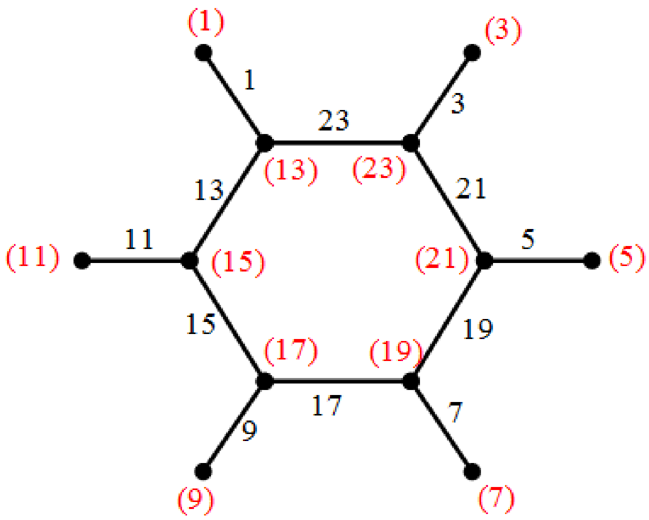

Figure 2.11. An edge-odd graceful labeling of $SF(6,1)$

Theorem 2.6.[5] $SF(n,m)$ is an edge-odd graceful graph when n is odd, m is even and $n|m$.

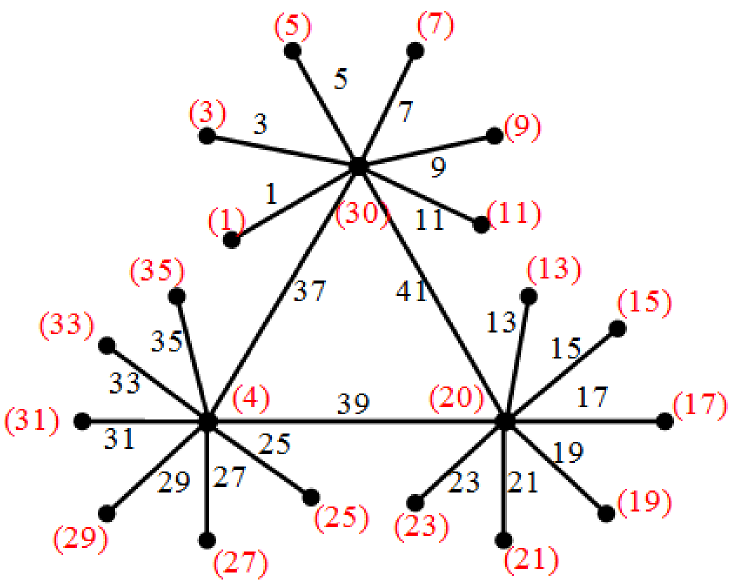

Figure 2.12. An edge-odd graceful labeling of $SF(3,6)$

Theorem 2.7.[5] W_{n+1} is an edge-odd graceful graph when n, the number of edges is even.

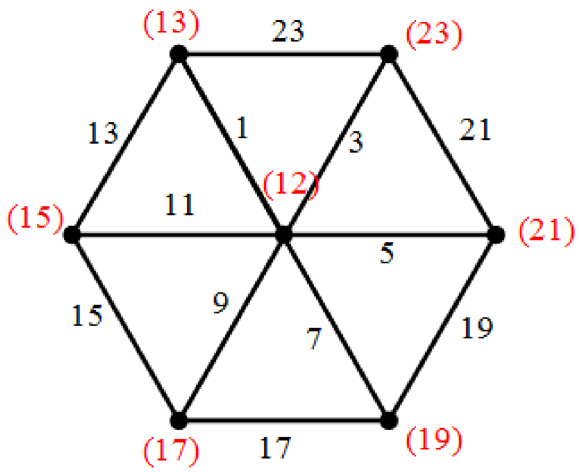

Figure 2.13. An edge-odd graceful labeling of W_7

Theorem 2.8.[7] $\mathrm{Prism}(C_n)$ is an edge-odd graceful graph whenever $n \geq 3$.

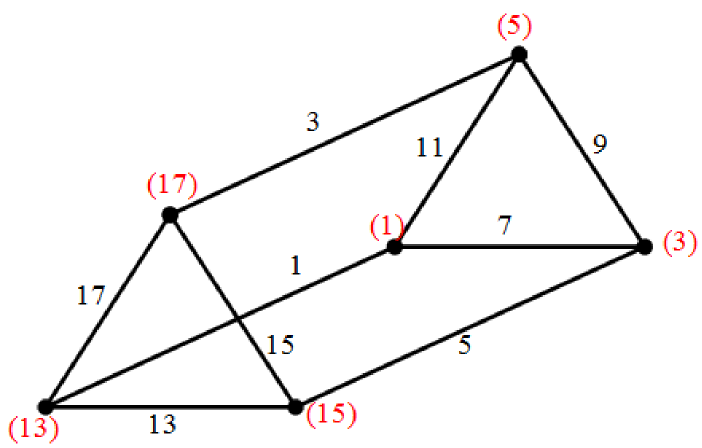

Figure 2.14. An edge-odd graceful labeling of $\mathrm{Prism}(C_3)$

Theorem 2.9.[7] $\mathrm{Shaft}(n, 1)$ is an edge-odd graceful graph whenever n is odd and $n \geq 3$.

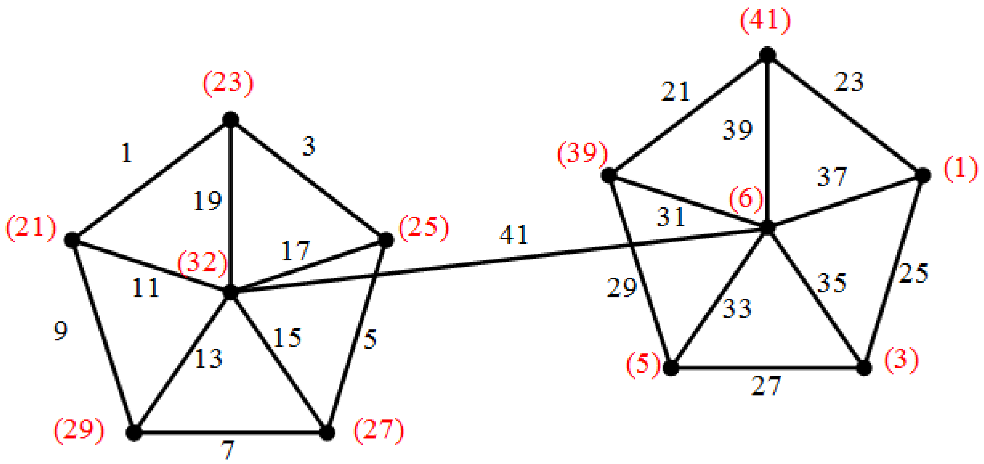

Figure 2.15. An edge-odd graceful labeling of $\mathrm{Shaft}(5,1)$

Theorem 2.10.[7] $\mathrm{XPrism}(C_n)$ is an edge-odd graceful graph for $n \geq 3$.

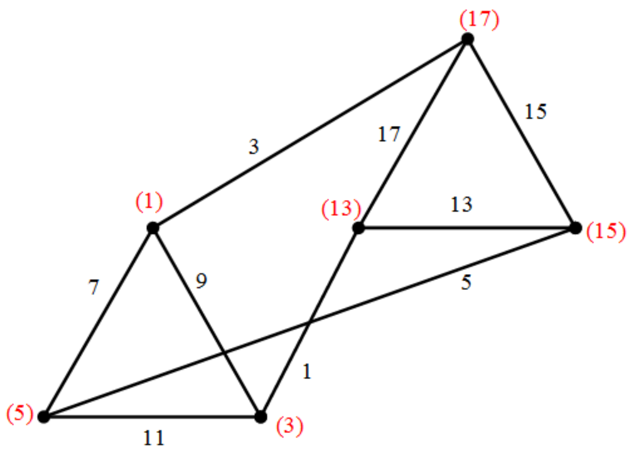

Figure 2.16. An edge-odd graceful labeling of $\mathrm{XPrism}(C_3)$

Theorem 2.11.[7] $\text{Prism}(S_n)$ is an edge-odd graceful graph for $n \geq 3$.

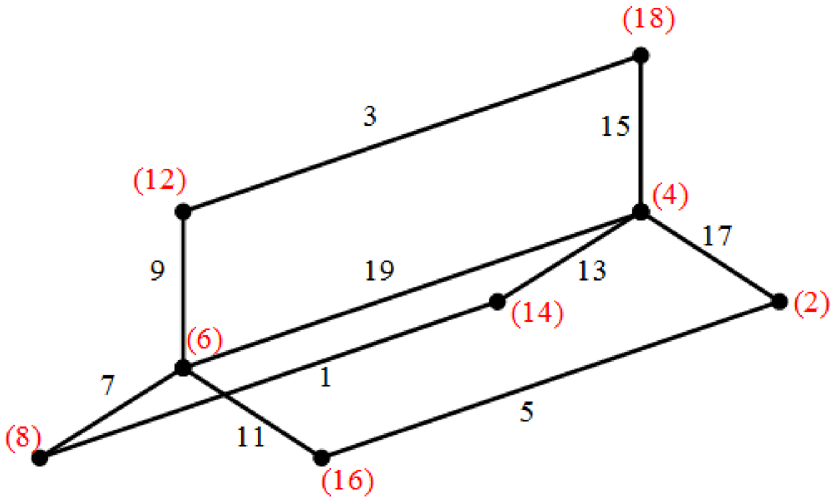

Figure 2.17. An edge-odd graceful labeling of $\text{Prism}(S_3)$

CHAPTER III

$\mathrm{Prism}_3(C_n)$

In this chapter, we generalize one of Wongpradit's results on $\mathrm{Prism}(C_n)$ by adding one more copy of a cycle C_n into $\mathrm{Prism}(C_n)$, that is, we consider $\mathrm{Prism}_3(C_n)$. First, for each integer $n \geq 3$, we give an algorithm to label each edge and then we can show that this algorithm induces an edge-odd graceful labeling for $\mathrm{Prism}_3(C_n)$.

Algorithm 3.1. let $k = 3$ and $n \geq 3$. Then, $q = \left|E\left(\mathrm{Prism}_3(C_n)\right)\right| = 5n$. Define $f: E\left(\mathrm{Prism}_3(C_n)\right) \to \{1, 3, 5, \dots, 10n - 1\}$ by

i. $f(u_1^1 u_1^2) = 1$,
ii. $f(u_1^2 u_1^3) = 3$,
iii. $f(u_{i+1}^1 u_{i+1}^2) = 4n - 4i + 1$, for $i \in \{1, 2, 3, \dots, n-1\}$,
iv. $f(u_{i+1}^2 u_{i+1}^3) = 4n - 4i + 3$, for $i \in \{1, 2, 3, \dots, n-1\}$,
v. $f(u_i^2 u_{i+1}^2) = 4n + 2i - 1$, for $i \in \{1, 2, 3, \dots, n-1\}$,
vi. $f(u_1^2 u_n^2) = 6n - 1$,
vii. $f(u_i^3 u_{i+1}^3) = 6n + 4i - 3$, for $i \in \{1, 2, 3, \dots, n-1\}$,
viii. $f(u_i^1 u_{i+1}^1) = 6n + 4i - 1$, for $i \in \{1, 2, 3, \dots, n-1\}$,
ix. $f(u_1^3 u_n^3) = 10n - 3$,
x. $f(u_1^1 u_n^1) = 10n - 1$.

Example 3.1 From Algorithm 3.1, we can label all edges of $\mathrm{Prism}_3(C_5)$ as shown in Figure 3.1.

