CHAPTER III

SUPERSYMMETRIC EXTENSION OF THE STANDARD MODEL

In this chapter we review the extension of the Standard Model in a supersymmetric way. The organization of this chapter is as follows. We start with the theoretical motivation for introducing supersymmetric theories fucusing on the mass hierarchy problem in Section 3.1. The necessary notations and conventions are then given in Section 3.2 following the Lecture Notes by Martin [28]. Before learning how to construct a supersymmetric Lagrangian in Section 3.5, we need to describe the building blocks of supersymmetric theories known as supermultiplets and supersymmetry transformations; this is done in Sections 3.3 and 3.4. In Section 3.6, the necessity and form of interactions of the supersymmetry breaking are discussed. The supersymmetric extension of the Standard Model known as the Minimal Supersymmetric Standard Model (MSSM) is then defined in Section 3.7. Lastly, the mass and mixing matrices of the sparticles, specifically neutralinos, gauginos and selectrons, are given in Sections 3.8 and 3.9.

3.1 Why supersymmetry

The mass hierarchy problem

The mass hierarchy problem in the Standard Model arises when radiative corrections to the Higgs boson mass are calculated. Phenomenologically the Higgs mass must be of order of the electroweak scale, $\mathcal{O}(100\text{ GeV})$. However, its radiative corrections are quadratically dependent on the UV cutoff scale Λ, the energy scale at which new physics is expected to start playing an important role. If we replace Λ with the grand unification (GUT) scale, $M_{\text{GUT}} \sim 10^{16}$ GeV, or Planck scale, $M_P \sim 10^{18}$ GeV, then the value of the Higgs mass would be enourmously larger than it should be (28 orders of magnitude or more). In order to renormalize the quadratic divergences of the mass squared away, we need to add counterterms to the Lagrangian and fine-tune the parameters to cancel the loop corrections with an extreme precision. Moreover, this adjustment must be made at each order in perturbation theory. Such a fine-tuning process seems rather unnatural, and therefore leads to a problem known as the fine-tuning problem.

To investigate the problem of quadratic divergences in detail, let us consider a theory with a single fermion ψ coupled to a complex Higgs scalar ϕ. The scalar potential for the Higgs boson is

$$V = m_H^2|\phi|^2 + \lambda|\phi|^4. \tag{3.1}$$

This leads to spontaneous symmetry breaking with a non-vanishing Higgs vacuum expectation value, $\langle\phi\rangle = \sqrt{-m_H^2/2\lambda} \equiv v/\sqrt{2}$, at the minimum of the potential if $m_H^2 < 0$. Let us write $\phi = (h+v)/\sqrt{2}$ where h is the physical Higgs boson with $\langle h\rangle = 0$. If the Higgs field couples to a fermion with a Yukawa coupling λ_f, after the symmetry breaking the fermion will acquire a mass $m_f = \lambda_f v/\sqrt{2}$. Now, consider the Higgs self-energy which receives a contribution from a fermion loop as shown in Fig. 3.1,

$$-i\Sigma_\phi(p^2) \;=\; \left(i\frac{\lambda_f}{\sqrt{2}}\right)^2 (i)^2(-1)\int \frac{d^4k}{(2\pi)^4}\frac{Tr[(\gamma^\mu k_\mu + m_f)(\gamma^\nu(k_\nu - p_\nu) + m_f)]}{[(p-k)^2 - m_f^2](k^2 - m_f^2)}.$$

Perform an integration by using a momentum cuttoff Λ, we obtain the correction to the Higgs mass squared,

$$(\delta m_h^2)_f = -\frac{\lambda_f^2}{8\pi^2}\left[\Lambda^2 - 6m_f^2\log\left(\frac{\Lambda}{m_f}\right) + 2m_f^2\right] + \ldots, \tag{3.2}$$

where the ellipses represent terms which vanish when $\Lambda \to \infty$. Explicitly, the Higgs boson mass diverges quadratically in the cutoff scale, and so we need to fine-tune to the mass parameter in the counter terms to a high precision in order to obtain a physical Higgs mass.

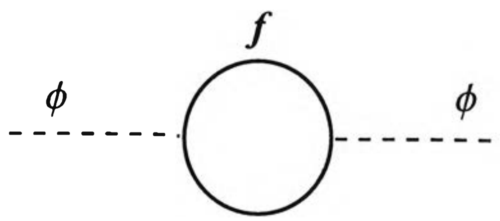

Figure 3.1: The Higgs self-energy diagram with a fermion loop.

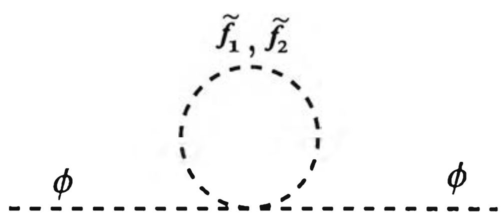

Figure 3.2: The Higgs self-energy diagram with a boson loop.

This problem can be solved by introducing two complex scalar fields [29, 30, 31], $\tilde{f}_1$ and $\tilde{f}_2$, with masses $m_{\tilde{f}_1}$ and $m_{\tilde{f}_2}$ respectively. Their interactions with the Higgs field are described by

$$\mathcal{L}_{\phi\tilde{f}} = \tilde{\lambda}_f|\phi|^2\left(|\tilde{f}_1|^2 + |\tilde{f}_2|^2\right). \tag{3.3}$$

From this Lagrangian, the Higgs self-energy receives a scalar loop contribution, see Fig. 3.2,

$$\begin{aligned}(\delta m_h^2)_{\tilde{f}}^{(1)} &= i\tilde{\lambda}_f\int\frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2 - m_{\tilde{f}_1}^2} + \frac{i}{k^2 - m_{\tilde{f}_2}^2}\right] \\ &= -\frac{\tilde{\lambda}_f}{16\pi^2}\left[2\Lambda^2 - 2m_{\tilde{f}_1}^2\log\left(\frac{\Lambda}{m_{\tilde{f}_1}}\right) - 2m_{\tilde{f}_2}^2\log\left(\frac{\Lambda}{m_{\tilde{f}_2}}\right)\right] + \ldots \end{aligned} \tag{3.4}$$

From Eqs. (3.2) and (3.4), we see that the quadratic divergences from fermionic and bosonic contributions cancel each other if

$$\tilde{\lambda}_f = -\lambda_f^2. \tag{3.5}$$

Notice that the cancellation of quadratic divergences occurs independent of the masses m_f, $m_{\tilde{f}_1}$, and $m_{\tilde{f}_2}$. Such a cancellation is a desirable result, since without the quadratic divergences the tuning of the mass parameter in the counter terms is very much less severe.

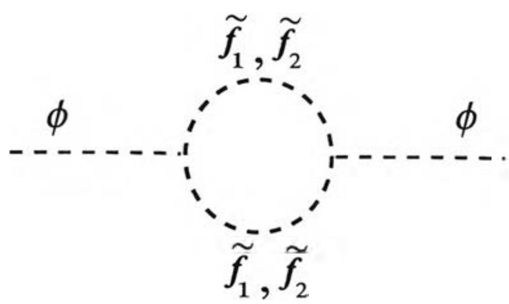

Figure 3.3: Additional loop diagram contribution after the spontaneous symmetry breaking.

After the spontaneous symmetry breaking, the Higgs mass squared also receives a contribution from the loop diagram in Fig. 3.3,

$$(\delta m_h^2)^{(2)}_{\tilde{f}} = -\frac{\tilde{\lambda}_f^2 v^2}{16\pi^2}\left[-1 + 2\log\left(\frac{\Lambda}{m_{\tilde{f}_1}}\right) + 2\log\left(\frac{\Lambda}{m_{\tilde{f}_2}}\right)\right] + \dots . \tag{3.6}$$

Assuming the relation (3.5) and taking $m_{\tilde{f}_1} = m_{\tilde{f}_2} = m_{\tilde{f}}$, the total contribution to the Higgs mass is

$$\begin{aligned}(\delta m_h^2)_{\text{total}} &= \frac{\lambda_f^2}{4\pi^2}\left[3m_f^2\log\left(\frac{\Lambda}{m_f}\right) - m_{\tilde{f}}^2\log\left(\frac{\Lambda}{m_{\tilde{f}}}\right) - 2m_f^2\log\left(\frac{\Lambda}{m_{\tilde{f}}}\right)\right] + \dots \\ &= \frac{\lambda_f^2}{4\pi^2}\left[(m_f^2 - m_{\tilde{f}}^2)\log\left(\frac{\Lambda}{m_{\tilde{f}}}\right) + 3m_f^2\log\left(\frac{m_{\tilde{f}}}{m_f}\right)\right] + \dots . \end{aligned} \tag{3.7}$$

With this construction, the quadratic divergences are now automatically canceled to all orders of perturbation. It should also be noticed that the total corrections to the Higgs boson mass squared will vanish altogether if $m_f = m_{\tilde{f}}$.

