Chapter 5

Conclusions and Discussions

Since the realization of Bose-Einstein condensation in an alkali gas system in 1995 [29] and [30]. There is an overwhelm interesting in this field. Since BEC provide a testing ground for the macroscopic many body theory. Also the implications are wide range such as atom laser. Then we decide to study the ground state properties of BEC in rubidium gas trapped in anisotropic magnetic field.

First we review Feynman path integral. In an imaginary time the Feynman propagator is the density matrix. Furthermore we seek for an approximation. There is a variational method first introduced by Feynman himself. This method is proved to be useful in many problems such as the works of V. Sa-yakanit [57, 58]. Next we review the existing theory which based on the mean field approach. Results show the crucial role played by the interactions. Since this approach yield an acceptable results we will compare our results to these results.

It is known that the trap used in the experiment has the form of harmonic potential and we choose the pseudopotential (hard-sphere potential) for the interaction of the constituent particles. Then we apply the variational method to the system. We obtain the approximated density matrix. In the limit $\beta \rightarrow \infty$ correspond to $T = 0$ we can extract the information of the ground state properties. The ground state wavefunction contains in the trial propagator.

The ground state energy is exactly the same as that of Baym and Pethick [51]. Then we compare with the numerical result [49]. Our result is a bit higher than theirs work. The deviation increase with increasing interactions (correspond to increasing in N), as can be seen in Table 5.1. The large deviation when N is high may be improved if the kinetic term is neglected. This is called the Thomas-Fermi approximation, which is valid in the limit that the interaction term is larger than kinetic term.

N	$\frac{E_1}{N}$	% diff of $\frac{E_1}{N}$	$\left(\frac{E_1}{N}\right)_{\text{kin}}$	% diff of kin term	$\left(\frac{E_1}{N}\right)_{\text{ho}}$	% diff of h.o. term	$\left(\frac{E_1}{N}\right)_{\text{int}}$	% diff of int. term
100	2.66 (2.66)	0.00	1.05 (1.06)	0.94	1.38 (1.39)	0.71	0.22 (0.21)	4.76
200	2.87 (2.86)	0.34	0.96 (0.98)	2.04	1.52 (1.52)	0.00	0.37 (0.36)	2.77
500	3.34 (3.30)	1.21	0.82 (0.86)	4.65	1.84 (1.81)	1.65	0.68 (0.63)	7.93
1000	3.92 (3.84)	2.08	0.70 (0.76)	7.89	2.21 (2.15)	2.79	1.00 (0.93)	7.52
2000	4.76 (4.61)	3.25	0.59 (0.66)	10.6	2.74 (2.64)	3.78	1.43 (1.32)	8.33
5000	6.40 (6.12)	4.57	0.45 (0.54)	16.6	3.75 (3.57)	5.04	2.20 (2.02)	8.91
10000	8.19 (7.76)	5.54	0.36 (0.45)	20.0	4.84 (4.57)	5.90	2.98 (2.74)	8.75
15000	9.51 (8.98)	5.90	0.31 (0.41)	24.3	5.64 (5.31)	6.21	3.55 (3.26)	8.89
20000	10.6 (9.98)	6.21	0.28 (0.38)	26.3	6.30 (5.91)	6.59	4.01 (3.68)	8.96

Table 5.1: The percent deviation of our results compare with Dalfovo and Stringari [49]. The numbers in the parenthesis are taken from Table 3.1.

In the process of minimizing energy, we can determine the variational parameters. Substituting these parameters into the expression of the ground state wavefunction, we obtain the wavefunction. We can see that when the interaction increase the wavefunction is flatten. In other words the interactions cause the condensate cloud to expand in the direction where the restoring forces are weakest.

Though our results are coincide with the the mean field approach, its applicability is restricted. Since our calculation based on the $T = 0$ limit, the result will be failed at higher temperature. This means that we need to extend our model to the finite temperature. And this can be done by concerning the identical particles of the system. This means that we have to include the permutation of particles into the density matrix. There are works of Brosens et. al. [59, 60] and Tempere et.al [61] which include the permutation of the identical particles. Theirs approach has an advantage since it allows us to incorporate any two-body interactions into the description at an arbitrary temperature. Hence its application is more general than the Gross-Pitaevskii (GP) model. GP equation has limited to the contact interaction, so the range and shape of theq interaction is excluded from the calculation.

From our work, the variational path integral has been proven to be the potential alternative. In this work the trial action has only one-body interaction. To improve the results, we have to choose a better trial action such as those contain two-body interaction and can be solved exactly.