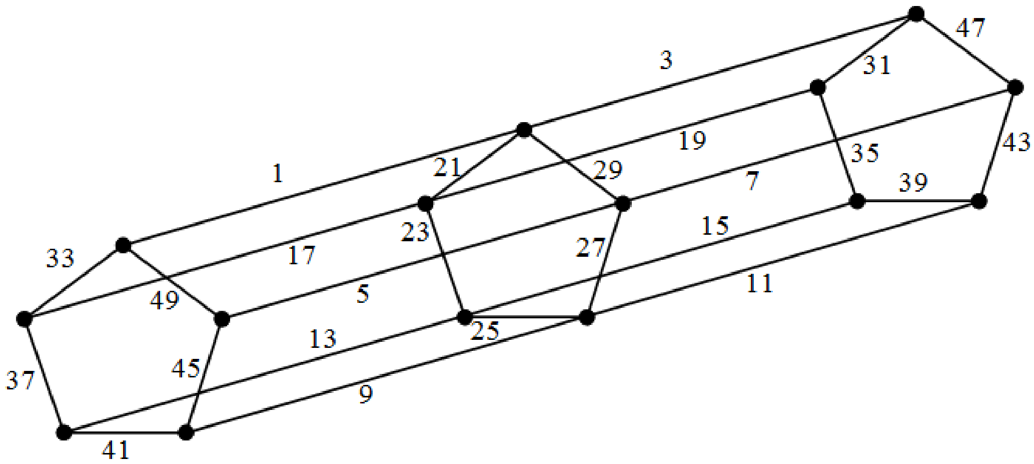

Figure 3.1. An edge-label for $\mathrm{Prism}_3(C_5)$

Theorem 3.1. *Let $n \geq 3$. The edge labeling of $Prism_3(C_n)$ given by Algorithm 3.1 is an edge-odd graceful labeling.*

Proof. To prove that f in Algorithm 3.1 is a bijection from $E(G)$ to $\{1, 3, 5, \dots, 10n - 1\}$, we consider the following. From Algorithm 3.1, we can see that there are $2n$ edges joining each copies of C_n labeled by a $2n$-element set $\{1, 3, 5, \dots, 4n - 1\}$, n edges of the second copy of C_n labeled by an n-element set $\{4n + 1, 4n + 3, 4n + 5, \dots, 6n - 1\}$, $2n$ edges of the first and the third copies of C_n labeled by a $2n$-element set $\{6n + 1, 6n + 3, 6n + 5, \dots, 10n - 1\}$. Thus, f defined in Algorithm 3.1 is a bijection from $E(G)$ to $\{1, 3, 5, \dots, 10n - 1\}$.

Next, from Algorithm 3.1, we have

$$\begin{aligned} f^+(u_1^1) &= \big(f(u_1^1u_n^1) + f(u_1^1u_2^1) + f(u_1^1u_1^2)\big)(\mathrm{mod}\ 10n) \\ &= (16n + 3)\ (\mathrm{mod}\ 10n) \\ &= 6n + 3; \\ f^+(u_n^1) &= \big(f(u_1^1u_n^1) + f(u_{n-1}^1u_n^1) + f(u_n^1u_n^2)\big)(\mathrm{mod}\ 10n) \\ &= (20n - 1)(\mathrm{mod}\ 10n) \\ &= 10n - 1; \\ f^+(u_i^1) &= (f(u_i^1u_i^2) + f(u_i^1u_{i+1}^1) + f(u_{i-1}^1u_i^1))\ (\mathrm{mod}\ 10n) \\ &= (16n + 4i - 1)(\mathrm{mod}\ 10n) \\ &= 6n + 4i - 1 \text{ for } i \in \{2, 3, 4, \dots, n - 1\}; \\ f^+(u_1^2) &= \big(f(u_1^2u_n^2) + f(u_1^2u_2^2) + f(u_1^1u_1^2) + f(u_1^2u_1^3)\big)(\mathrm{mod}\ 10n) \\ &= (10n + 4)(\mathrm{mod}\ 10n) = 4; \end{aligned}$$

$$\begin{aligned} f^+(u_n^2) &= \big(f(u_1^2u_n^2)+f(u_{n-1}^2u_n^2)+f(u_n^1u_n^2)+f(u_n^2u_n^3)\big)(\text{mod } 10n) \\ &= (12n+8)(\text{mod } 10n) = 2n+8; \\ f^+(u_i^2) &= \Big(f(u_{i-1}^2u_i^2)+f(u_i^2u_{i+1}^2)+f(u_i^1u_i^2)+f(u_i^2u_i^3)\Big)(\text{mod } 10n) \\ &= (16n-4i+8)(\text{mod } 10n) \\ &= 6n-4i+8 \text{ for } i \in \{2,3,4,\dots,n-1\}; \\ f^+(u_1^3) &= \big(f(u_1^3u_n^3)+f(u_1^3u_2^3)+f(u_1^2u_1^3)\big)(\text{mod } 10n) \\ &= (16n+1)(\text{mod } 10n) \\ &= 6n+1; \\ f^+(u_n^3) &= \big(f(u_1^3u_n^3)+f(u_{n-1}^3u_n^3)+f(u_n^2u_n^3)\big)(\text{mod } 10n) \\ &= (20n-3)(\text{mod } 10n) \\ &= 10n-3; \\ f^+(u_i^3) &= \Big(f(u_i^2u_i^3)+f(u_i^3u_{i+1}^3)+f(u_{i-1}^3u_i^3)\Big)(\text{mod } 10n) \\ &= (16n+4i-3)(\text{mod } 10n) \\ &= 6n+4i-3 \text{ for } i \in \{2,3,4,\dots,n-1\}. \end{aligned}$$

We can see that $f^+(u_1^1)$, $f^+(u_n^1)$, $f^+(u_1^2)$, $f^+(u_n^2)$, $f^+(u_1^3)$ and $f^+(u_n^3)$ are all distinct. Let $F_1 = \{f^+(u_1^1), f^+(u_n^1), f^+(u_1^2), f^+(u_n^2), f^+(u_1^3), f^+(u_n^3)\}$, $F_2 = \{6n+4i-1 \mid i \in \{2,3,4,\dots,n-1\}\}$, $F_3 = \{6n-4i+8 \mid i \in \{2,3,4,\dots,n-1\}\}$ and $F_4 = \{6n+4i-3 \mid i \in \{2,3,4,\dots,n-1\}\}$.

Since $\max F_2 = 10n-5$, $\max F_3 = 6n$, $\max F_4 = 10n-7$, $\min F_2 = 6n+7$, $\min F_3 = 2n+12$ and $\min F_4 = 6n+5$ and the numbers in F_2, F_3 and F_4 are consecutive, we can conclude that $F_1 \cap F_j = \emptyset$ for $j \in \{2,3,4\}$. Next, we notice that all elements in F_3 are even integers, while all elements in F_2 and F_4 are odd integers. Thus, $F_2 \cap F_3$ and $F_3 \cap F_4$ are empty. Finally, let $a \in F_2$ and $b \in F_4$, i.e., $a = 6n+4i-1$ and $b = 6n+4l-3$ for some $i, l \in \{2,3,4,\dots,n-1\}$. Assume that $a = b$. We have $4(l-i) = 2$ which leads to a contradiction because both i and l are integer. That is $F_2 \cap F_4 = \emptyset$.

Therefore, f defined by Algorithm 3.1 is an edge-odd graceful labeling for $\text{Prism}_3(C_n)$. □

Example 3.2. From the edge label in Example 3.1, the induced vertex label of $\text{Prism}_3(C_5)$ is shown below.

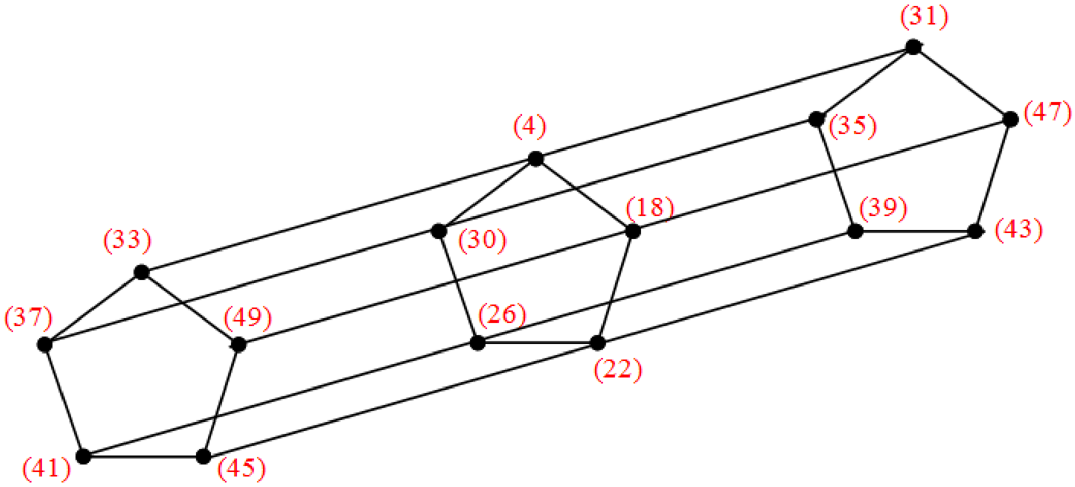

Figure 3.2. A vertex-label for $\mathrm{Prism}_3(C_5)$

CHAPTER IV

$\mathrm{Prism}_k(C_3)$

In this chapter, we generalize one of the Wongpradit's results on $\mathrm{Prism}(C_n)$ by considering $n = 3$ and adding finite copies of C_3 into $\mathrm{Prism}(C_3)$, that is, we consider $\mathrm{Prism}_k(C_3)$. First, for each integer $k \geq 3$, we divide into four cases. Each case, we give an algorithm to label each edge of $\mathrm{Prism}_k(C_3)$ and then we can show that the algorithm in that case induces the edge-odd graceful labeling for $\mathrm{Prism}_k(C_3)$. Throughout this chapter, we let $k \geq 3$.

Algorithm 4.1. Let $k \equiv 0 (\mathrm{mod}\ 4)$ and $n = 3$. Then, $q = \left|E\left(\mathrm{Prism}_k(C_3)\right)\right| = 6k - 3$. Define $f: E\left(\mathrm{Prism}_k(C_3)\right) \to \{1, 3, 5, \dots, 12k - 7\}$ by

i. $f\left(u_1^i u_1^{i+1}\right) = 2i - 1$, for $i \in \{1, 2, 3, \dots, k-1\}$,
ii. $f\left(u_2^i u_2^{i+1}\right) = 2k + 2i - 3$, for $i \in \{1, 2, 3, \dots, k-1\}$,
iii. $f\left(u_3^i u_3^{i+1}\right) = 4k + 2i - 5$, for $i \in \{1, 2, 3, \dots, k-1\}$,
iv. $f\left(u_1^i u_2^i\right) = 6k + 2i - 7$, for $i \in \{1, 2, 3, \dots, k\}$,
v. $f\left(u_2^i u_3^i\right) = 8k + 2i - 7$, for $i \in \{1, 2, 3, \dots, k\}$,
vi. $f\left(u_1^i u_3^i\right) = 10k + 2i - 7$, for $i \in \{1, 2, 3, \dots, k\}$.

Example 4.1. From Algorithm 4.1, we can label all edges of $\mathrm{Prism}_8(C_3)$ as shown in Figure 4.1.

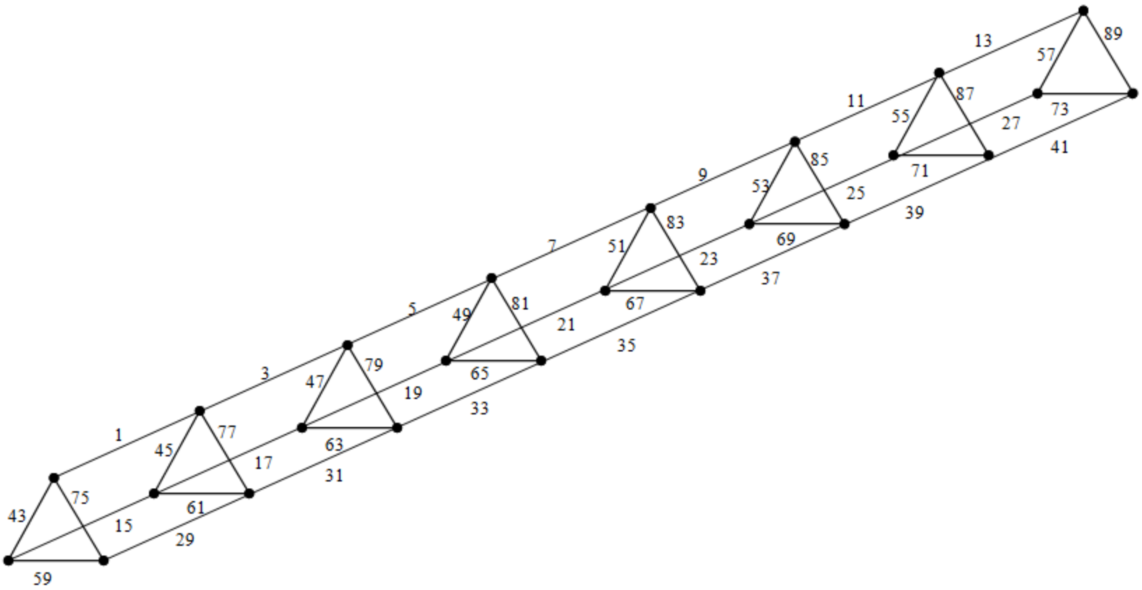

Figure 4.1. An edge-label for $\mathrm{Prism}_8(C_3)$

Theorem 4.1. Let $k \equiv 0 (\mathrm{mod}\, 4)$. The edge labeling of $\mathrm{Prism}_k(C_3)$ given by Algorithm 4.1 is an edge odd graceful labeling.