In conclusion, what we have found is that for each fermion interacting with the Higgs field, if we add two scalar bosons (or equivalently a single complex scalar boson) whose coupling constants and masses are related to those of the fermion to the theory in an appropriate way, then the corrections to the Higgs boson mass squared diverges at most logarithmically, and the fine-tuning problem is now solved. Such a situation automatically occurs if the theory possesses a special kind of symmetry, known as supersymmetry. In addition, if supersymmetry is an exact symmetry, then the masses of fermions and bosons are degenerate and it might be possible that the theory contains no divergences at all.

3.2 Dirac, Weyl and Majorana spinors

Supersymmetry is naturally formulated using two-component Weyl spinors for fermions rather than four-component Dirac or Majorana spinors. The reason for this is that the Weyl spinor is the smallest possible spinor representation of the Lorentz group in four dimensions. An immediate consequence of this is that it is possible to construct an irreducible representation of supersymmetry which contains only chiral spinors. Therefore, in a supersymmetric theory, we can treat left-handed and right-handed parts of Dirac fermions separately, since they must have completely different electroweak gauge interactions as they do in the Standard Model. In this section, the notations and conventions for Dirac, Majorana, and Weyl spinors will be given.

A four-component Dirac spinor Ψ_D with mass M is described by the Lagrangian

$$\mathcal{L}_{\text{Dirac}} = i\bar{\Psi}_D\gamma^\mu\partial_\mu\Psi_D - M\bar{\Psi}_D\Psi_D. \tag{3.8}$$

Here, γ^μ are 4×4 gamma matrices satisfying the anti-commutation relations $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ where $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is a spacetime metric tensor. In the Weyl or chiral representation, the γ matrices are given by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{3.9}$$

where

$$\sigma^0 = \bar{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \sigma^1 = -\bar{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \tag{3.10}$$

$$\sigma^2 = -\bar{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma^3 = -\bar{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3.11}$$

As the γ-matrices can be expressed in the 2×2 block representation, the spinor Ψ_D can also be decomposed into blocks by using two-component, complex, anti-commuting objects (ξ_α) and $(\chi^\dagger)^{\dot\alpha}$ with $\alpha, \dot\alpha = 1, 2$:

$$\Psi_D = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}. \tag{3.12}$$

These two-component spinors are the smallest representations of the Lorentz group, and therefore form the building blocks for all bigger representations. The spinor indices are raised and lowered using the antisymmetric symbol ϵ according to

$$\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta; \quad \xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta; \quad \xi^\dagger_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\xi^{\dagger\dot\beta}; \quad \xi^{\dagger\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\xi^\dagger_{\dot\beta}, \tag{3.13}$$

where

$$\epsilon^{\alpha\beta} = -\epsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \epsilon^{\dot\alpha\dot\beta} = -\epsilon_{\dot\alpha\dot\beta}. \tag{3.14}$$

In this representation, we let

$$P_L\Psi_D = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}; \qquad P_R\Psi_D = \begin{pmatrix} 0 \\ \chi^{\dagger\dot\alpha} \end{pmatrix} \tag{3.15}$$

where P_L and P_R in (3.15) are the left- and right-handed projection operators defined by

$$P_L = \frac{(1-\gamma^5)}{2}, \qquad P_R = \frac{(1+\gamma^5)}{2}, \tag{3.16}$$

with γ^5, in the Weyl representation, defined by

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3.17}$$

The upper two-component object ξ is thus called a left-handed Weyl spinor while the lower one $\chi^\dagger$ is a right-handed Weyl spinor. They are said to have left and right chiralities respectively. The notations for all fermions are chosen so that left-handed Weyl spinors do not carry a dagger and carry undotted spinor indices, while right-handed ones do carry a dagger and carry dotted spinor indices. In addition, the hermitian conjugate of a left-handed Weyl spinor is a right-handed Weyl spinor and vice versa, since they transform in the same way under the Lorentz group. Note that the matrix γ^5 satisfies the following properties

$$\begin{aligned}(\gamma^5)^\dagger &= \gamma^5 &(3.18)\\ (\gamma^5)^2 &= 1 &(3.19)\\ \{\gamma^5,\gamma^\mu\} &= 0. &(3.20)\end{aligned}$$

When dotted (or undotted) indices are contracted, it is helpful to drop these indices by introducing shorthand notations. Anyway, summing over indices for dotted and undotted indices are defined differently. Summing over undotted indices are performed with an upper index for the first spinor and a lower index for the second one, for example,

$$\begin{aligned}\xi\chi &\equiv \xi^\alpha\chi_\alpha = -\chi_\alpha\xi^\alpha\\ &= \xi^\alpha\epsilon_{\alpha\beta}\chi^\beta = -\chi^\beta\epsilon_{\alpha\beta}\xi^\alpha = \chi^\beta\epsilon_{\beta\alpha}\xi^\alpha = \chi^\beta\xi_\beta \equiv \chi\xi. \qquad (3.21)\end{aligned}$$

This is in contrast with summing over dotted indices which performs with a lower index for the first spinor and an upper index for the second one,

$$\begin{aligned}\xi^\dagger\chi^\dagger &\equiv \xi^\dagger_{\dot\alpha}\chi^{\dagger\dot\alpha} = -\chi^{\dagger\dot\alpha}\xi^\dagger_{\dot\alpha}\\ &= \xi^\dagger_{\dot\alpha}\epsilon^{\dot\alpha\dot\beta}\chi^\dagger_{\dot\beta} = -\chi^\dagger_{\dot\beta}\epsilon^{\dot\alpha\dot\beta}\xi^\dagger_{\dot\alpha} = \chi^\dagger_{\dot\beta}\epsilon^{\dot\beta\dot\alpha}\xi^\dagger_{\dot\alpha} = \chi^\dagger_{\dot\alpha}\xi^{\dagger\dot\alpha} \equiv \chi^\dagger\xi^\dagger. \qquad (3.22)\end{aligned}$$

Note that the minus sign is due to the anticommuting property of the spinors.

With this index convention, the matrices σ and $\bar\sigma$ carry both undotted and dotted spinor indices as they provide a mixing of left-handed and right-handed Weyl spinors, i.e.,

$$(\sigma^\mu)_{\alpha\dot\alpha}, \qquad (\bar\sigma^\mu)^{\dot\alpha\alpha}.$$

Moreover, they are related to each other via

$$\begin{aligned}(\bar\sigma^\mu)^{\dot\alpha\alpha} &= \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}(\sigma^\mu)_{\beta\dot\beta};\\ (\sigma^\mu)_{\alpha\dot\alpha} &= \epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}(\bar\sigma^\mu)^{\dot\beta\beta}. \qquad (3.23)\end{aligned}$$

Then we also have another abbreviated notation

$$\xi^\dagger \bar{\sigma}^\mu \chi \;=\; \xi^\dagger_{\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \chi_\alpha = \xi^{\dagger\dot{\alpha}} (\sigma^\mu)_{\alpha\dot{\alpha}} \chi^\alpha = -\chi^\alpha (\sigma^\mu)_{\alpha\dot{\alpha}} \xi^{\dagger\dot{\alpha}} = -\chi \sigma^\mu \xi^\dagger. \quad (3.24)$$

Using Eqs. (3.21), (3.22), and (3.24), the Dirac Lagrangian (3.8) can be written in terms of two-component Weyl spinors as

$$\mathcal{L}_{\text{Dirac}} = i\xi^\dagger \bar{\sigma}^\mu \partial_\mu \xi + i\chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi - M(\xi\chi + \xi^\dagger \chi^\dagger). \quad (3.25)$$

A four-component Majorana spinor is a Dirac spinor with the condition $\chi = \xi$, or explicitly,

$$\Psi_M = \begin{pmatrix} \xi_\alpha \\ \xi^{\dagger\dot{\alpha}} \end{pmatrix}. \quad (3.26)$$

It can be seen that the Majorana spinor has only two independent components, so it is equivalent to a Weyl spinor. The Lagrangian for a Majorana fermion of mass M is

$$\mathcal{L}_{\text{Majorana}} = \frac{i}{2} \bar{\Psi}_M \gamma^\mu \partial_\mu \Psi_M - \frac{M}{2} \bar{\Psi}_M \Psi_M \quad (3.27)$$

which can be written in terms of a two-component Weyl spinor as

$$\mathcal{L}_{\text{Majorana}} = i\xi^\dagger \bar{\sigma}^\mu \partial_\mu \xi - \frac{1}{2} M(\xi\xi + \xi^\dagger \xi^\dagger). \quad (3.28)$$

It can be shown that the propagator for a Majorana spinor is the same as that for a Dirac spinor.

3.3 Chiral and vector supermultiplets

Supersymmetry (SUSY) is an extension of the Poincaré group to a super-Lie group, by adding to the set of generators, the fermionic generators transforming as Weyl spinors and possibly some other bosonic generators. In particle physics phenomenology, people are interested only in $N = 1$ supersymmetry in which only two fermionic generators (forming a Weyl spinor) and their hermitian conjugates are added. Like the Poincaré group, $N = 1$ supersymmetry has many irreducible representations, called supermultiplets. Each supermultiplet contains equal numbers of bosonic and fermionic degrees of freedom.

In a supersymmetric theory, particles are combined into supermultiplets, which within a supermultiplet, have both bosonic and fermionic components known as superpartners of each other. The particles in the same supermultiplet must have the same mass and quantum numbers except the spins which differ by 1/2 unit. In constructing a supersymmetric extension of the Standard Model, there are two types of supermultiplets relevant: chiral and vector supermultiplets.

Chiral or matter or scalar supermultiplets

Each chiral supermultiplet contains a complex scalar field, ϕ, a Weyl fermion, ψ, and an auxiliary complex field, F. The field F is called auxiliary since it has no kinetic term in the Lagrangian and can be eliminated from the theory by using its equations of motion. Moreover, it can be seen that the physical fermionic and bosonic degrees of freedom in a chiral supermultiplet are equal. Obviously, since a complex scalar boson has two degrees of freedom and each Weyl fermion has two helicity states, then each has its number of degrees of freedom equal to two.

Note that the name chiral supermultiplet came from the presence of the chiral (Weyl) spinor in the supermultiplet.

Vector or gauge supermultiplets

Each massless vector supermultiplet contains a spin-1 gauge boson, A^a_μ, a Weyl fermion, λ^a, and a real auxiliary field, D. A massless gauge boson has two helicity states, so the number of bosonic degrees of freedom is two. Its superpartner, a massless Weyl fermion, again has two helicity states and therefore two fermionic degrees of freedom. Gauge bosons must transform as the adjoint representation of the gauge group, so their fermionic partners called gauginos and the auxiliary field D must also. Since the adjoint representation of a gauge group is its own conjugate, then the left-handed and right-handed components of fermions must have the same gauge transformation properties.

The gauge transformations of the vector supermultiplet fields are

$$\delta_{\text{gauge}} A_\mu^a = -\partial_\mu \Lambda^a + g^{(a)} f^{abc} A_\mu^b \Lambda^c \tag{3.29}$$

$$\delta_{\text{gauge}} \lambda^a = g^{(a)} f^{abc} \lambda^b \Lambda^c, \tag{3.30}$$

$$\delta_{\text{gauge}} D^a = g^{(a)} f^{abc} D^b \Lambda^c, \tag{3.31}$$

where Λ^a are the infinitesimal gauge transformation parameters, $g^{(a)}$ is the gauge coupling, and f^{abc} are the totally antisymmetric structure constants of the gauge group. Here, the latin indices run over the adjoint representation of the gauge group.

Now suppose that a chiral supermultiplet transforms under the gauge group in a representation with hermitian generators $(T^a)_i{}^j$ satisfying the commutation relations $[T^a, T^b] = i f^{abc} T^c$. Since all components in a chiral supermultiplet must be in the same representation of the gauge group, their transformation properties are then given by

$$\delta_{\text{gauge}} X_i = i g^{(a)} \Lambda^a (T^a X)_i \tag{3.32}$$

with X_i denotes a scalar ϕ_i, or a fermion ψ_i, or an auxiliary field F_i.

3.4 Supersymmetry transformations

In this section, we state the supersymmetry transformations of all component fields without the derivation as follows:

$$\delta \phi_i = \epsilon \psi_i, \tag{3.33}$$

$$\delta (\psi_i)_\alpha = i(\sigma^\mu \epsilon^\dagger)_\alpha D_\mu \phi_i + \epsilon_\alpha F_i, \tag{3.34}$$

$$\delta F_i = i\epsilon^\dagger \bar{\sigma}^\mu D_\mu \psi_i + \sqrt{2} g^{(a)} (T^a \phi)_i \epsilon^\dagger \lambda^{\dagger a}, \tag{3.35}$$

$$\delta A^a_\mu = -\frac{1}{\sqrt{2}}\left[\epsilon^\dagger\bar{\sigma}_\mu\lambda^a + \lambda^{\dagger a}\bar{\sigma}_\mu\epsilon\right], \tag{3.36}$$

$$\delta\lambda^a_\alpha = -\frac{i}{2\sqrt{2}}\left(\sigma^\mu\bar{\sigma}^\nu\epsilon\right)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\epsilon_\alpha D^a, \tag{3.37}$$

$$\delta D^a = \frac{i}{\sqrt{2}}\left[\epsilon^\dagger\bar{\sigma}^\mu D_\mu\lambda^a - D_\mu\lambda^{\dagger a}\bar{\sigma}^\mu\epsilon\right]. \tag{3.38}$$

Here, ϵ is an infinitesimal, constant, anticommuting, two-component Weyl spinor which parameterizes the supersymmetry transformation. D_μ is the gauge covariant derivative which is different for chiral and vector superfields due to their differences in the representations of the gauge group. For example, for the gaugino fields λ^a,

$$D_\mu\lambda^a = \partial_\mu\lambda^a - g^{(a)}f^{abc}A^b_\mu\lambda^c, \tag{3.39}$$

while for $X_i = \phi_i$ or ψ_i,

$$D_\mu X_i = \partial_\mu X_i + ig^{(a)}A^a_\mu(T^aX)_i. \tag{3.40}$$

Lastly, $F^a_{\mu\nu}$ in Eq. (3.37) is the usual field strength tensor defined by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g^{(a)}f^{abc}A^b_\mu A^c_\nu. \tag{3.41}$$

Moreover, from the supersymmetry transformations above we also see that ϵ must have a dimension of $(\text{mass})^{-1/2}$ while F and D have the dimension of $(\text{mass})^2$.

In the next section, where we will write down the general supersymmetric Lagrangian, we will see that the form of interactions is highly restriced by the requirement that the action be invariant under supersymmetry transformations up to total spacetime derivatives.

3.5 Supersymmetric Lagrangians

In this section, we will provide the construction of the supersymmetric Lagrangians. We will begin with the simplest Lagrangian density which contains only the kinetic terms of all fields,

$$\mathcal{L}_{\text{KE}} = D^\mu\phi^{*i}D_\mu\phi_i + i\psi^{\dagger i}\bar{\sigma}^\mu D_\mu\psi_i - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\bar{\sigma}^\mu D_\mu\lambda^a. \tag{3.42}$$

The index i runs over all gauge and flavor degrees of freedom of the chiral superfields while the index a runs over the adjoint representation of the gauge group.

Next, the Lagrangian density for the auxiliary fields F and D is given by

$$\mathcal{L}_{\text{auxiliary}} = F^*F + \frac{1}{2}D^aD^a. \tag{3.43}$$

It is clear that the auxiliary fields do not propagate since they have no kinetic terms.

The interactions among the component fields of chiral and gauge supermultiplets are specified by supersymmetry and gauge invariance

$$\mathcal{L}_{\text{int}} = -\sqrt{2}g^{(a)}\left[(\phi^*T^a\psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a\phi)\right] + g^{(a)}(\phi^*T^a\phi)D^a. \tag{3.44}$$

Observe that the interaction strengths are fixed in terms of the gauge coupling constants, so there is no new adjustable parameters.

Now consider the interactions among chiral superfields. The most general form of the interactions that preserve supersymmetry is

$$\mathcal{L}_W = -\frac{1}{2}W^{ij}\psi_i\psi_j + W^iF_i + \text{h.c.} \tag{3.45}$$

where W^{ij} and W^i are functions of the scalar fields ϕ_i, but not their complex conjugate ϕ^{*i}; thus they are said to be analytic or holomorphic functions of ϕ_i. By power counting W^{ij} and W^i must have dimensions of (mass) and (mass)^2 respectively. Actually, the functions W^{ij} and W^i are related to each other through the so-called superpotential, W, by

$$W^{ij} = \frac{\partial^2 W}{\partial\phi_i\partial\phi_j} \tag{3.46}$$

and

$$W^i = \frac{\partial W}{\partial\phi_i}. \tag{3.47}$$

The superpotential is also an analytic function of the complex scalar fields ϕ, and thus cannot contain derivative interactions. The most general form of the superpotential for a renormalizable theory can be written as

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k, \tag{3.48}$$

where M^{ij} is a symmetric mass matrix for the fermion fields, and y^{ijk} is the Yukawa coupling of a scalar and two fermion fields which must be totally symmetric under the permutations of indices i, j, k.

From Eqs. (3.43), (3.44), and (3.45), we can find the equations of motion of F and D fields,

$$\begin{aligned} F_i &= -W_i^*, & (3.49)\\ D^a &= -g^{(a)}(\phi^* T^a \phi). & (3.50) \end{aligned}$$

Therefore, both auxiliary fields can be eliminated from the Lagrangian by expressing them in terms of scalar fields.