Proof. To prove that f in Algorithm 4.1 is a bijection from $E(G)$ to $\{1,3,5,\ldots,12k-7\}$, we consider the following. From Algorithm 4.1, we can see that $3k-3$ edges joining each copy of C_3 are labeled by a $3k-3$-element set $\{1,3,5,\ldots,6k-7\}$, k edges joining u_1^i and u_2^i of each copy of C_3 are labeled by a k-element set $\{6k-5,6k-3,6k-1,\ldots,8k-7\}$, k edges joining u_2^i and u_3^i of each copy of C_3 are labeled by a k-element set $\{8k-5,8k-3,8k-1,\ldots,10k-7\}$ and k edges joining u_1^i and u_3^i of each copy of C_3 are labeled by a k-element set $\{10k-5,$ $10k-3,10k-1,\ldots,12k-7\}$. Thus, f defined in Algorithm 4.1 is a bijection from $E(G)$ to $\{1,3,5,\ldots,12k-7\}$.

Next, from Algorithm 4.1, we have

$$\begin{aligned} f^+(u_1^1) &= \left(f(u_1^1u_1^2)+f(u_1^1u_2^1)+f(u_1^1u_3^1)\right) (\mathrm{mod}\ 12k-6) \\ &= (16k-9)(\mathrm{mod}\ 12k-6) = 4k-3; \\ f^+(u_2^1) &= \left(f(u_2^1u_2^2)+f(u_1^1u_2^1)+f(u_2^1u_3^1)\right)(\mathrm{mod}\ 12k-6) \\ &= (16k-11)(\mathrm{mod}\ 12k-6) = 4k-5; \\ f^+(u_3^1) &= \left(f(u_3^1u_3^2)+f(u_1^1u_3^1)+f(u_2^1u_3^1)\right)(\mathrm{mod}\ 12k-6) \\ &= (22k-13)(\mathrm{mod}\ 12k-6) = 10k-7; \end{aligned}$$

$$f^+(u_1^k) = \left(f(u_1^{k-1}u_1^k) + f(u_1^k u_2^k) + f(u_1^k u_3^k)\right) (\text{mod } 12k-6)$$
$$= (22k-17)(\text{mod } 12k-6) = 10k-11;$$
$$f^+(u_2^k) = \left(f(u_2^{k-1}u_2^k) + f(u_1^k u_2^k) + f(u_2^k u_3^k)\right) (\text{mod } 12k-6)$$
$$= (22k-19)(\text{mod } 12k-6) = 10k-13;$$
$$f^+(u_3^k) = \left(f(u_3^{k-1}u_3^k) + f(u_1^k u_3^k) + f(u_2^k u_3^k)\right) (\text{mod } 12k-6)$$
$$= (28k-21)(\text{mod } 12k-6) = 4k-9;$$
$$f^+(u_1^i) = \left(f(u_1^{i-1}u_1^i) + f(u_1^i u_1^{i+1}) + f(u_1^i u_2^i) + f(u_1^i u_3^i)\right) (\text{mod } 12k-6)$$
$$= (16k+8i-18)\ (\text{mod } 12k-6)$$
$$= 4k+8i-12 \text{ for } i \in \{2,3,4,\dots,k-1\};$$
$$f^+(u_2^i) = \left(f(u_2^{i-1}u_2^i) + f(u_2^i u_2^{i+1}) + f(u_1^i u_2^i) + f(u_2^i u_3^i)\right) (\text{mod } 12k-6)$$
$$= (18k+8i-22)(\text{mod } 12k-6)$$
$$= (6k+8i-16)(\text{mod } 12k-6) \text{ for } i \in \{2,3,4,\dots,k-1\};$$
$$f^+(u_3^i) = \left(f(u_3^{i-1}u_3^i) + f(u_3^i u_3^{i+1}) + f(u_1^i u_3^i) + f(u_2^i u_3^i)\right) (\text{mod } 12k-6)$$
$$= (26k+8i-26)(\text{mod } 12k-6)$$
$$= 2k+8i-14 \text{ for } i \in \{2,3,4,\dots,k-1\};$$

We can see that $f^+(u_1^1), f^+(u_2^1), f^+(u_3^1), f^+(u_1^k), f^+(u_2^k)$ and $f^+(u_3^k)$ are all distinct. Let $Q_1 = \{f^+(u_1^1), f^+(u_2^1), f^+(u_3^1), f^+(u_1^k), f^+(u_2^k), f^+(u_3^k)\}$, $Q_2 = \{4k+8i-12 \mid i \in \{2,3,4,\dots,k-1\}\}$, $Q_3 = \{(6k+8i-16)(\text{mod } 12k-6) \mid i \in \{2,3,4,\dots,k-1\}\}$ and $Q_4 = \{2k+8i-14 \mid i \in \{2,3,4,\dots,k-1\}\}$.

We notice that all elements in Q_1 are odd integers, while all elements in Q_2, Q_3 and Q_4 are even integers, we conclude that $Q_1 \cap Q_j = \emptyset$ for $j \in \{2,3,4\}$. Since $k \equiv 0\ (\text{mod } 4)$, $k = 4m$ for some $m \in \mathbb{N}$. Then,

$$Q_2 = \{4(4m)+8i-12 \mid i \in \{2,3,4,\dots,4m-1\}\}$$
$$= \{8(2m+i-2)+4 \mid i \in \{2,3,4,\dots,4m-1\}\} \text{ and}$$
$$Q_4 = \{2(4m)+8i-14 \mid i \in \{2,3,4,\dots,4m-1\}\}$$
$$= \{8(m+i-2)+2 \mid i \in \{2,3,4,\dots,4m-1\}\}.$$

Notice that elements in Q_2 and Q_4 are arithmetic progression with common difference 8, we also can see that each element in Q_2 is congruent to 4 modulo 8 and each element in Q_4 is congruent to 2 modulo 8.

Next, consider Q_3, then

$$Q_3 = \{(6(4m)+8i-16)(\text{mod } 12(4m)-6) \mid i \in \{2,3,4,\dots,4m-1\}\}$$
$$= \{(24m+8i-16)(\text{mod } 48m-6) \mid i \in \{2,3,4,\dots,4m-1\}\},$$

We can see that

$$Q_3 = \{(24m + 8i - 16) \mid i \in \{2, 3, 4, \dots, 3m + 1\}\} \cup \{(24m + 8i - 16)(\text{mod } 48m - 6) \mid i \in \{3m + 2, 3m + 3, 3m + 4, \dots, 4m - 1\}\}$$

$$Q_3 = \{24m, 24m + 8, 24m + 16, \dots, 48m - 24, 48m - 16, 48m - 8\} \cup \{6, 14, 22, \dots, 8m - 34, 8m - 26, 8m - 18\}$$

$$Q_3 = Q_{31} \cup Q_{32}.$$

Notice that elements in Q_{31} and Q_{32} are arithmetic progression with common difference 8, we also can see that each element in Q_{31} is congruent to 0 modulo 8 and each element in Q_{32} is congruent to 6 modulo 8.

Thus, Q_2, Q_3 and Q_4 are distinct. Therefore, f defined by Algorithm 4.1 is an edge-odd graceful labeling for $\text{Prism}_k(C_3)$, where $k \equiv 0 \pmod 4$. □

Example 4.2. From the edge label in Example 4.1, the induced vertex label of $\text{Prism}_8(C_3)$ is shown below.

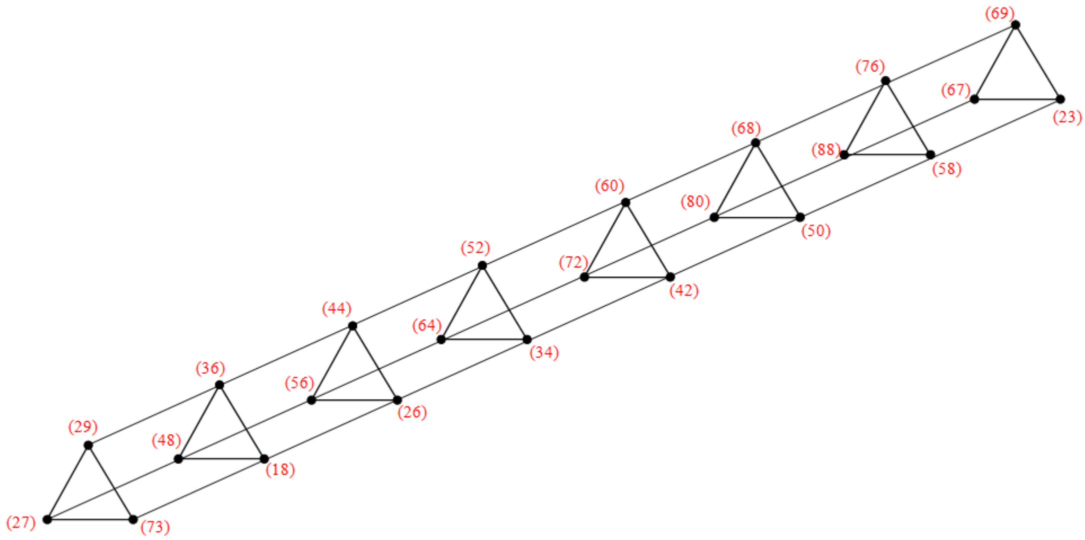

Figure 4.2. A vertex-label for $\text{Prism}_8(C_3)$

Algorithm 4.2. Let $k \equiv 1 \pmod 4$ and $n = 3$. Then, $q = |E(\text{Prism}_k(C_3))| = 6k - 3$. Define $f: E(\text{Prism}_k(C_3)) \to \{1, 3, 5, \dots, 12k - 7\}$ by

i. $f(u_1^i u_1^{i+1}) = 6i - 5$, for $i \in \{1, 2, 3, \dots, k - 1\}$,

ii. $f(u_2^i u_2^{i+1}) = 6i - 3$, for $i \in \{1, 2, 3, \dots, k - 1\}$,

iii. $f(u_3^i u_3^{i+1}) = 6i - 1$, for $i \in \{1, 2, 3, \dots, k - 1\}$,

iv. $f(u_1^i u_2^i) = 8k - 2i - 5$, for $i \in \{1, 2, 3, \dots, k\}$,

v. $f(u_2^i u_3^i) = 12k - 2i - 5$, for $i \in \{1, 2, 3, \dots, k\}$,

vi. $f(u_1^i u_3^i) = 10k - 2i - 5$, for $i \in \{1, 2, 3, \dots, k\}$.

Example 4.3. From Algorithm 4.2, we can label all edges of $\mathrm{Prism}_9(C_3)$ as shown in Figure 4.3.