Combining terms in the Lagrangian involving only scalar fields, one gets the scalar potential for a supersymmetric theory which consists of F-terms and D-terms,

$$V = F^{*i}F_i + \frac{1}{2}D^aD^a = W_i^*W^i + \frac{1}{2}(g^{(a)})^2(\phi^*T^a\phi)^2 \tag{3.51}$$

or explicitly,

$$\begin{aligned} V(\phi,\phi^*) = \; & M_{ik}^*M^{kj}\phi^{*i}\phi_j + \frac{1}{2}M^{in}y_{jkn}^*\phi_i\phi^{*j}\phi^{*k} + \frac{1}{2}M_{in}^*y^{jkn}\phi^{*i}\phi_j\phi_k \\ & + \frac{1}{4}y^{ijn}y_{kln}^*\phi_i\phi_j\phi^{*k}\phi^{*l} + \frac{1}{2}(g^{(a)})^2(\phi^*T^a\phi)^2. \end{aligned} \tag{3.52}$$

3.6 Soft supersymmetry breaking interactions

It is clear that supersymmetry cannot be an exact symmetry because if it is so, the Standard Model particles and their superpartners must be degenerate in mass. However, no experimental evidences of such superpartners have ever been observed at all. So if Nature is really supersymmetric, then supersymmetry must be broken at the energy scale higher than that of the electroweak symmetry breaking, and the mass-splitting within a supermultiplet must occur. Although the mechanism of supersymmetry breaking is not well understood yet, we can add by hand, for phenomenological purposes, the explicit supersymmetry breaking terms to the Lagrangian. To be consistent with the motivations for introducing supersymmetry, these terms are constrained not to reintroduce the quadratic divergences to the Higgs mass squared. Such explicit breaking interactions are called soft supersymmetry breaking. The couplings associated with the softly broken supersymmetry Lagrangian are called the soft parameters and must have positive mass dimensions.

The possible renormalizable soft supersymmetry breaking Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{soft}} &= -\frac{1}{2}(M_\lambda \lambda^a \lambda^a + \text{h.c.}) - \left(m^2\right)^i_j \phi^{j*}\phi_i \\ &\quad - \left(\frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{h.c.}\right). \end{aligned} \tag{3.53}$$

The first term consists of the gaugino masses M_λ which remove the degeneracy between gauginos and gauge bosons. The next one is the scalar mass squared term $(m^2)^i_j$ which gives masses to all scalar fields. The last two terms are called bilinear and trilinear soft breaking terms with mass dimensions of couplings b^{ij} and a^{ijk} equal to 2 and 1 respectively. Note that the bilinear and trilinear terms have the same form as the fermion mass terms M^{ij} and the Yukawa coupling terms y^{ijk} in the superpotential (3.48), so they will be allowed by gauge invariance if a corresponding superpotential term is allowed. It has been shown rigorously that $\mathcal{L}_{\text{soft}}$ given by (3.53) is free from quadratic divergences of scalar masses to all orders in perturbation theory [32].

3.7 The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is defined to be the minimal supersymmetric extension of the Standard Model with minimal gauge group and minimal particle content. Therefore the MSSM respects the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group as the Standard Model does.

The Lagrangian of the MSSM is given by

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_W + \mathcal{L}_{\text{soft}} \tag{3.54}$$

where

$$\mathcal{L}_0 = \mathcal{L}_{\text{KE}} + \mathcal{L}_{\text{auxiliary}} + \mathcal{L}_{\text{int}}. \tag{3.55}$$

When the particle content of the MSSM and their representations of the gauge group are specified, all terms in the Lagrangian $\mathcal{L}_0$ will be completely determined. In the next subsection, the known matter and gauge fields in the Standard Model will be assigned to the supermultiplets in the MSSM. Afterwards, the superpotential and soft supersymmetry breaking terms will be discussed.

3.7.1 The particle content

The Minimal Supersymmetric Standard Model is called so because it is a supersymmetric version of the Standard Model that has minimum particle content necessary to give rise to all Standard Model particles. In order to identify the particle content of the MSSM, firstly, we must promote all fields in the Standard Model to the appropriate supermultiplets, give their names, and postulate new particles with spins differing by 1/2 unit known as the superpartners or sparticles associated with each supermultiplet. For the quarks and leptons in the Standard Model, they normally have left-handed and right-handed parts which transform differently under the gauge groups. To place them in chiral supermultiplets, however, we have to make all fermion fields left-handed. This is because all chiral supermultiplets are conventionally defined in terms of left-handed Weyl spinors. To put a right-handed fermion into a supermultiplet, we observe that the hermitian conjugate of a right-handed Weyl spinor has left chirality, so the supermultiplet containing a right-handed fermion is just a complex conjugate of a chiral supermultiplet.

The spin-0 supersymmetric partners of the quarks and leptons are named as squarks and sleptons ("s" at the beginning stands for "scalar"). They are denoted by the same symbols as their corresponding fermions with a tilde. For example, the superpartners of the left- and right-handed electrons, e_L and e_R, are called left-handed and right-handed selectrons and are denoted by $\tilde{e}_L$ and $\tilde{e}_R$ respectively. It is vital noting that the handedness presenting here does nothing about the helicity of the selectrons since they are just scalar fields; actually, it refers to that of their corresponding electrons instead. The notations with bar are used for the names of chiral supermultiplets associated to the right-handed fermions as shown in Table 3.1. For example, $\bar{e}$ denotes the $SU(2)_L$-singlet supermultiplet containing $e_R^\dagger$ and its superpartner $\tilde{e}_R^*$. For neutrinos, ν, which are always left-handed, their superpartners are sneutrinos and are simply denoted by $\tilde{\nu}$.

Besides introducing a number of supersymmetric partners of all known particles of the Standard Model, another interesting feature of the supersymmetric theory is the Higgs sector. It can be seen obviously that the Higgs boson must belong to a chiral supermultiplet. We may guess that just imposing its spin-1/2 superpartner into the same supermultiplet our work on the Higgs sector will be finished. However, the situation is not that easy. In the Standard Model, a single Higgs doublet can generate masses to both up- and down-type quarks upon the electroweak symmetry breaking. In doing so, the up-type quarks acquire masses through the terms which couple them to the conjugate of the Higgs doublet. In the MSSM, unfortunately, such terms are not allowed, for the superpotential must be an analytic funcion of chiral superfields. Therefore, only a single Higgs doublet is not enough for giving rise to masses of both types of quarks and so we need to introduce another Higgs doublet with opposite hypercharge to that of the first one. In this thesis, the $SU(2)_L$ Higgs doublets with $Y = \pm 1/2$ are named as H_u and H_d respectively. These names followed from their responsibility to generate mass to each type of quarks. Moreover, it is conventional to denote the weak isospin components of H_u as $(H_u^+,\ H_u^0)$ which have electric charges equal to 1 and 0. Similarly, H_d has components $(H_d^0,\ H_d^-)$ with electric charges 0 and -1 respectively. In general, the names for fermionic superpartners are obtained by adding "-ino" at the end of corresponding bosons' names in the Standard Model. Hence the supersymmetric partners of Higgs particles are called higgsinos. They are denoted by $\widetilde{H}_u^+, \widetilde{H}_u^0$ for isospin components of $\widetilde{H}_u$, and $\widetilde{H}_d^0, \widetilde{H}_d^-$ for those of $\widetilde{H}_d$. In all, the chiral supermultiplets in the Minimal Supersymmetric Standard Model are summarized in Table 3.1.

Table 3.1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model.

Names	Notations	spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks (3 families)	Q $\bar{u}$ $\bar{d}$	$(\tilde{u}_L\ \tilde{d}_L)$ $\tilde{u}_R^*$ $\tilde{d}_R^*$	$(u_L\ d_L)$ $u_R^\dagger$ $d_R^\dagger$	$(\mathbf{3},\ \mathbf{2},\ \frac{1}{6})$ $(\bar{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$ $(\bar{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons,leptons (3 families)	L $\bar{e}$	$(\tilde{\nu}\ \tilde{e}_L)$ $\tilde{e}_R^*$	$(\nu\ e_L)$ $e_R^\dagger$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$ $(\bar{\mathbf{1}},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u H_d	$(H_u^+\ H_u^0)$ $(H_d^0\ H_d^-)$	$(\widetilde{H}_u^+\ \widetilde{H}_u^0)$ $(\widetilde{H}_d^0\ \widetilde{H}_d^-)$	$(\mathbf{1},\ \mathbf{2},\ +\frac{1}{2})$ $(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$

The vector gauge bosons of the Standard Model must clearly be promoted to vector supermultiplets. Their spin-1/2 superpartners are called gauginos. The $SU(3)_C$ vector bosons well-known as gluons, g, have superpartners named gluinos,

$\tilde{g}$ while the superpartners of the electroweak gauge bosons, W^+, W^-, W^0 and B are $\widetilde{W}^+, \widetilde{W}^-, \widetilde{W}^0$ and $\widetilde{B}$, called winos and bino respectively. All of these vector supermultiplets are listed in Table 3.2.