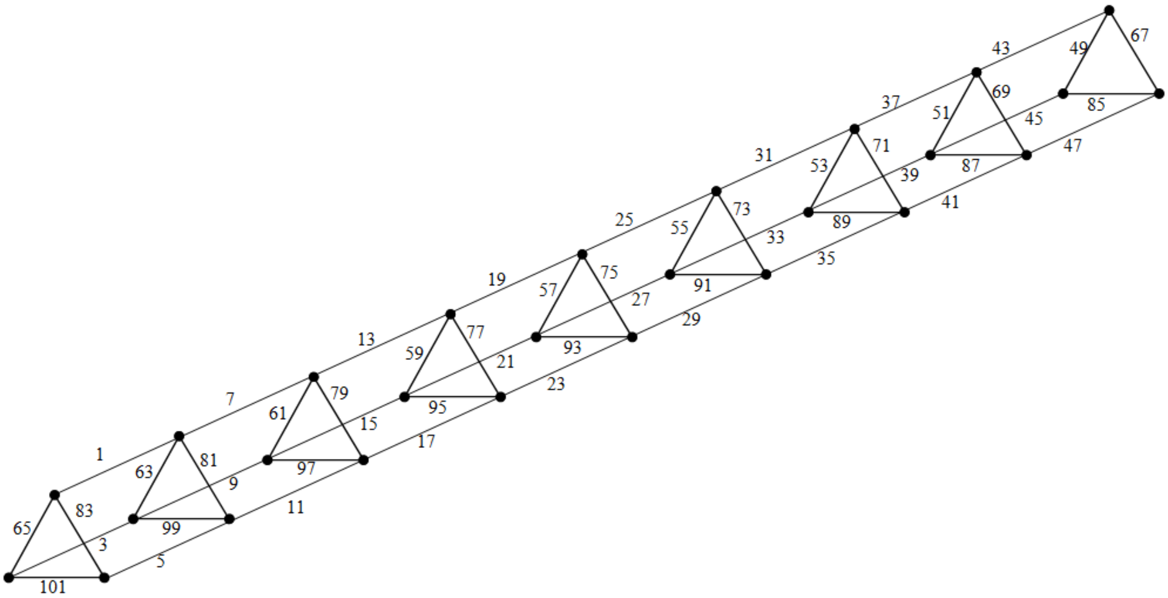

Figure 4.3. Edge-label for $\mathrm{Prism}_9(C_3)$

Theorem 4.2 Let $k \equiv 1 (\mathrm{mod}\ 4)$. The edge labeling of $\mathrm{Prism}_k(C_3)$ given by Algorithm 4.2 is an edge odd graceful labeling.

Proof. To prove that f in Algorithm 4.2 is a bijection from $E(G)$ to $\{1, 3, 5, \ldots, 12k - 7\}$, we consider the following. From Algorithm 4.2, we can see that $3k - 3$ edges joining each copy of C_3 are cyclicly labeled by a $3k - 3$-element set $\{1, 3, 5, \ldots, 6k - 7\}$, k edges joining u_1^i and u_2^i of each copy of C_3 are labeled by a k-element set $\{6k - 5, 6k - 3, 6k - 1, \ldots, 8k - 7\}$, k edges joining u_1^i and u_3^i of each copy of C_3 are labeled by a k-element set $\{8k - 5, 8k - 3, 8k - 1, \ldots, 10k - 7\}$ and k edges joining u_2^i and u_3^i of each copy of C_3 are labeled by a k-element set $\{10k - 5, 10k - 3, 10k - 1, \ldots, 12k - 7\}$. Thus, f defined in Algorithm 4.2 is a bijection from $E(G)$ to $\{1, 3, 5, \ldots, 12k - 7\}$.

Next, from Algorithm 4.2, we have

$$f^+(u_1^1) = \left(f(u_1^1 u_1^2) + f(u_1^1 u_2^1) + f(u_1^1 u_3^1)\right) (\mathrm{mod}\ 12k - 6)$$

$$
\begin{aligned}
&= (18k-13)(\text{mod } 12k-6) = 6k-7;\\
f^{+}(u_2^1) &= \left(f(u_2^1u_2^2)+f(u_1^1u_2^1)+f(u_2^1u_3^1)\right)(\text{mod } 12k-6)\\
&= (20k-11)(\text{mod } 12k-6) = 8k-5;\\
f^{+}(u_3^1) &= \left(f(u_3^1u_3^2)+f(u_1^1u_3^1)+f(u_2^1u_3^1)\right)(\text{mod } 12k-6)\\
&= (22k-9)(\text{mod } 12k-6) = 10k-3;\\
f^{+}(u_1^k) &= \left(f\left(u_1^{k-1}u_1^k\right)+f\left(u_1^ku_2^k\right)+f\left(u_1^ku_3^k\right)\right)(\text{mod } 12k-6)\\
&= (20k-21)(\text{mod } 12k-6) = 8k-15;\\
f^{+}(u_2^k) &= \left(f\left(u_2^{k-1}u_2^k\right)+f\left(u_1^ku_2^k\right)+f\left(u_2^ku_3^k\right)\right)(\text{mod } 12k-6)\\
&= (22k-19)(\text{mod } 12k-6) = 10k-13;\\
f^{+}(u_3^k) &= \left(f\left(u_3^{k-1}u_3^k\right)+f\left(u_1^ku_3^k\right)+f\left(u_2^ku_3^k\right)\right)(\text{mod } 12k-6)\\
&= (24k-17)(\text{mod } 12k-6) = 12k-11;\\
f^{+}(u_1^i) &= \left(f\left(u_1^{i-1}u_1^i\right)+f\left(u_1^iu_1^{i+1}\right)+f\left(u_1^iu_2^i\right)+f\left(u_1^iu_3^i\right)\right)(\text{mod } 12k-6)\\
&= (18k+8i-26)\ (\text{mod } 12k-6)\\
&= (6k+8i-20)(\text{mod } 12k-6) \text{ for } i\in\{2,3,4,\dots,k-1\};\\
f^{+}(u_2^i) &= \left(f\left(u_2^{i-1}u_2^i\right)+f\left(u_2^iu_2^{i+1}\right)+f\left(u_1^iu_2^i\right)+f\left(u_2^iu_3^i\right)\right)(\text{mod } 12k-6)\\
&= (20k+8i-22)(\text{mod } 12k-6)\\
&= (8k+8i-16)(\text{mod } 12k-6) \text{ for } i\in\{2,3,4,\dots,k-1\};\\
f^{+}(u_3^i) &= \left(f\left(u_3^{i-1}u_3^i\right)+f\left(u_3^iu_3^{i+1}\right)+f\left(u_1^iu_3^i\right)+f\left(u_2^iu_3^i\right)\right)(\text{mod } 12k-6)\\
&= (22k+8i-18)(\text{mod } 12k-6)\\
&= (10k+8i-12)(\text{mod } 12k-6) \text{ for } i\in\{2,3,4,\dots,k-1\};
\end{aligned}
$$

We can see that $f^+(u_1^1), f^+(u_2^1), f^+(u_3^1), f^+\left(u_1^k\right), f^+\left(u_2^k\right)$ and $f^+(u_3^k)$ are all distinct. Let $R_1 = \{f^+(u_1^1), f^+(u_2^1), f^+(u_3^1), f^+\left(u_1^k\right), f^+\left(u_2^k\right), f^+(u_3^k)\}$, $R_2 = \{(6k+8i-20)(\text{mod } 12k-6) \mid i\in\{2,3,4,\dots,k-1\}\}$, $R_3 = \{(8k+8i-16)(\text{mod } 12k-6) \mid i\in\{2,3,4,\dots,k-1\}\}$ and $R_4 = \{(10k+8i-12)(\text{mod } 12k-6) \mid i\in\{2,3,4,\dots,k-1\}\}$.

We notice that all elements in R_1 are odd integers, while all elements in R_2, R_3 and R_4 are even integers, we conclude that $R_1 \cap R_j = \emptyset$ for $j\in\{2,3,4\}$.

Since $k \equiv 1(\text{mod } 4)$, $k = 4m+1$ for some $m\in\mathbb{N}$.

If $m = 1$, $R_2 = \{26, 34, 42\}$, $R_3 = \{40, 48, 2\}$ and $R_4 = \{0, 8, 16\}$.

If $m = 2$, $R_2 = \{50, 58, 66, 74, 82, 90, 98\}$, $R_3 = \{72, 80, 88, 96, 2, 10, 18\}$ and

$R_4 = \{94, 0, 8, 16, 24, 32, 40\}$.

If $m \geq 3$, then

$$R_2 = \{(6(4m+1) + 8i - 20)(\text{mod } 12(4m+1) - 6) \mid i \in \{2, 3, 4, \dots, 4m\}\}$$
$$= \{24m + 8i - 14)(\text{mod } 48m + 6) \mid i \in \{2, 3, 4, \dots, 4m\}\},$$

we can see that

$$R_2 = \{24m + 8i - 14 \mid i \in \{2, 3, 4, \dots, 3m + 2\}\}$$
$$\cup \{(24m + 8i - 14)(\text{mod } 48\text{m} + 6) \mid i \in \{3m + 3, 3m + 4, 3m + 5, \dots, 4m\}$$
$$= \{24m + 2, 24m + 10, 24m + 18, \dots, 48m - 14, 48m - 6, 48m + 2\}$$
$$\cup \{4, 12, 20, \dots, 8m - 36, 8m - 28, 8m - 20\}$$
$$= R_{21} \cup R_{22}.$$

Notice that elements in R_{21} and R_{22} are arithmetic progression with common difference 8. We also can see that each element in R_{21} is congruent to 2 modulo 8 and each element in R_{22} is congruent to 4 modulo 8, $\min R_{21} = 24m + 2$, $\max R_{21} = 48m + 2$, $\min R_{22} = 4$ and $\max R_{22} = 8m - 20$.

Next, consider R_3, then

$$R_3 = \{(8(4m+1) + 8i - 16)(\text{mod } 12(4m+1) - 6) \mid i \in \{2, 3, 4, \dots, 4m\}\}$$
$$= \{(32m + 8i - 8)(\text{mod } 48m + 6) \mid i \in \{2, 3, 4, \dots, 4m\}\},$$

we can see that

$$R_3 = \{32m + 8i - 8 \mid i \in \{2, 3, 4, \dots, 2m + 1\}\}$$
$$\cup \{(32m + 8i - 8)(\text{mod } 48m + 6) \mid i \in \{2m + 2, 2m + 3, 2m + 4, \dots, 4m\}\}$$
$$= \{32m + 8, 32m + 16, 32m + 24, \dots, 48m - 16, 48m - 8, 48m\}$$
$$\cup \{2, 10, 18, \dots, 16m - 30, 16m - 22, 16m - 14\}$$
$$= R_{31} \cup R_{32}.$$

Notice that elements in R_{31} and R_{32} are arithmetic progression with common difference 8. We also can see that each element in R_{31} is congruent to 0 modulo 8 and each element in R_{32} is congruent to 2 modulo 8, $\min R_{31} = 32m + 8$, $\max R_{31} = 48m$, $\min R_{32} = 2$ and $\max R_{32} = 16m - 14$.

Finally,

$$R_4 = \{(10(4m+1) + 8i - 12)(\text{mod } 12(4m+1) - 6) \mid i \in \{2, 3, 4, \dots, 4m\}\}$$
$$= \{(40m + 8i - 2)(\text{mod } 48m + 6) \mid i \in \{2, 3, 4, \dots, 4m\}\},$$

we can see that

$$R_4 = \{40m + 8i - 2 \mid i \in \{2, 3, 4, \dots, m\}\}$$
$$\cup \{(40m + 8i - 2)(\text{mod } 48m + 6) \mid i \in \{m + 1, m + 2, m + 3, \dots, 4m\}\}$$
$$= \{40m + 14, 40m + 22, 40m + 30, \dots, 48m - 18, 48m - 10, 48m - 2\}$$
$$\cup \{0, 8, 16, \dots, 24m - 24, 24m - 16, 24m - 8\}$$
$$= R_{41} \cup R_{42}.$$

Notice that elements in R_{41} and R_{42} are arithmetic progression with common difference 8. We also can see that each element in R_{41} is congruent to 6 modulo 8 and each element in R_{42} is congruent to 0 modulo 8, $\min R_{41} = 40m + 14$, $\max R_{41} = 48m - 2$, $\min R_{42} = 0$ and $\max R_{42} = 24m - 8$.

Thus, for all $m \geq 1$, R_2, R_3 and R_4 are distinct. Therefore, f defined by Algorithm 4.2 is an edge-odd graceful labeling for $\text{Prism}_k(C_3)$, where $k \equiv 1 \pmod 4$.

□

Example 4.4. From the edge label in Example 4.3, the induced vertex label of $\text{Prism}_9(C_3)$ is shown below.