Table 3.2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

Names	spin 1/2	spin 1	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\widetilde{W}^{\pm}$ $\widetilde{W}^0$	$W^{\pm}$ W^0	$(\mathbf{1}, \mathbf{3}, 0)$
binos, B boson	$\widetilde{B}$	B	$(\mathbf{1}, \mathbf{1}, 0)$

3.7.2 Superpotential and supersymmetric interactions

The renormalizable, $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariant superpotential for the MSSM is given by

$$W_{\mathrm{MSSM}} = \bar{\tilde{u}}\mathbf{y_u}\widetilde{Q}H_u - \bar{\tilde{d}}\mathbf{y_d}\widetilde{Q}H_d - \bar{\tilde{e}}\mathbf{y_e}\widetilde{L}H_d + \mu H_u H_d. \tag{3.56}$$

The dimensionless Yukawa couplings $\mathbf{y_u}, \mathbf{y_d}, \mathbf{y_e}$ are 3×3 matrices in the generation space. In the above equation, the gauge and family indices are all suppressed. The indices of the term $\mu H_u H_d$ can be written explicitly as $\mu(H_u)_\alpha(H_d)_\beta\epsilon^{\alpha\beta}$ where $\alpha, \beta = 1, 2$ are the $SU(2)_L$ weak isospin indices. Similarly, the term $\bar{\tilde{u}}\mathbf{y_u}\widetilde{Q}H_u$ can be written explicitly as $\bar{\tilde{u}}^i_a(\mathbf{y_u})_i{}^j\widetilde{Q}^a_{j\alpha}(H_u)_\beta\epsilon^{\alpha\beta}$ where $i, j = 1, 2, 3$ are family indices, and $a = 1, 2, 3$ is the $SU(3)_C$ color index.

The first three terms in (3.56) are the supersymmetric generalization of the scalar-fermion-fermion Yukawa interactions in the Standard Model. Besides the ordinary Yukawa interactions, Higgs-quark-quark and Higgs-lepton-lepton, they also include the squark-higgsino-quark and slepton-higgsino-lepton interactions. There are also scalar quartic interactions: (squark)^4, (slepton)^4, (squark)^2(slepton)^2, (squark)^2(Higgs)^2, and (slepton)^2(Higgs)^2 with the strength proportional to $y^{ijn}y^*_{kln}$, see Eq. (3.52). For example, the strength of $H_d^{0*}H_d^0(\tilde{e}_L^*\tilde{e}_L + \tilde{e}_R^*\tilde{e}_R)$ couplings is y_e^2.

The parameter μ which has dimension of (mass) appears in the higgsino fermion mass terms

$$\mathcal{L}_W \supset -\mu(\widetilde{H}_u^+\widetilde{H}_d^- - \widetilde{H}_u^0\widetilde{H}_d^0) \tag{3.57}$$

and the scalar Higgs (mass)^2 terms in the scalar potential. As such, μ is often called the Higgs mass parameter. Moreover, the μ-term and the Yukawa couplings are combined to give rise to (scalar)^3 interactions, see Eq. (3.52), of the form

$$\begin{aligned} \mathcal{L}_W \quad \supset \quad & \mu^*(\bar{\tilde{u}}\mathbf{y_u}\tilde{u}H_d^{0*} + \bar{\tilde{d}}\mathbf{y_d}\tilde{d}H_u^{0*} + \bar{\tilde{e}}\mathbf{y_e}\tilde{e}H_u^{0*} \\ & + \bar{\tilde{u}}\mathbf{y_u}\tilde{d}H_d^{-*} + \bar{\tilde{d}}\mathbf{y_d}\tilde{u}H_u^{+*} + \bar{\tilde{e}}\mathbf{y_e}\tilde{\nu}H_u^{+*}) + \text{h.c.}\,. \end{aligned} \tag{3.58}$$

After the electroweak symmetry is broken and the neutral scalar components of H_u and H_d get vacuum expectation values (VEVs), the Yukawa matrices yield the masses and ordinary CKM mixing angles of the quarks and leptons. In addition, the first line in Eq. (3.58) and scalar quartic couplings of the form (squark)^2(Higgs)^2 and (slepton)^2(Higgs)^2 will provide the mass squared matrices for squarks and sleptons.

We now turn to consider the interaction terms described by the Lagrangian of the form (3.44). The first two terms couple the gauginos to (squark, quark), (slepton, lepton), and (Higgs, higgsino) pairs. The gluinos couple only to the (squark, quark) pairs with the gauge coupling g_3. The winos couple to the (Higgs, higgsino) and left-handed (squark, quark), (slepton, lepton) pairs with the gauge coupling g. Lastly, the bino couples to the (scalar, fermion) pairs with the strength proportional to the gauge coupling g' and the weak hypercharges Y. The last term in $\mathcal{L}_{\text{int}}$ corresponds to various (scalar)^4 interactions with the strength proportional to $(g^{(a)})^2$. Some of these terms also contribute to the mass squared matrices of squarks and sleptons after the electroweak symmetry breaking.

3.7.3 Soft supersymmetry breaking in the MSSM

The complete set of soft supersymmetry breaking terms in the MSSM is given by the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{soft}}^{\text{MSSM}} &= -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\right) + \text{h.c.} \\ &- \left(\bar{\tilde{u}}\mathbf{a_u}\widetilde{Q}H_u - \bar{\tilde{d}}\mathbf{a_d}\widetilde{Q}H_d - \bar{\tilde{e}}\mathbf{a_e}\widetilde{L}H_d\right) + \text{h.c.} \\ &-\widetilde{Q}^\dagger \mathbf{m}^2_{\widetilde{\mathbf{Q}}_\mathbf{L}}\widetilde{Q} - \widetilde{L}^\dagger \mathbf{m}^2_{\widetilde{\mathbf{L}}_\mathbf{L}}\widetilde{L} - \bar{\tilde{u}}\mathbf{m}^2_{\tilde{\mathbf{u}}_\mathbf{R}}\bar{\tilde{u}}^\dagger - \bar{\tilde{d}}\mathbf{m}^2_{\tilde{\mathbf{d}}_\mathbf{R}}\bar{\tilde{d}}^\dagger - \bar{\tilde{e}}\mathbf{m}^2_{\tilde{\mathbf{e}}_\mathbf{R}}\bar{\tilde{e}}^\dagger \\ &-m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{h.c.}). \end{aligned} \tag{3.59}$$

The first line of (3.59) describes the gluino, wino, and bino mass terms with complex masses M_3, M_2, and M_1 respectively. The second line contains the (scalar)^3 interactions with 3×3 complex matrices in the family space $\mathbf{a_u}, \mathbf{a_d}, \mathbf{a_e}$. The third line describes the squark and slepton mass terms. Here, $\mathbf{m}^2_{\widetilde{\mathbf{Q}}_\mathbf{L}}, \mathbf{m}^2_{\widetilde{\mathbf{L}}_\mathbf{L}}, \mathbf{m}^2_{\tilde{\mathbf{u}}_\mathbf{R}}, \mathbf{m}^2_{\tilde{\mathbf{d}}_\mathbf{R}}, \mathbf{m}^2_{\tilde{\mathbf{e}}_\mathbf{R}}$ are 3×3 hermitian mass squared matrices in the family space corresponding to $(m^2)^i_j$ in Eq. (3.53). Finally, the last line consists of the mass squared terms, $m^2_{H_u}$ and $m^2_{H_d}$ terms, and the bilinear terms (b terms) of the Higgs scalars. A good review on the theory and experimental implications of the soft supersymmetry breaking in the MSSM is given in [33].

3.7.4 Electroweak symmetry breaking in the MSSM

Now that we have already specified the particle content and constructed the Lagrangian with softly broken supersymmetry of the Minimal Supersymmetric Standard Model. The rest of this chapter will be devoted to the mass matrices of the neutralinos, charginos and selectrons which will be used further in the next chapter.

Before proceeding to the next section, we now list some important results of the electroweak symmetry breaking in the MSSM. First, the neutral components of the Higgs doublets get non-zero vacuum expectation values (VEVs),

$$\langle H_u^0 \rangle = v_u, \qquad \langle H_d^0 \rangle = v_d \tag{3.60}$$

after the symmetry breaking. By redefining the Higgs fields, we can always choose v_u and v_d to be real and positive. As in the Standard Model, these VEVs are connected to the masses of the W and Z gauge bosons,

$$m_W^2 = \frac{g^2 v^2}{2}, \tag{3.61}$$

$$m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2, \tag{3.62}$$

where

$$v^2 = v_u^2 + v_d^2. \tag{3.63}$$

The ratio of the two VEVs is traditionally written as

$$\tan\beta \equiv \frac{v_u}{v_d}. \tag{3.64}$$

Since v_u and v_d are real and positive, then we have $0 < \beta < \frac{\pi}{2}$. At the tree-level, the masses of quarks and leptons are determined by the Yukawa couplings of the superpotential and the parameter β as follows:

$$y_u = \frac{m_u}{v_u} = \frac{g m_u}{\sqrt{2} m_W \sin\beta}; \qquad y_{d,e} = \frac{m_{d,e}}{v_d} = \frac{g m_{d,e}}{\sqrt{2} m_W \cos\beta}, \tag{3.65}$$

for the first generation.

Moreover, from now on we assume that the trilinear couplings are flavor diagonal and are proportional to the corresponding Yukawa couplings as

$$a_f = A_f y_f \tag{3.66}$$

so that the only complex parameters of the soft SUSY breaking Lagrangian are A_f and μ. The discussion of their origin will be postponed until the next chapter.

3.8 Neutralinos and charginos

One of the most important features of the MSSM is that after the electroweak symmetry is broken, in addition to the explicit breaking of supersymmetry, sparticles with the same electric charge can mix together. Then the sparticles listed in Tables 3.1 and 3.2 are no longer the mass eigenstates of the theory. In this section, we discuss the masses and mixing of the electroweak gauginos and higgsinos. Afterwards, those of the selectrons will be given in the next section.

After the electroweak symmetry breaking, the higgsinos and the gauginos mix together. There are two cases of mixing among them: the neutral one and the charged one. The neutral higgsinos ($\widetilde{H}_u^0$ and $\widetilde{H}_d^0$) and the neutral gauginos ($\widetilde{B}$ and $\widetilde{W}^0$) combine to form neutral mass eigenstates called neutralinos denoted by $\widetilde{N}_i$, $i = 1, 2, 3, 4$. The charged higgsinos ($\widetilde{H}_u^+$ and $\widetilde{H}_d^-$) mix with winos ($\widetilde{W}^+$ and $\widetilde{W}^-$) to form two mass eigenstates $\widetilde{C}_i^\pm$ ($i = 1, 2$) called charginos with charges ± 1.

3.8.1 The neutralino mass matrix

The neutralino mass matrix receives contributions from three sources:

1. From the explicit gaugino mass terms in the soft supersymmetry breaking Lagrangian

$$\mathcal{L}_{\text{soft}}^{\text{MSSM}} \supset -\frac{1}{2}M_2\widetilde{W}^0\widetilde{W}^0 - \frac{1}{2}M_1\widetilde{B}\widetilde{B} + \text{h.c.}. \tag{3.67}$$

2. From the superpotential, see Eq. (3.57),

$$\mathcal{L}_W \supset \mu\widetilde{H}_u^0\widetilde{H}_d^0 + \text{h.c.}. \tag{3.68}$$

3. From the Higgs-higgsino-gaugino interactions in (3.44) with the neutral Higgs fields replaced by their VEVs,

$$\mathcal{L}_{\text{int}} \supset -\sqrt{2}g\left[\left(H_u^*\tau^3\widetilde{H}_u\right)\widetilde{W}^0 + \left(H_d^*\tau^3\widetilde{H}_d\right)\widetilde{W}^0\right]$$

$$
\begin{aligned}
& -\sqrt{2}g'\left[\left(H_u^*\left(+\frac{1}{2}\right)\widetilde{H}_u\right)\widetilde{B}+\left(H_d^*\left(+\frac{1}{2}\right)\widetilde{H}_d\right)\widetilde{B}\right]+\text{h.c.} \\
\supset\ & -\frac{\sqrt{2}g}{2}\left[-H_u^{0*}\widetilde{H}_u^0\widetilde{W}^0+H_d^{0*}\widetilde{H}_d^0\widetilde{W}^0\right]-\frac{\sqrt{2}g'}{2}\left[H_u^{0*}\widetilde{H}_u^0\widetilde{B}\right. \\
& \left.-H_d^{0*}\widetilde{H}_d^0\widetilde{B}\right]+\text{h.c.} \\
\supset\ & \frac{g}{\sqrt{2}}\left[-v_u\widetilde{H}_u^0\widetilde{W}^0+v_d\widetilde{H}_d^0\widetilde{W}^0\right]-\frac{g'}{\sqrt{2}}\left[v_u\widetilde{H}_u^0\widetilde{B}\right. \\
& \left.-v_d\widetilde{H}_d^0\widetilde{B}\right]+\text{h.c.} \qquad (3.69)
\end{aligned}
$$

where we have replaced the Higgs fields by their VEVs on the last line.

From Eqs. (3.62), (3.63), (3.64) and

$$
\sin\theta_W=\frac{g'}{\sqrt{g^2+g'^2}},\qquad \cos\theta_W=\frac{g}{\sqrt{g^2+g'^2}}, \qquad (3.70)
$$

where θ_W is the electroweak mixing angle known as the Weinberg angle, we have the relations

$$
\begin{aligned}
s_\beta s_W m_Z &= \frac{v_u g'}{\sqrt{2}}, \quad & s_\beta c_W m_Z &= \frac{v_u g}{\sqrt{2}}, \\
c_\beta s_W m_Z &= \frac{v_d g'}{\sqrt{2}}, \quad & c_\beta c_W m_Z &= \frac{v_d g}{\sqrt{2}}. \qquad (3.71)
\end{aligned}
$$

Here s_β, c_β and s_W, c_W stand for $\sin\beta, \cos\beta$ and $\sin\theta_W, \cos\theta_W$ respectively. From all above, therefore, in the gauge-eigenstate basis $\psi^0=(\widetilde{B},\widetilde{W}^0,\widetilde{H}_d^0,\widetilde{H}_u^0)$, the neutralino mass terms in the Lagrangian are

$$
\mathcal{L}\supset-\frac{1}{2}(\psi^0)^T\mathbf{M}_{\widetilde{N}}\psi^0+\text{h.c.} \qquad (3.72)
$$

with the mass matrix

$$
\mathbf{M}_{\widetilde{N}}=\begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix}. \qquad (3.73)
$$

This mass matrix $\mathbf{M}_{\widetilde{N}}$ is a complex non-hermitian and symmetric matrix, which can be diagonalized by using an orthogonal matrix $\mathbf{N}$,

$$
\mathbf{M}_{\widetilde{N}}^{\text{diag}}=\mathbf{N}^{\mathrm{T}}\mathbf{M}_{\widetilde{N}}\mathbf{N}, \qquad (3.74)
$$

so that $\widetilde{N}_i = \mathbf{N}^{\mathrm{T}}_{ij}\psi^0_j$ are the mass eigenstates. The physical masses must be real and non-negative, and by convention, $m_{\widetilde{N}_1} < m_{\widetilde{N}_2} < m_{\widetilde{N}_3} < m_{\widetilde{N}_4}$. They are given in terms of the parameters M_1, M_2, μ and $\tan\beta$. The lightest neutralino, $\widetilde{N}_1$, is usually assumed to be the lightest supersymmetric particle (LSP).

3.8.2 The chargino mass matrix

The chargino mass matrix receives contributions from three parts:

1. From the gaugino mass terms in the soft supersymmetry breaking Lagrangian

$$\mathcal{L}^{\mathrm{MSSM}}_{\mathrm{soft}} \supset -\frac{1}{2}M_2\left[\widetilde{W}^1\widetilde{W}^1 + \widetilde{W}^2\widetilde{W}^2\right] + \mathrm{h.c.}.$$

Let

$$\widetilde{W}^{\pm} = \frac{1}{\sqrt{2}}\left(\widetilde{W}^1 \mp i\widetilde{W}^2\right). \tag{3.75}$$

Substitute this into $\mathcal{L}^{\mathrm{MSSM}}_{\mathrm{soft}}$, we get

$$\begin{aligned}\mathcal{L}^{\mathrm{MSSM}}_{\mathrm{soft}} &\supset -\frac{1}{4}M_2\left(\widetilde{W}^+\widetilde{W}^+ + \widetilde{W}^+\widetilde{W}^- + \widetilde{W}^-\widetilde{W}^+ + \widetilde{W}^-\widetilde{W}^-\right)\\ &\quad +\frac{1}{4}M_2\left(\widetilde{W}^+\widetilde{W}^+ - \widetilde{W}^+\widetilde{W}^- - \widetilde{W}^-\widetilde{W}^+ + \widetilde{W}^-\widetilde{W}^-\right) + \mathrm{h.c.}\\ &= -\frac{1}{2}M_2\left(\widetilde{W}^+\widetilde{W}^- + \widetilde{W}^-\widetilde{W}^+\right) + \mathrm{h.c.}.\end{aligned} \tag{3.76}$$

2. From the superpotential, see Eq. (3.57),

$$\mathcal{L}_W \supset -\mu\widetilde{H}^+_u\widetilde{H}^-_d + \mathrm{h.c.}. \tag{3.77}$$