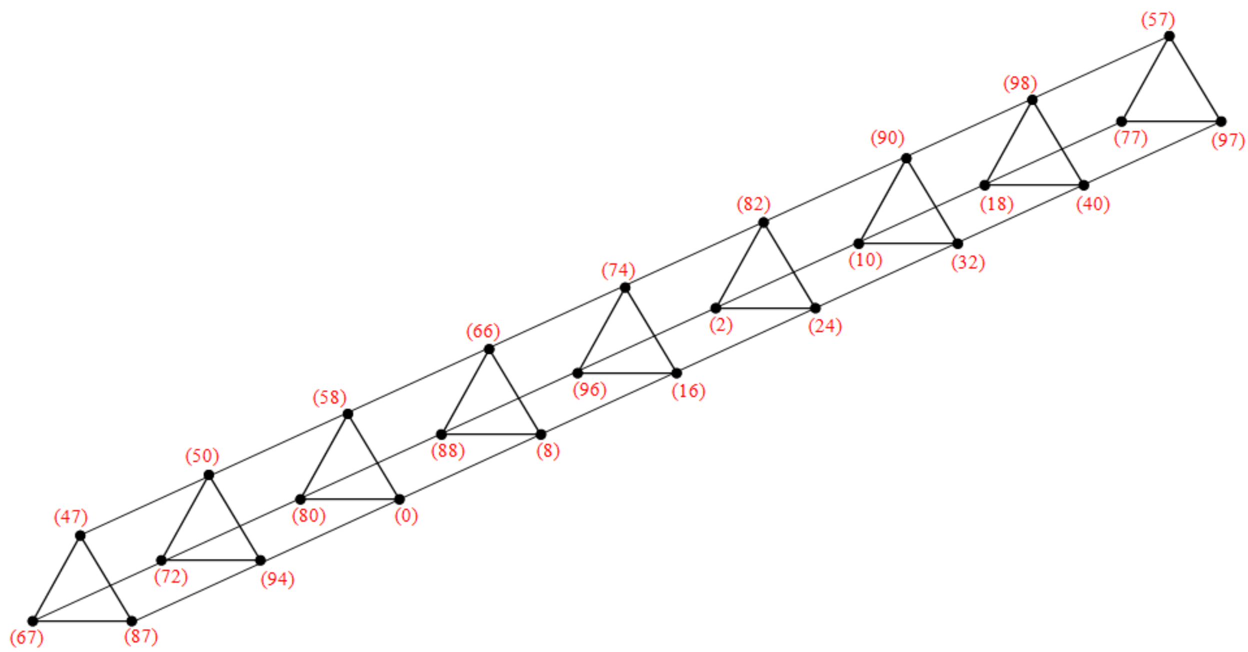

Figure 4.4. A vertex label for $\text{Prism}_9(C_3)$

Algorithm 4.3. Let $k \equiv 2 (\mathrm{mod}\ 4)$ and $n = 3$. Then, $q = |E(\mathrm{Prism}_k(C_3))| = 6k - 3$. Define $f: E(\mathrm{Prism}_k(C_3)) \to \{1, 3, 5, \ldots, 12k - 7\}$ by

i. $f(u_1^i u_1^{i+1}) = 6k + 2i - 1$, for $i \in \{1, 2, 3, \ldots, k - 1\}$,
ii. $f(u_2^i u_2^{i+1}) = 8k + 2i - 3$, for $i \in \{1, 2, 3, \ldots, k - 1\}$,
iii. $f(u_3^i u_3^{i+1}) = 10k + 2i - 5$, for $i \in \{1, 2, 3, \ldots, k - 1\}$,
iv. $f(u_1^i u_2^i) = 6i - 5$, for $i \in \{1, 2, 3, \ldots, k\}$,
v. $f(u_2^i u_3^i) = 6i - 1$, for $i \in \{1, 2, 3, \ldots, k\}$,
vi. $f(u_1^i u_3^i) = 6i - 3$, for $i \in \{1, 2, 3, \ldots, k\}$.

Example 4.5. From Algorithm 4.3, we can label all edges of $\mathrm{Prism}_6(C_3)$ as shown in Figure 4.5.

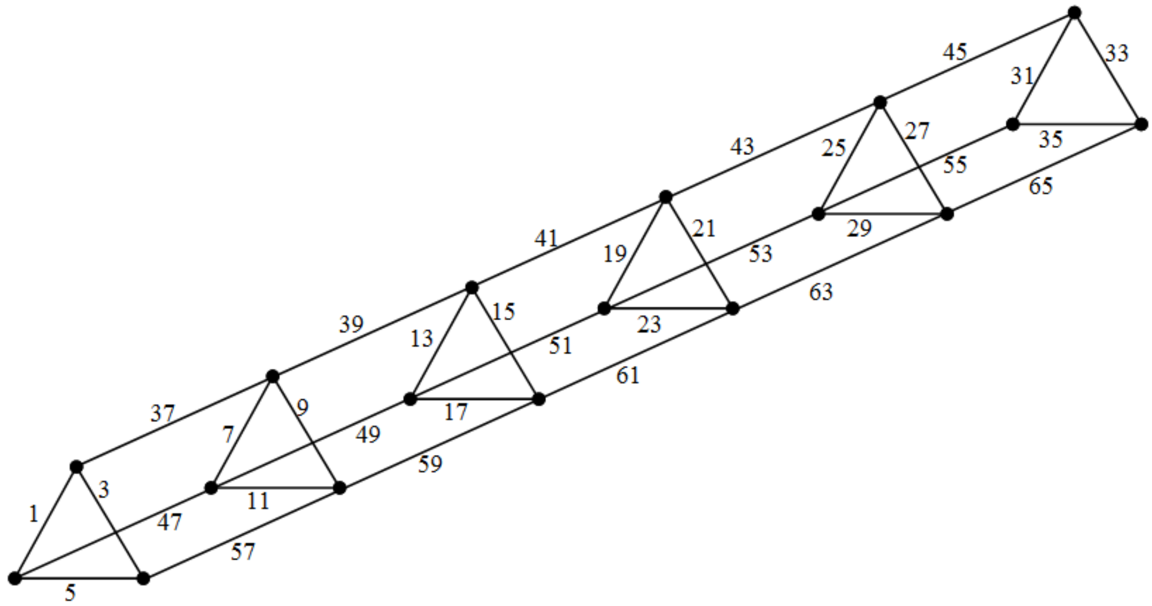

Figure 4.5. A vertex label for $\mathrm{Prism}_6(C_3)$

Theorem 4.3. Let $k \equiv 2 (\mathrm{mod}\ 4)$. The edge labeling of $\mathrm{Prism}_k(C_3)$ given by Algorithm 4.3 is an edge odd graceful labeling.

Proof. To prove that f in Algorithm 4.3 is a bijection from $E(G)$ to $\{1, 3, 5, \ldots, 12k - 7\}$, we consider the following. From Algorithm 4.3, we can see that $3k$ edges of each copy of C_3 are cyclicly labeled by a $3k$-element set $\{1, 3, 5, \ldots, 6k-1\}$ and $3k-3$ edges joining each copy of C_3 are labeled by a $3k-3$-element set $\{6k+1, 6k+3, 6k+5, \ldots, 12k-7\}$. Thus, f defined in Algorithm 4.3 is a bijection from $E(G)$ to $\{1, 3, 5, \ldots, 12k-7\}$.

Next, from Algorithm 4.3, we have

$$\begin{aligned}
f^+(u_1^1) &= \big(f(u_1^1u_1^2) + f(u_1^1u_2^1) + f(u_1^1u_3^1)\big) \pmod{12k-6} \\
&= (6k+5) \pmod{12k-6} = 6k+5; \\
f^+(u_2^1) &= \big(f(u_2^1u_2^2) + f(u_1^1u_2^1) + f(u_2^1u_3^1)\big) \pmod{12k-6} \\
&= (8k+5) \pmod{12k-6} = 8k+5; \\
f^+(u_3^1) &= \big(f(u_3^1u_3^2) + f(u_1^1u_3^1) + f(u_2^1u_3^1)\big) \pmod{12k-6} \\
&= (10k+5) \pmod{12k-6} = 10k+5; \\
f^+(u_1^k) &= \Big(f\big(u_1^{k-1}u_1^k\big) + f\big(u_1^ku_2^k\big) + f\big(u_1^ku_3^k\big)\Big) \pmod{12k-6} \\
&= (20k-11) \pmod{12k-6} = 8k-5; \\
f^+(u_2^k) &= \Big(f\big(u_2^{k-1}u_2^k\big) + f\big(u_1^ku_2^k\big) + f\big(u_2^ku_3^k\big)\Big) \pmod{12k-6} \\
&= (22k-11) \pmod{12k-6} = 10k-5; \\
f^+(u_3^k) &= \Big(f\big(u_3^{k-1}u_3^k\big) + f\big(u_1^ku_3^k\big) + f\big(u_2^ku_3^k\big)\Big) \pmod{12k-6} \\
&= (24k-11) \pmod{12k-6} = 1; \\
f^+(u_1^i) &= \Big(f\big(u_1^{i-1}u_1^i\big) + f\big(u_1^iu_1^{i+1}\big) + f\big(u_1^iu_2^i\big) + f\big(u_1^iu_3^i\big)\Big) \pmod{12k-6} \\
&= (12k+16i-12) \pmod{12k-6} \\
&= (16i-6) \pmod{12k-6} \text{ for } i \in \{2, 3, 4, \ldots, k-1\}; \\
f^+(u_2^i) &= \Big(f\big(u_2^{i-1}u_2^i\big) + f\big(u_2^iu_2^{i+1}\big) + f\big(u_1^iu_2^i\big) + f\big(u_2^iu_3^i\big)\Big) \pmod{12k-6} \\
&= (20k+8i-22) \pmod{12k-6} \\
&= (4k+16i-8) \pmod{12k-6} \text{ for } i \in \{2, 3, 4, \ldots, k-1\}; \\
f^+(u_3^i) &= \Big(f\big(u_3^{i-1}u_3^i\big) + f\big(u_3^iu_3^{i+1}\big) + f\big(u_1^iu_3^i\big) + f\big(u_2^iu_3^i\big)\Big) \pmod{12k-6} \\
&= (20k+16i-16) \pmod{12k-6} \\
&= (8k+16i-10) \pmod{12k-6} \text{ for } i \in \{2, 3, 4, \ldots, k-1\};
\end{aligned}$$

We can see that $f^+(u_1^1), f^+(u_2^1), f^+(u_3^1), f^+(u_1^k), f^+(u_2^k)$ and $f^+(u_3^k)$ are all distinct. Let $S_1 = \{f^+(u_1^1), f^+(u_2^1), f^+(u_3^1), f^+(u_1^k), f^+(u_2^k), f^+(u_3^k)\}$, $S_2 = \{(16i-6) \pmod{12k-6} \mid i \in \{2, 3, 4, \ldots, k-1\}\}$, $S_3 = \{(4k+16i-8) \pmod{12k-6} \mid i \in \{2, 3, 4, \ldots, k-1\}\}$ and $S_4 = \{(8k+16i-10) \pmod{12k-6} \mid i \in \{2, 3, 4, \ldots, k-1\}\}$. We notice that all elements in S_1 are odd integers, while all elements in S_2, S_3 and S_4 are even integers, we conclude that $S_1 \cap S_j = \emptyset$ for $j \in \{2,3,4\}$.

Since $k \equiv 2(\text{mod } 4)$, $k = 4m + 2$ for some $m \in \mathbb{N}$.

If $m = 1$, $S_2 = \{26, 42, 58, 8\}$, $S_3 = \{48, 64, 14, 30\}$ and $S_4 = \{4, 20, 36, 52\}$.

If $m \geq 2$, then

$$\begin{aligned} S_2 &= \{(16i-6)(\text{mod } 12(4m+2)-6) \mid i \in \{2,3,4,\ldots,4m+1\}\} \\ &= \{(16i-6)(\text{mod } 48m+18) \mid i \in \{2,3,4,\ldots,4m+1\}\}, \end{aligned}$$

we can see that

$$\begin{aligned} S_2 &= \{16i-6 \mid i \in \{2,3,4,\ldots,3m+1\}\} \\ &\quad \cup \{(16i-6)(\text{mod } 48\text{m}+18) \mid i \in \{3m+2, 3m+3, 3m+ \\ &\quad\quad 4,\ldots,4m+1\} \\ &= \{26, 42, 58, \ldots, 48m-22, 48m-6, 48m+10\} \\ &\quad \cup \{8, 24, 40, \ldots, 16m-40, 16m-24, 16m-8\} \\ &= S_{21} \cup S_{22}. \end{aligned}$$

Notice that elements in S_{21} and S_{22} are arithmetic progression with common difference 8. We also can see that each element in S_{21} is congruent to 2 modulo 8 and each element in S_{22} is congruent to 0 modulo 8, $\min S_{21} = 26$, $\max S_{21} = 48m + 10$, $\min S_{22} = 8$ and $\max S_{21} = 16m - 8$.

Next, consider S_3, then

$$\begin{aligned} S_3 &= \{(4(4m+2)+16i-8)(\text{mod } 12(4m+2)-6) \mid i \in \{2,3,\ldots,4m+1\}\} \\ &= \{(16m+16i)(\text{mod } 48m+18) \mid i \in \{2,3,4,\ldots,4m+1\}\}, \end{aligned}$$

we can see that

$$\begin{aligned} S_3 &= \{16m+16i \mid i \in \{2,3,4,\ldots,2m+1\}\} \\ &\quad \cup \{(16m+16i)(\text{mod } 48m+18) \mid i \in \{2m+2, 2m+3, 2m+4, \ldots, \\ &\quad\quad 4m+1\}\} \\ &= \{16m+32, 16m+48, 16m+64, \ldots, 48m-16, 48m, 48m+16\} \\ &\quad \cup \{14, 30, 46, \ldots, 32m-34, 32m-18, 32m-2\} \\ &= S_{31} \cup S_{32} \end{aligned}$$

Notice that elements in S_{31} and S_{32} are arithmetic progression with common difference 8. We also can see that each element in S_{31} is congruent to 0 modulo 8 and each element in S_{32} is congruent to 6 modulo 8, $\min S_{31} = 16m + 32$, $\max S_{31} = 48m + 16$, $\min S_{32} = 14$ and $\max S_{32} = 32m - 2$.

Finally,

$$S_4 = \{(8(4m+2)+16i-10)(\text{mod } 12(4m+2)-6) \mid i \in \{2,3,4,\dots,4m+1\}\}$$
$$= \{(32m+16i+6)(\text{mod } 48m+18) \mid i \in \{2,3,4,\dots,4m+1\}\},$$

we can see that

$$S_4 = \{32m+16i+6 \mid i \in \{2,3,4,\dots,m\}\}$$
$$\cup \{(32m+16i+6)(\text{mod } 48m+18) \mid i \in \{m+1, m+2, m+3,\dots,4m+1\}\}$$
$$= \{32m+38, 32m+54, 32m+70, \dots, 48m-26, 48m-10, 48m+6\}$$
$$\cup \{4, 20, 36, \dots, 48m-28, 48m-12, 48m+4\}$$
$$= S_{41} \cup S_{42}.$$

Notice that elements in S_{41} and S_{42} are arithmetic progression with common difference 8. We also can see that each element in S_{41} is congruent to 6 modulo 8 and each element in S_{42} is congruent to 4 modulo 8, $\min S_{41} = 32m+38$, $\max S_{41} = 48m+6$, $\min S_{42} = 4$ and $\max S_{42} = 48m+4$.

Thus, for all $m \geq 1$, S_2, S_3 and S_4 are distinct. Therefore, f defined by Algorithm 4.3 is an edge-odd graceful labeling for $\text{Prism}_k(C_3)$, where $k \equiv 2 \pmod 4$.

□

Example 4.6. From the edge label in Example 4.5, the induced vertex label of $\text{Prism}_6(C_3)$ is shown below.

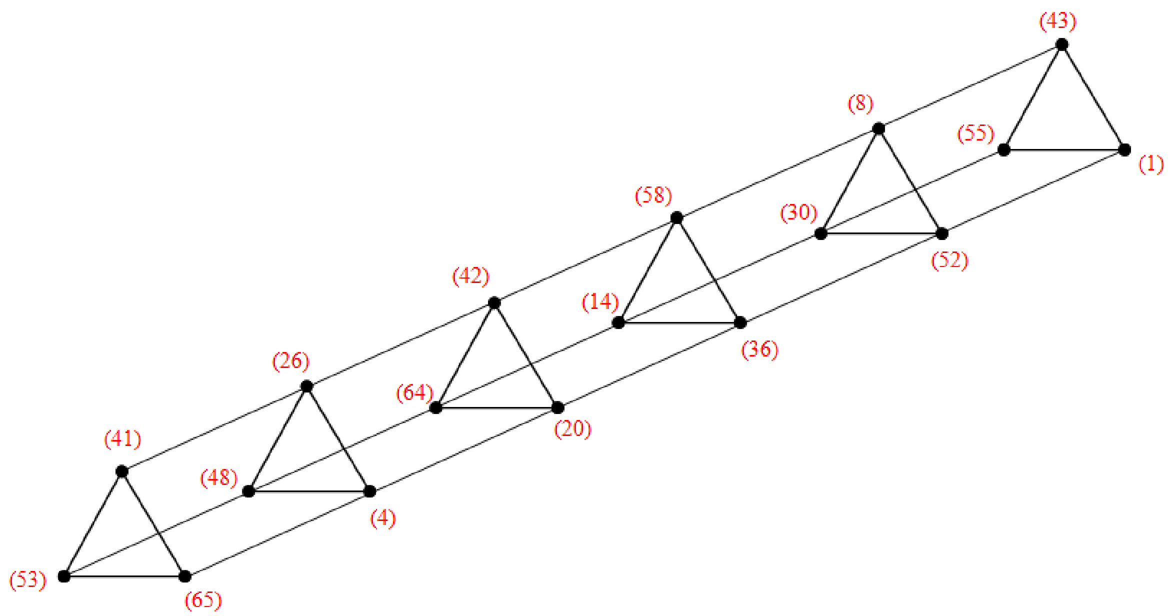

Figure 4.6. Vertex-label for $\text{Prism}_6(C_3)$

Algorithm 4.4. Let $k \equiv 3(\text{mod } 4)$ and $n = 3$. Then, $q = \left|E\left(\text{Prism}_k(C_3)\right)\right| = 6k - 3$. Define $f: E\left(\text{Prism}_k(C_3)\right) \to \{1, 3, 5, \dots, 12k - 7\}$ by

i. $f\left(u_1^i u_1^{i+1}\right) = 2i - 1$, for $i \in \{1, 2, 3, \dots, k - 1\}$,

ii. $f\left(u_2^i u_2^{i+1}\right) = 2k + 2i - 3$, for $i \in \{1, 2, 3, \dots, k - 1\}$,

iii. $f\left(u_3^i u_3^{i+1}\right) = 4k + 2i - 5$, for $i \in \{1, 2, 3, \dots, k - 1\}$,

iv. $f\left(u_1^i u_2^i\right) = 12k - 6i - 5$, for $i \in \{1, 2, 3, \dots, k\}$,

v. $f\left(u_2^i u_3^i\right) = 12k - 6i - 1$, for $i \in \{1, 2, 3, \dots, k\}$,

vi. $f\left(u_1^i u_3^i\right) = 12k - 6i - 3$, for $i \in \{1, 2, 3, \dots, k\}$.

Example 4.7. From Algorithm 4.4, we can label all edges of $\text{Prism}_7(C_3)$ as shown in Figure 4.7.

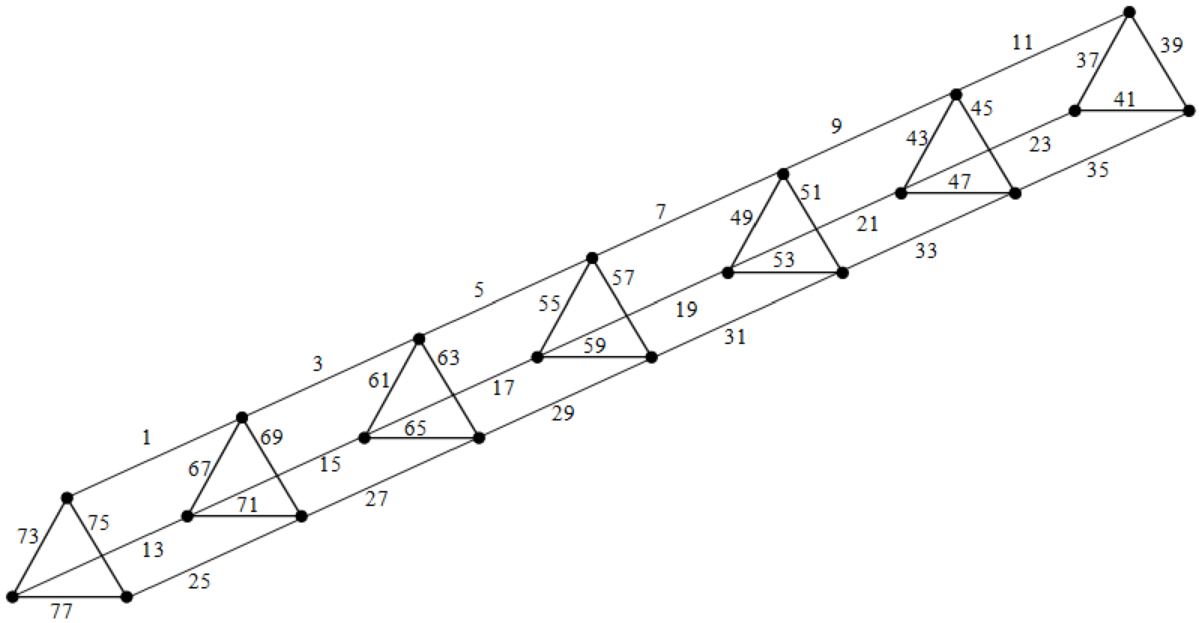

Figure 4.7. Edge-label for $\text{Prism}_7(C_3)$

Theorem 4.4 Let $k \equiv 3(\text{mod } 4)$. The edge labeling of $\text{Prism}_k(C_3)$ given by Algorithm 4.4 is an edge-odd graceful labeling.

Proof. To prove that f in Algorithm 4.4 is a bijection from $E(G)$ to $\{1, 3, 5, \dots, 12k - 7\}$, we consider the following. From Algorithm 4.4, we can see that $3k - 3$ edges joining each copy of C_3 are cyclicly labeled by a $3k-3$-element set $\{1,3,5,\dots,6k-7\}$ and $3k$ edges of each copy of C_3 are cyclicly labeled by a $3k$-element set $\{6k-5,6k-3,6k-1,\dots,12k-7\}$. Thus, f defined in Algorithm 4.4 is a bijection from $E(G)$ to $\{1,3,5,\dots,12k-7\}$.

Next, from Algorithm 4.4, we have

$$\begin{aligned}
f^+(u_1^1) &= \left(f(u_1^1u_1^2)+f(u_1^1u_2^1)+f(u_1^1u_3^1)\right)(\text{mod } 12k-6)\\
&= (24k-19)(\text{mod } 12k-6) = 12k-13;\\
f^+(u_2^1) &= \left(f(u_2^1u_2^2)+f(u_1^1u_2^1)+f(u_2^1u_3^1)\right)(\text{mod } 12k-6)\\
&= (26k-19)(\text{mod } 12k-6) = 2k-7;\\
f^+(u_3^1) &= \left(f(u_3^1u_3^2)+f(u_1^1u_3^1)+f(u_2^1u_3^1)\right)(\text{mod } 12k-6)\\
&= (28k-19)(\text{mod } 12k-6) = 4k-7;\\
f^+(u_1^k) &= \left(f\left(u_1^{k-1}u_1^k\right)+f\left(u_1^ku_2^k\right)+f\left(u_1^ku_3^k\right)\right)(\text{mod } 12k-6)\\
&= (14k-11)(\text{mod } 12k-6) = 2k-5;\\
f^+(u_2^k) &= \left(f\left(u_2^{k-1}u_2^k\right)+f\left(u_1^ku_2^k\right)+f\left(u_2^ku_3^k\right)\right)(\text{mod } 12k-6)\\
&= (16k-11)(\text{mod } 12k-6) = 4k-5;\\
f^+(u_3^k) &= \left(f\left(u_3^{k-1}u_3^k\right)+f\left(u_1^ku_3^k\right)+f\left(u_2^ku_3^k\right)\right)(\text{mod } 12k-6)\\
&= (18k-11)(\text{mod } 12k-6) = 6k-5;\\
f^+(u_1^i) &= \left(f\left(u_1^{i-1}u_1^i\right)+f\left(u_1^iu_1^{i+1}\right)+f\left(u_1^iu_2^i\right)+f\left(u_1^iu_3^i\right)\right)(\text{mod } 12k-6)\\
&= (24k-8i-12)\ (\text{mod } 12k-6)\\
&= (12k-8i-6) \text{ for } i\in\{2,3,4,\dots,k-1\};\\
f^+(u_2^i) &= \left(f\left(u_2^{i-1}u_2^i\right)+f\left(u_2^iu_2^{i+1}\right)+f\left(u_1^iu_2^i\right)+f\left(u_2^iu_3^i\right)\right)(\text{mod } 12k-6)\\
&= (28k-8i-14)(\text{mod } 12k-6)\\
&= (16k-8i-8)(\text{mod } 12k-6) \text{ for } i\in\{2,3,4,\dots,k-1\};\\
f^+(u_3^i) &= \left(f\left(u_3^{i-1}u_3^i\right)+f\left(u_3^iu_3^{i+1}\right)+f\left(u_1^iu_3^i\right)+f\left(u_2^iu_3^i\right)\right)(\text{mod } 12k-6)\\
&= (32k-8i-16)(\text{mod } 12k-6)\\
&= (8k-8i-4) \text{ for } i\in\{2,3,4,\dots,k-1\};
\end{aligned}$$

We can see that $f^+(u_1^1), f^+(u_2^1), f^+(u_3^1), f^+\left(u_1^k\right), f^+\left(u_2^k\right)$ and $f^+(u_3^k)$ are all distinct. Let $T_1=\{f^+(u_1^1), f^+(u_2^1), f^+(u_3^1), f^+\left(u_1^k\right), f^+\left(u_2^k\right), f^+(u_3^k)\}$, $T_2=\{(12k-8i-6)\mid i\in\{2,3,4,\dots,k-1\}\}$, $T_3=\{(16k-8i-8)(\text{mod } 12k-6)\mid i\in\{2,3,4,\dots,k-1\}\}$ and $T_4=\{(8k-8i-4)\mid i\in\{2,3,4,\dots,k-1\}\}$. We notice th all elements in T_1 are odd integers, while all elements in T_2, T_3 and T_4 are even integers, we conclude that $T_1 \cap T_j = \emptyset$ for $j \in \{2,3,4\}$.