3. From the Higgs-higgsino-gaugino couplings in (3.44) with the neutral Higgs scalars replaced by their VEVs,

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} &\supset -\sqrt{2}g\left[\left(H^*_u\tau^1\widetilde{H}_u\right)\widetilde{W}^1 + \left(H^*_u\tau^2\widetilde{H}_u\right)\widetilde{W}^2\right]\\ &\quad -\sqrt{2}g\left[\left(H^*_d\tau^1\widetilde{H}_d\right)\widetilde{W}^1 + \left(H^*_d\tau^2\widetilde{H}_d\right)\widetilde{W}^2\right] + \mathrm{h.c.}\\ &\supset -\frac{g}{\sqrt{2}}\left[H^{0*}_u\widetilde{H}^+_u\widetilde{W}^1 + iH^{0*}_u\widetilde{H}^+_d\widetilde{W}^2 + H^{0*}_d\widetilde{H}^-_d\widetilde{W}^1 - iH^{0*}_d\widetilde{H}^-_d\widetilde{W}^2\right] + \mathrm{h.c.},\end{aligned}$$

or in terms of $\widetilde{W}^{\pm}$,

$$\begin{aligned}\mathcal{L}_{\rm int} \supset & -\frac{g}{2}v_u\left[\widetilde{H}_u^+\left(\widetilde{W}^+ + \widetilde{W}^-\right) - \widetilde{H}_u^+\left(\widetilde{W}^+ - \widetilde{W}^-\right)\right] \\ & -\frac{g}{2}v_d\left[\widetilde{H}_d^-\left(\widetilde{W}^+ + \widetilde{W}^-\right) + \widetilde{H}_d^-\left(\widetilde{W}^+ - \widetilde{W}^-\right)\right] + \text{h.c.} \\ = & -\frac{g}{2}\left[2v_u\widetilde{H}_u^+\widetilde{W}^- + 2v_d\widetilde{H}_d^-\widetilde{W}^+\right] + \text{h.c.}\end{aligned} \tag{3.78}$$

where we have replaced the Higgs fields by their VEVs. Using Eqs. (3.61), (3.63), and (3.64), the Lagrangian (3.78) becomes

$$\mathcal{L}_{\rm int} = -\frac{1}{2}\cdot 2\left[\sqrt{2}s_\beta m_W\widetilde{H}_u^+\widetilde{W}^- + \sqrt{2}c_\beta m_W\widetilde{H}_d^-\widetilde{W}^+\right] + \text{h.c.}. \tag{3.79}$$

In conclusion, in the gauge-eigenstate basis $\psi^{\pm} = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$, the chargino mass terms are

$$\mathcal{L} \supset -\frac{1}{2}(\psi^{\pm})^T\mathbf{M}_{\widetilde{C}}\psi^{\pm} + \text{h.c.} \tag{3.80}$$

where

$$\mathbf{M}_{\widetilde{C}} = \begin{pmatrix} 0 & \mathbf{X}^T \\ \mathbf{X} & 0 \end{pmatrix}; \qquad \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2}s_\beta m_W \\ \sqrt{2}c_\beta m_W & \mu \end{pmatrix}. \tag{3.81}$$

The matrix $\mathbf{X}$ is not hermitian, not symmetric and not real because μ is generally complex. It can be diagonalized by using a biunitary transformation [34]

$$\mathbf{X}_{\rm D} = \mathbf{U}'^*\mathbf{X}\mathbf{V}^{-1}, \tag{3.82}$$

where $\mathbf{U}'$ and $\mathbf{V}$ are unitary matrices and $\mathbf{X}_{\rm D}$ is a diagonal matrix but not yet real. $\mathbf{U}'$ and $\mathbf{V}$ satisfy the relations

$$\mathbf{V}\left(\mathbf{X}^\dagger\mathbf{X}\right)\mathbf{V}^{-1} = \mathbf{U}'^*\left(\mathbf{X}\mathbf{X}^\dagger\right)(\mathbf{U}'^*)^{-1} = \text{diag}\left(|m_{\widetilde{C}_1}|^2, |m_{\widetilde{C}_2}|^2\right). \tag{3.83}$$

Paying attention to only the $SU(2)$ part, the matrix $\mathbf{U}'$ can be parameterized such that

$$\mathbf{U}' = \begin{pmatrix} \cos\frac{\theta_1}{2} & e^{i\phi_1}\sin\frac{\theta_1}{2} \\ -e^{-i\phi_1}\sin\frac{\theta_1}{2} & \cos\frac{\theta_1}{2} \end{pmatrix} \tag{3.84}$$

where the mixing angles θ_1 and ϕ_1 are given by

$$\tan\theta_1 = \frac{2\sqrt{2}m_W\left[M_2^2\cos^2\beta + |\mu|^2\sin^2\beta + |\mu|M_2\sin 2\beta\cos\varphi_\mu\right]^{1/2}}{M_2^2 - |\mu|^2 - 2m_W^2\cos 2\beta} \tag{3.85}$$

and

$$\tan\phi_1 = \frac{|\mu|\sin\varphi_\mu \sin\beta}{M_2\cos\beta + |\mu|\cos\varphi_\mu \sin\beta} \tag{3.86}$$

with $\mu = |\mu|\exp(i\varphi_\mu)$. Similarly, $\mathbf{V}$ can be parameterized as

$$\mathbf{V} = \begin{pmatrix} \cos\frac{\theta_2}{2} & e^{-i\phi_2}\sin\frac{\theta_2}{2} \\ -e^{i\phi_2}\sin\frac{\theta_2}{2} & \cos\frac{\theta_2}{2} \end{pmatrix} \tag{3.87}$$

where the mixing angles θ_2 and ϕ_2 satisfy the relations

$$\tan\theta_2 = \frac{2\sqrt{2}m_W\left[M_2^2\sin^2\beta + |\mu|^2\cos^2\beta + |\mu|M_2\sin 2\beta\cos\varphi_\mu\right]^{1/2}}{M_2^2 - |\mu|^2 + 2m_W^2\cos 2\beta} \tag{3.88}$$

and

$$\tan\phi_2 = \frac{-|\mu|\sin\varphi_\mu\cos\beta}{M_2\sin\beta + |\mu|\cos\varphi_\mu\cos\beta}. \tag{3.89}$$

To make $\mathbf{X}_\mathrm{D}$ real so that its diagonal elements can be interpreted as masses, note that $\mathbf{U}'$ and $\mathbf{V}$ obtained above belong to the $SU(2)$ part of $U(2)$, so we still have our freedom to multiply them by a $U(1)$ factor. We do this only on $\mathbf{U}'$ by letting $\mathrm{U} = \mathbf{H}\mathbf{U}'$ with the U(1) factor

$$\mathbf{H} = \begin{pmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{pmatrix}. \tag{3.90}$$

The phases γ_1 and γ_2 are chosen such that the matrix

$$\mathbf{U}^*\mathbf{X}\mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix} \tag{3.91}$$

is real. Thus the phases γ_1 and γ_2 are the phases of the diagonal elements in Eq. (3.82). The doubly degenerate eigenvalues of the mass matrix (3.91) are

$$\begin{aligned} m_{\widetilde{C}_1}^2, m_{\widetilde{C}_2}^2 &= \frac{1}{2}(M_2^2 + |\mu|^2 + 2m_W^2) \mp \frac{1}{2}\left[(M_2^2 - |\mu|^2)^2 + 4m_W^4\cos^2 2\beta\right. \\ &\quad \left. + 4m_W^2(M_2^2 + |\mu|^2 + 2M_2\mu\cos\varphi_\mu\sin 2\beta)\right]^{1/2}. \end{aligned} \tag{3.92}$$

Therefore, the mass eigenstates are related to the gauge eigenstates according to

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V}\begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}; \quad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U}\begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}. \tag{3.93}$$

It can be noticed that the mixing matrices for the positive charge states and the negative charge states are different.

3.9 Masses and mixing of the selectrons

As mentioned above, the breaking of supersymmetry and electroweak symmetry results in the mixing of sleptons. In this thesis, we will however consider only the selectrons' case, i.e., $\tilde{e}_L$ mixes with $\tilde{e}_R$.