Since $k \equiv 3 \pmod 4$, $k = 4m + 3$ for some $m \in \mathbb{N}$. Then,

$$T_2 = \{12(4m+3) - 8i - 6 \mid i \in \{2,3,4,\dots,4m+2\}\}$$
$$= \{8(6m - i + 3) + 6 \mid i \in \{2,3,4,\dots,4m+2\}\} \text{ and}$$
$$T_4 = \{8(4m+3) - 8i - 4 \mid i \in \{2,3,4,\dots,4m+2\}\}$$
$$= \{8(4m - i + 2) + 4 \mid i \in \{2,3,4,\dots,4m+2\}\}.$$

Notice that elements in T_2 and T_4 are arithmetic progression with common difference 8. We also can see that each element in T_2 is congruent to 6 modulo 8 and each element in T_4 is congruent to 4 modulo 8.

Next, consider T_3, then

$$T_3 = \{(16(4m+3) - 8i - 8) (\text{mod } 12(4m+3) - 6) \mid i \in \{2,3,4,\dots,4m+2\}\}$$
$$= \{(64m - 8i + 40) (\text{mod } 48m + 30) \mid i \in \{2,3,4,\dots,4m+2\}\},$$

We can see that

$$T_3 = \{(64m - 8i + 40) (\text{mod } 48m + 30) \mid i \in \{2,3,4,\dots,2m+1\}\} \cup \{(64m - 8i + 40) \mid i \in \{2m+2, 2m+3, 2m+4, \dots, 4m+2\}\}$$
$$T_3 = \{16m - 6, 16m - 14, 16m - 22, \dots, 18, 10, 2\} \cup \{48m + 24, 48m + 16, 48m + 8, \dots, 32m + 40, 32m + 32, 32m + 24\}$$
$$T_3 = T_{31} \cup T_{32}.$$

Notice that elements in T_{31} and T_{32} are arithmetic progression with common difference 8. We also can see that each element in T_{31} is congruent to 2 modulo 8 and each element in T_{32} is congruent to 0 modulo 8.

Thus, T_2, T_3 and T_4 are distinct. Therefore, f defined by Algorithm 4.4 is an edge-odd graceful labeling for $\text{Prism}_k(C_3)$, where $k \equiv 3 \pmod 4$. □

Example 4.8. From the edge label in Example 4.7, the induced vertex label of $\text{Prism}_7(C_3)$ is shown below.

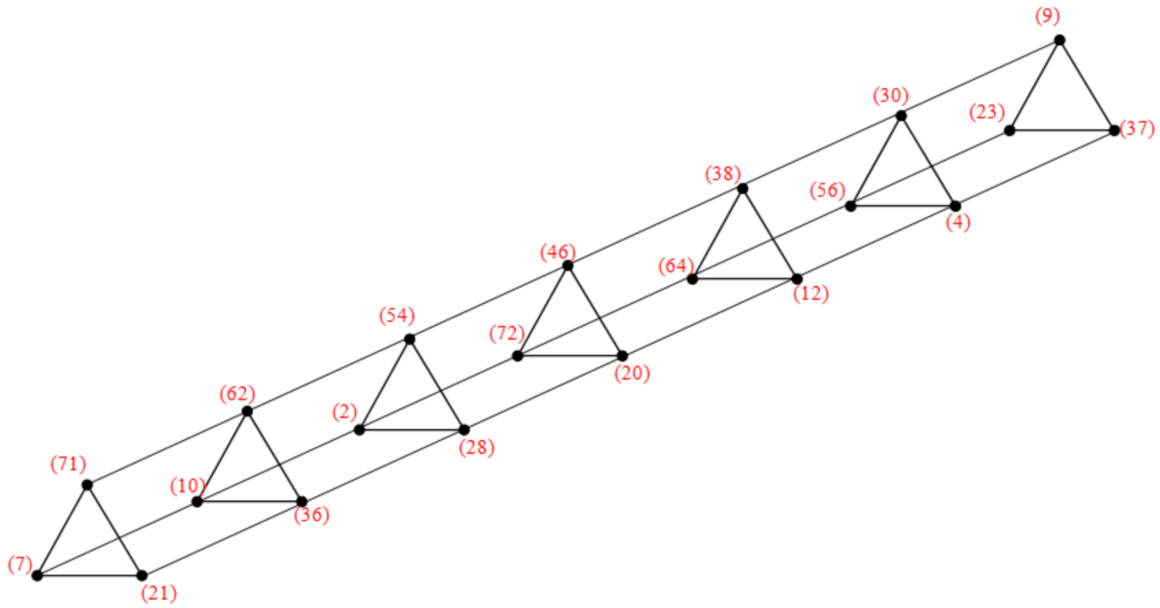

Figure 4.8. A vertex label for $\mathbf{Prism}_7(C_3)$

CHAPTER V

CONCLUSION AND DISCUSSION

We can construct algorithms to label each edge of $\mathrm{Prism}_3(C_n)$ and $\mathrm{Prism}_k(C_3)$ in such a way that both are edge-odd graceful graphs. One should notice that the edge-odd graceful labeling function for the considered graphs may not be unique. We also would like to note that one may try to construct an edge-label function for $\mathrm{Prism}_k(C_n)$ that induces the edge-odd graceful labeling for $\mathrm{Prism}_k(C_n)$. In addition, one may consider prisms of other edge-odd graceful graphs and see whether they will be edge-odd graceful or not. For example, the prism of sunflower, $\mathrm{Prism}(SF(m,n))$.

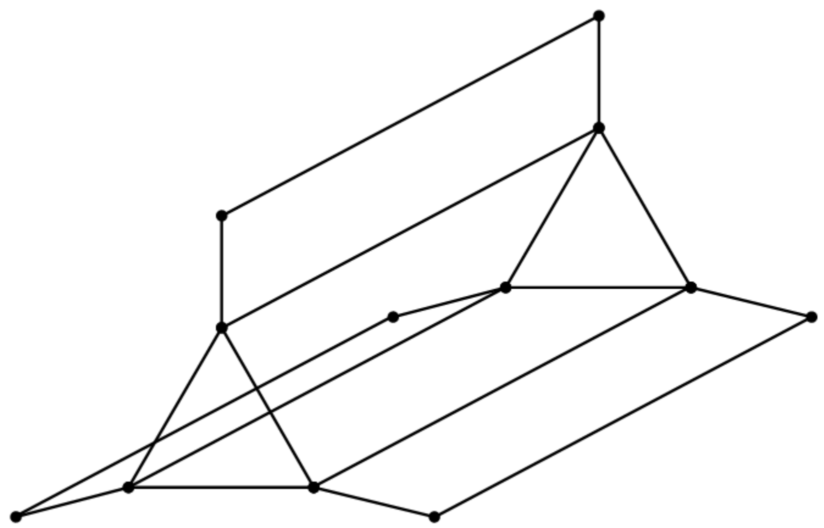

Figure 5.1. The prism of sunflower, $\mathrm{Prism}(SF(3,1))$

Moreover, to illustrate our results numerically, we can use the JAVA program to find the edge-labeling as well as the vertex-labeling according to those five algorithms presented in the Chapters 3 and 4. The following example shows the user interface of our developed program.

Example 5.1. User interface for constructing the edge-labels and the vertex-labels of $\mathrm{Prism}_3(C_n)$.

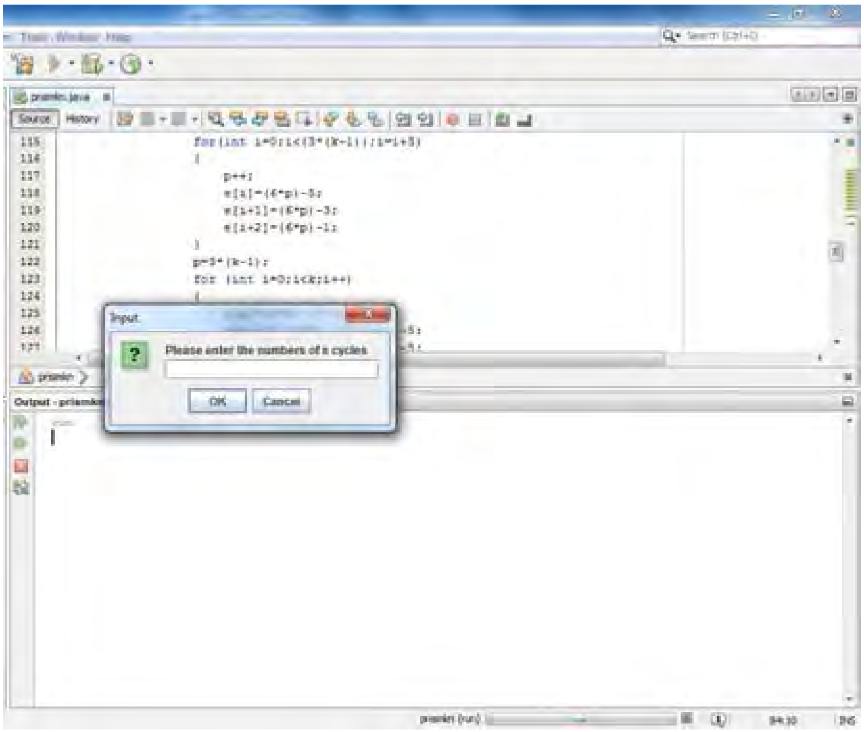

Figure 5.2. The panel to input the value of n for $\mathrm{Prism}_3(C_n)$

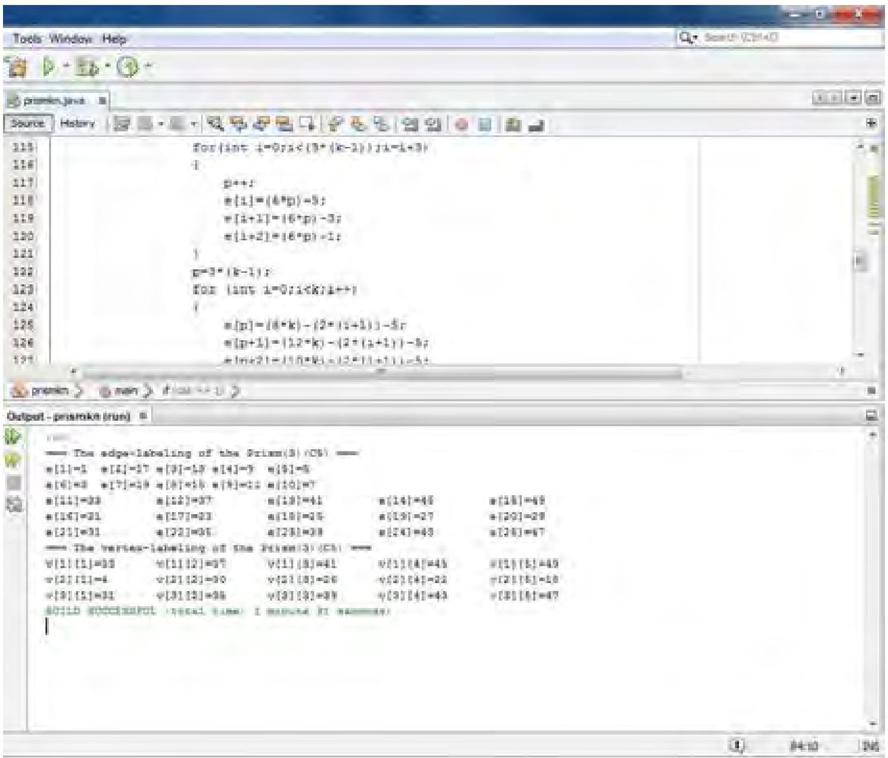

Figure 5.3. The edge-labels and the vertex-labels of $\mathrm{Prism}_3(C_5)$

Example 5.2. User interface for constructing the edge-labels and the vertex-labels of $\mathrm{Prism}_k(C_3)$.

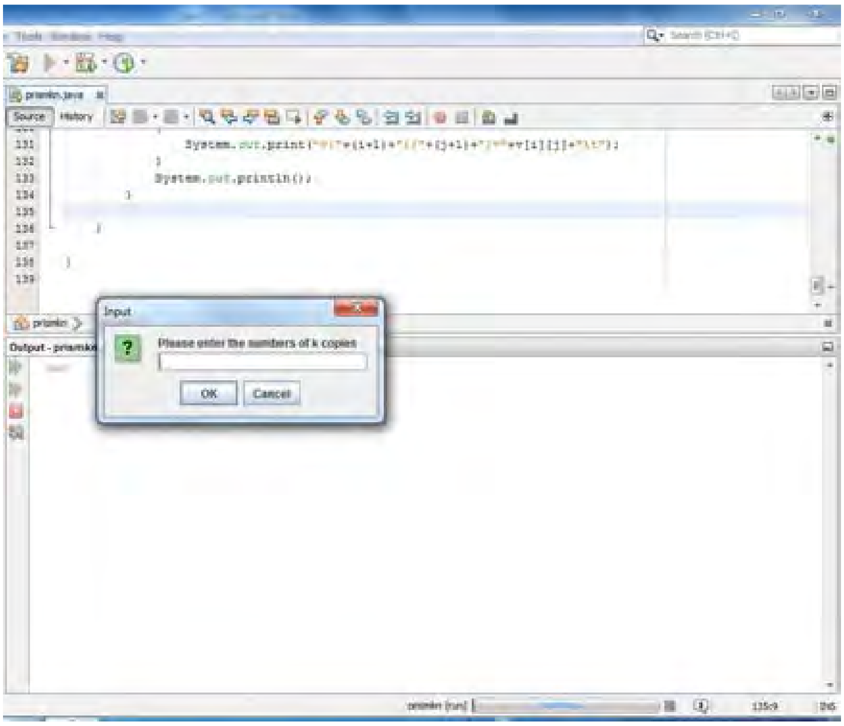

Figure 5.4. The panel to input the value of k for $\mathrm{Prism}_k(C_3)$

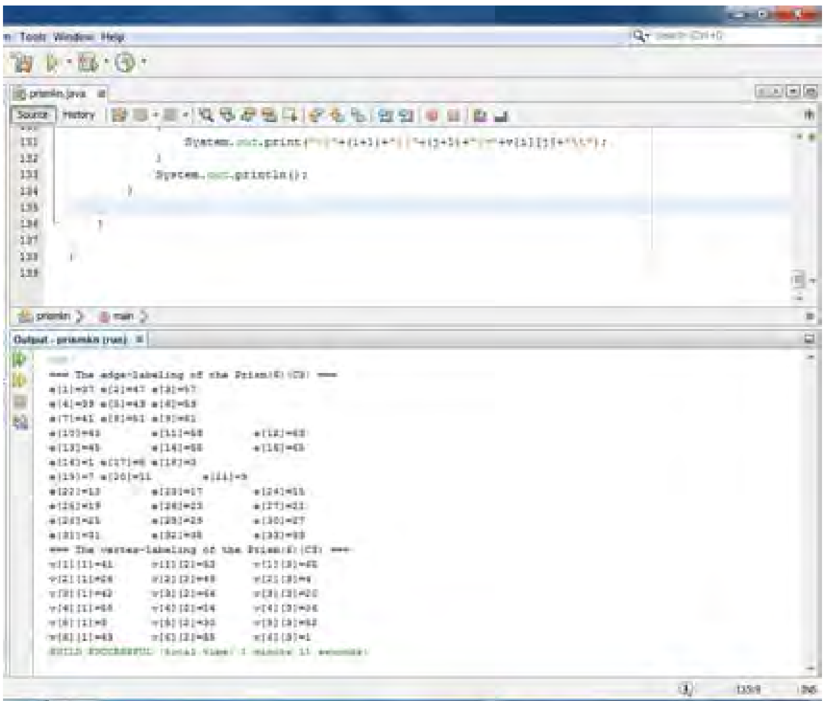

Figure 5.5. The edge-labels and the vertex-labels of $\mathrm{Prism}_6(C_3)$

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APPENDICES

JAVA language code for labeling $\mathbf{Prism}_k(C_n)$ where $k = 3$ or $n = 3$.

```
import javax.swing.JOptionPane;
public class prismkn
{
    public static void main(String[] args)
    {
        int cas = Integer.parseInt(JOptionPane.showInputDialog("Please select case ::\n1.
Prism(3)(Cn)\n2. Prism(k)(C3)"));
        if (cas==1)
        {
            int n = Integer.parseInt(JOptionPane.showInputDialog("Please enter the
numbers of n cycles"));
            int q=5*n;
            int v[][]=new int[3][n];
            int e[]=new int[q];

            //find edges
            e[0]=1;
            e[n]=3;
            for(int i=1;i<n;i++)
            {
                e[i]=4*n-4*i+1;
                e[n+i]=4*n-4*i+3;
            }
            int a=2*n;
            int b=3*n;
            int c=4*n;
```

```
for(int i=1;i<n;i++)
{
    e[a]=6*n+4*i-1;
    e[b]=4*n+2*i-1;
    e[c]=6*n+4*i-3;
    a++;b++;c++;
}
e[3*n-1]=10*n-1;
e[4*n-1]=6*n-1;
e[5*n-1]=10*n-3;

//find vertexs
v[0][0]=(e[0]+e[2*n]+e[3*n-1])%(2*q);
v[1][0]=(e[0]+e[n]+e[3*n]+e[4*n-1])%(2*q);
v[2][0]=(e[n]+e[4*n]+e[5*n-1])%(2*q);
for(int i=1;i<n;i++)
{
    v[0][i]=(e[i]+e[2*n+i-1]+e[2*n+i])%(2*q);
    v[1][i]=(e[i]+e[n+i]+e[3*n+i-1]+e[3*n+i])%(2*q);
    v[2][i]=(e[n+i]+e[4*n+i-1]+e[4*n+i])%(2*q);
}

//print edges
System.out.println("=== The edge-labeling of the Prism(3)(C"+n+") ===");
for(int i=0;i<q;i++)
{
    System.out.print("e["+(i+1)+"]="+e[i]+"\t");
    if(0==(i+1)%n)
    {
        System.out.println();
    }
```

```
            }

            //print vertexs
            System.out.println("=== The vertex-labeling of the Prism(3)(C"+n+") ===");
            for(int i=0;i<3;i++)
            {
                for(int j=0;j<n;j++)
                {
                    System.out.print("v["+(i+1)+"]["+(j+1)+"]="+v[i][j]+"\t");
                }
                System.out.println();
            }
        }
        else if(cas==2)
        {
            int k = Integer.parseInt(JOptionPane.showInputDialog("Please enter the
numbers of k copies"));
            int q=(6*k)-3;
            int v[][]=new int[k][3];
            int e[]=new int[q];

            //find edges
            if ((k % 4)==0)
            {
                int p=0;
                for(int i=0;i<(3*(k-1));i=i+3)
                {
                    p++;
                    e[i]=(2*p)-1;
                    e[i+1]=(2*k)+(2*p)-3;
                    e[i+2]=(4*k)+(2*p)-5;
```

```
    }
    p=3*(k-1);
    for (int i=0;i<k;i++)
    {
        e[p]=(6*k)+(2*(i+1))-7;
        e[p+1]=(8*k)+(2*(i+1))-7;
        e[p+2]=(10*k)+(2*(i+1))-7;
        p=p+3;
    }
}//((k % 4)==0)
else if((k % 4)==1)
{
    int p=0;
    for(int i=0;i<(3*(k-1));i=i+3)
    {
        p++;
        e[i]=(6*p)-5;
        e[i+1]=(6*p)-3;
        e[i+2]=(6*p)-1;
    }
    p=3*(k-1);
    for (int i=0;i<k;i++)
    {
        e[p]=(8*k)-(2*(i+1))-5;
        e[p+1]=(12*k)-(2*(i+1))-5;
        e[p+2]=(10*k)-(2*(i+1))-5;
        p=p+3;
    }
}//((k % 4)==1)
else if ((k % 4)==2)
{
```

```
    int p=0;
    for(int i=0;i<(3*(k-1));i=i+3)
    {
        p++;
        e[i]=(6*k)+(2*p)-1;
        e[i+1]=(8*k)+(2*p)-3;
        e[i+2]=(10*k)+(2*p)-5;
    }
    p=3*(k-1);
    for (int i=0;i<k;i++)
    {
        e[p]=(6*(i+1))-5;
        e[p+1]=(6*(i+1))-1;
        e[p+2]=(6*(i+1))-3;
        p=p+3;
    }
}//((k % 4)==2)
else if ((k % 4)==3)
{
    int p=0;
    for(int i=0;i<(3*(k-1));i=i+3)
    {
        p++;
        e[i]=(2*p)-1;
        e[i+1]=(2*k)+(2*p)-3;
        e[i+2]=(4*k)+(2*p)-5;
    }
    p=3*(k-1);
    for (int i=0;i<k;i++)
    {
        e[p]=(12*k)-(6*(i+1))-5;
```

```
        e[p+1]=(12*k)-(6*(i+1))-1;
        e[p+2]=(12*k)-(6*(i+1))-3;
        p=p+3;
    }
}//((k % 4)==3)

//print edges
System.out.println("=== The edge-labeling of the Prism("+k+")(C3) ===");
for(int i=0;i<(6*k)-3;i++)
{
    System.out.print("e["+(i+1)+"]="+e[i]+"\t");
    if(0==(i+1)%3)
    {
        System.out.println();
    }
}

//find vertexs
v[0][0]=(e[0]+e[3*(k-1)]+e[3*(k-1)+2])%(2*q);
v[0][1]=(e[1]+e[3*(k-1)]+e[3*(k-1)+1])%(2*q);
v[0][2]=(e[2]+e[3*(k-1)+1]+e[3*(k-1)+2])%(2*q);
v[k-1][0]=(e[3*(k-2)]+e[6*(k-1)+2]+e[6*(k-1)])%(2*q);
v[k-1][1]=(e[3*(k-2)+1]+e[6*(k-1)]+e[6*(k-1)+1])%(2*q);
v[k-1][2]=(e[3*(k-2)+2]+e[6*(k-1)+1]+e[6*(k-1)+2])%(2*q);
for(int i=1;i<k-1;i++)
{
    v[i][0]=(e[3*i-3]+e[3*i]+e[3*k+3*i-3]+e[3*k+3*i-1])%(2*q);
    v[i][1]=(e[3*i-2]+e[3*i+1]+e[3*k+3*i-3]+e[3*k+3*i-2])%(2*q);
    v[i][2]=(e[3*i-1]+e[3*i+2]+e[3*k+3*i-1]+e[3*k+3*i-2])%(2*q);
}
```

```
            //print vertexs
            System.out.println("=== The vertex-labeling of the Prism("+k+")(C3) ===");
            for(int i=0;i<k;i++)
            {
                for(int j=0;j<3;j++)
                {
                    System.out.print("v["+(i+1)+"]["+(j+1)+"]="+v[i][j]+"\t");
                }
                System.out.println();
            }
        }
    }
}
```

BIOGRAPHY

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