Let consider terms in Lagrangians which contribute to the selectron mass matrix. Note that all neutral Higgs scalars in the coupling terms will be replaced by their VEVs. The selectron mass matrix receives contributions from three sources:

1. From the explicit slepton mass terms and the (scalar)^3 couplings in the soft supersymmetry breaking Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{\text{MSSM}} &\supset (\tilde{\bar{e}}\mathbf{a_e}\widetilde{L}H_d + \text{h.c.}) - \widetilde{L}^\dagger \mathbf{m}^2_{\tilde{\mathbf{L}}_\mathbf{L}}\widetilde{L} - \tilde{\bar{e}}\mathbf{m}^2_{\tilde{\mathbf{e}}_\mathbf{R}}\tilde{\bar{e}}^\dagger \\ &\supset -A_e y_e \tilde{e}_R^* \tilde{e}_L H_d^0 - A_e^* y_e \tilde{e}_L^* \tilde{e}_R H_d^{0*} - m^2_{\tilde{e}_L}\tilde{e}_L^*\tilde{e}_L - m^2_{\tilde{e}_R}\tilde{e}_R^*\tilde{e}_R \\ &= -A_e y_e v_d \tilde{e}_R^* \tilde{e}_L - A_e^* y_e v_d \tilde{e}_L^* \tilde{e}_R - m^2_{\tilde{e}_L}\tilde{e}_L^*\tilde{e}_L - m^2_{\tilde{e}_R}\tilde{e}_R^*\tilde{e}_R \\ &= -A_e m_e \tilde{e}_R^* \tilde{e}_L - A_e^* m_e \tilde{e}_L^* \tilde{e}_R - m^2_{\tilde{e}_L}\tilde{e}_L^*\tilde{e}_L - m^2_{\tilde{e}_R}\tilde{e}_R^*\tilde{e}_R \qquad (3.94)\end{aligned}$$

   with $m_e = y_e v_d$.

2. From the superpotential

$$\begin{aligned}W_{\text{MSSM}} &\supset -\tilde{\bar{e}}\mathbf{y_e}\widetilde{L}H_d + \mu H_u H_d \\ &= y_e \tilde{e}_R^* \tilde{e}_L H_d^0 + \mu\left(H_u^+ H_d^- - H_u^0 H_d^0\right).\end{aligned}$$

   This leads to the scalar potential

$$\begin{aligned}V^W &\supset \mu y_e H_u^0 \tilde{e}_R \tilde{e}_L^* + \mu^* y_e H_u^{0*} \tilde{e}_R^* \tilde{e}_L + y_e^2 \left|H_d^0\right|^2 (\tilde{e}_L^*\tilde{e}_L + \tilde{e}_R^*\tilde{e}_R) \\ &= \mu m_e \tan\beta\ \tilde{e}_R \tilde{e}_L^* + \mu^* m_e \tan\beta\ \tilde{e}_R^* \tilde{e}_L + m_e^2(\tilde{e}_L^*\tilde{e}_L + \tilde{e}_R^*\tilde{e}_R) \qquad (3.95)\end{aligned}$$

3. From the D-term in the scalar potential

$$V \supset \frac{1}{2}\sum_a (g^{(a)})^2 (\phi^* T^a \phi)^2$$

$$\begin{aligned}
&\supset \frac{g^2}{2}\left[H_u^*\tau^3 H_u + H_d^*\tau^3 H_d + \tilde{e}_L^*\tau^3\tilde{e}_L\right]^2 \\
&\quad \frac{g'^2}{2}\left[H_u^*\left(+\frac{1}{2}\right)H_u + H_d^*\left(-\frac{1}{2}\right)H_d + \tilde{e}_L^* Y \tilde{e}_L + \tilde{e}_R^*(+1)\tilde{e}_R\right]^2 \\
&\supset 2\cdot\frac{1}{4}\left(v_d^2 - v_u^2\right)\tilde{e}_L^*\left[g^2\tau^3 - g'^2 Y\right]\tilde{e}_L - 2\cdot\frac{1}{4}\left(v_d^2 - v_u^2\right)g'^2 Y \tilde{e}_R^*\tilde{e}_R.
\end{aligned} \tag{3.96}$$

Here τ^3 is the third component of the weak isospin of the left-handed electron. From Eq. (3.64), after some algebra one gets

$$\cos 2\beta = \frac{v_d^2 - v_u^2}{v_u^2 + v_d^2}. \tag{3.97}$$

Using Eqs. (3.62), (3.70) and the above identity, then (3.96) becomes

$$\begin{aligned}
V &\supset \frac{m_Z^2\cos 2\beta}{g^2+g'^2}\,\tilde{e}_L^*\left[\left(g^2+g'^2\right)\tau^3 - g'^2\left(\tau^3+Y\right)\right]\tilde{e}_L \\
&\quad + m_Z^2\cos 2\beta\frac{g'^2}{g^2+g'^2}Q_e\tilde{e}_R^*\tilde{e}_R \\
&= m_Z^2\cos 2\beta\tilde{e}_L^*\left[\tau^3 - Q_e\sin^2\theta_W\right]\tilde{e}_L + m_Z^2\cos 2\beta\cdot Q_e\sin^2\theta_W\tilde{e}_R^*\tilde{e}_R.
\end{aligned} \tag{3.98}$$

To sum up, the mass terms of the selectrons is expressed in the Lagrangian as

$$\mathcal{L}_{\tilde{e}} = -\begin{pmatrix} \tilde{e}_L^* & \tilde{e}_R^* \end{pmatrix}\widetilde{\mathbf{M}}_e^2\begin{pmatrix} \tilde{e}_L \\ \tilde{e}_R \end{pmatrix}, \tag{3.99}$$

where the mass-squared matrix is

$$\widetilde{\mathbf{M}}_e^2 = \begin{pmatrix} m_{\tilde{e}_L}^2 + \cos 2\beta\left(\tau^3 - Q_e\sin^2\theta_W\right)m_Z^2 + m_e^2 & m_e\left(A_e^* + \mu\tan\beta\right) \\ m_e\left(A_e + \mu^*\tan\beta\right) & m_{\tilde{e}_R}^2 + \cos 2\beta\cdot Q_e\sin^2\theta_W m_Z^2 + m_e^2 \end{pmatrix}. \tag{3.100}$$

This mass-squared matrix can be diagonalized as

$$\mathbf{S}_e^\dagger\widetilde{\mathbf{M}}_e^2\mathbf{S}_e = \begin{pmatrix} M_{\tilde{e}_1}^2 & 0 \\ 0 & M_{\tilde{e}_2}^2 \end{pmatrix} \tag{3.101}$$

where $\mathbf{S}_e$ is a unitary matrix

$$\mathbf{S}_e = \begin{pmatrix} \cos\theta_e & -e^{-i\varphi_e}\sin\theta_e \\ e^{i\varphi_e}\sin\theta_e & \cos\theta_e \end{pmatrix}. \tag{3.102}$$

Hence, the mass eigenstates and the gauge eigenstates are related to each other according to

$$\begin{pmatrix} \tilde{e}_L \\ \tilde{e}_R \end{pmatrix} = \mathbf{S}_e \begin{pmatrix} \tilde{e}_1 \\ \tilde{e}_2 \end{pmatrix}. \tag{3.103}$$

The mixing angles θ_e and φ_e are obtained by solving the equations

$$\tan 2\theta_e = \frac{2m_e|A_e^* + \mu\tan\beta|}{\cos 2\beta(\tau^3 - 2Q_e\sin^2\theta_W)m_Z^2 + m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2} \tag{3.104}$$

and

$$\tan\varphi_e = -\frac{|\mu|\tan\beta\sin\varphi_\mu - |A_e|\sin\theta_{A_e}}{|\mu|\tan\beta\cos\varphi_\mu + |A_e|\sin\theta_{A_e}}, \tag{3.105}$$

where $\theta_{A_e} = \arg(A_e)$. The selectron masses are obtained as [35]

$$\begin{aligned} M_{\tilde{e}_1,\tilde{e}_2}^2 &= \frac{1}{2}(2m_e^2 + \cos 2\beta\cdot\tau^3 m_Z^2 + m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2) \\ &\mp\frac{1}{2}\left\{\left[\cos 2\beta(\tau^3 - 2Q_e\sin^2\theta_W)m_Z^2 + m_{\tilde{e}_L}^2 - m_{\tilde{e}_R}^2\right]^2\right. \\ &\left.+4m_e^2|A_e^* + \mu\cot\beta|^2\right\}^{\frac{1}{2}}. \end{aligned} \tag{3.106}$$

We finally note that the general form of the sfermion mass-squared matrix is

$$\widetilde{\mathrm{M}}_f^2 = \begin{pmatrix} m_{\tilde{f}_L}^2 + \cos 2\beta\left(\tau_f^3 - Q_f\sin^2\theta_W\right)m_Z^2 + m_f^2 & m_f\left(A_f^* + \mu R_f\right) \\ m_f\left(A_f + \mu^* R_f\right) & m_{\tilde{f}_R}^2 + \cos 2\beta\cdot Q_f\sin^2\theta_W m_Z^2 + m_f^2 \end{pmatrix} \tag{3.107}$$

where τ_f^3 is the third component of the weak isospin of left-handed fermion, and R_f is defined by

$$R_f = \begin{cases} \cot\beta & \text{for } \tau_f^3 = \frac{1}{2} \\ \tan\beta & \text{for } \tau_f^3 = -\frac{1}{2} \end{cases}. \tag{3.108}$$