CHAPTER I

INTRODUCTION

The Scottish botanist Robert Brown in 1829 has found that the small but microscopically visible particles such as pollen dust and very fine particles of minerals, when suspended in a fluid with approximately the same density, exhibit a constant and apparently irregular zigzag motion. For this reason, the motion is known as "Brownian motion" and the particles undergo this motion is called "Brownian particles". The physical theory of Brownian motion was first described by Einstein [6] in 1905 and later further developed by P. Langevin and others. The Brownian motion is one of the most important phenomena since it has subsequently always been included among the best direct proofs of the existence of the molecule. Einstein's theory had a great impact at that time on many branches in physics and also greatly influenced pure mathematics i.e., the theory of stochastic processes.

Although this Brownian motion theory can be used to describe many physical situations very well, it can not be used for the case where the Brownian particle is very small so that it needs the quantum-mechanical treatment. This comes from the fact that the usual Langevin's equation can not be applied in this case. By this reason, many authors have tried to find the way to constitute the theory of "quantum Brownian motion". The most common approaches have been based on quantum mechanical Langevin equations or associated quantum master equations which have been successfully used to describe damping phenomena. Unfortunately, the formal simplicity of quantum Langevin equation and master equation could provide concrete results only for a system which allows for a perturbative treatment of the coupling to the environment heat bath. Basically this restricts the approach to weakly damped system. On the other hand, the functional integral description of damped quantum system pioneered by Feynman and vernon [2,3] allows for a study of the quantum mechanical dynamics at arbitrarily low temperatures and for arbitrarily strong damping. In particular, Caldeira and Leggett [39] have presented a detailed study of quantum Brownian motion in the case of frequency-independent (Ohmic) dissipation.

The functional integral or path integral approach in quantum Brownian motion theory has led to an understanding of many phenomena. One of the important phenomena is the quantum-statistical decay for quantum dissipative systems [16]. This phenomenon deals with a quantum dissipative system which is placed in the external metastable potential e.g., a metastable quadratic-plus-cubic potential well. One of the important quantities in this phenomena is the escape rate i.e., the rate for which the system escape out of the metastable potential. The problem of escape from a metastable state plays a central role in many scientific areas including low-temperature physics, nuclear physics, chemical kinetics, and transport in biomolecules. The theoretical description in this phenomena has been continuously developed since the day of Arrhenius [28] in 1889 by many authors such as Kramer [37,41] in 1940. Soon after that, the quantum-mechanical version in this theory has been developed by various methods by many authors especially Caldeira and Leggett [17,40], who studied quantum tunneling in the presence of dissipation at zero temperature by the functional integral method. Out of several methods, the functional integral method has provided a unified description of this phenomena in the entire temperature range [16,36].

Normally, the problem is formulated when the system in question can be treated as a Brownian particle described by only one coordinate, and the environment is modeled by the set of uncoupled harmonic oscillators which is linearly coupling with Brownian particle. From this point of view, under the functional integral approach, the effective action of a Brownian particle will contain the nonlocal term described by the damping kernel. In this thesis, our system i.e., a vortex, which can be treated as a Brownian particle, is described by two coordinates instead of one. These two coordinates couple with each other via their velocity due to the presence of the Magnus force. In our system, since a vortex can escape in one direction only, the effective action of a vortex corresponding to this escaping coordinate will also contain the nonlocal term but, similar to the work of P.Ao and D.J. Thouless [19], this nonlocal term is not described by only one damping kernel as in the above case, it is also described by another damping kernel called "anomalous damping kernel". This anomalous damping kernel appears since the escaping coordinate of a vortex is coupled linearly with the velocity of another degree of freedom of a vortex.

The purpose of this thesis is to show how the anomalous damping kernel affects the important quantities and behaviors in the escape processes such as crossover temperature, escape rate, and localization of a vortex. The outline of our work is as follows:

In chapter II, some basic notions about the classical and quantum statistical mechanics concerning canonical distribution, partition function, and free energy, and the derivation of density operator and density matrix in quantum statistical mechanics will be reviewed. Moreover, the path integral formulation of the density matrix and its interpretation will be presented.

In chapter III, a review of the history of the foundation of Brownian motion and some of the important basic ideas about the classical Brownian motion concerning Gaussian random process, method of Rice, Fokker-Plank equation, and Langevin equation will be made. The limitation of classical Brownian motion will be discussed and the system-plus-reservoir model or "Caldeira-Leggett model" will be reviewed. Based on this model, a review of the derivation of reduced density matrix and reduced partition function under the functional integral approach in quantum Brownian motion theory will be made.

In chapter IV, the functional integral approach in quantum Brownian motion theory will be applied to study the problem of a vortex escaping out of a metastable potential by first reviewing the concept of the Magnus force and then stating the Hamiltonian of the problem. Second, reviewing the escape rate formula and explaining the processes in deriving the effective one-dimensional reduced partition function. Third, the escape rate formula of a vortex above the crossover temperature, which is divided into two cases due to the value of the frequency of the oscillation in the harmonic pinning potential in stable direction, is derived through the reduced partition function by the use of Affleck's formula [31]. Moreover, two important theorems concerning the existence of crossover temperature and the dependence of dissipation and Magnus force strength on the crossover temperature, which again depend significantly on the value of the frequency of the oscillation in the harmonic pinning potential in stable direction, will be stated and analyzed respectively. Finally, the localization and the effective mass of a vortex will be discussed.

In the last chapter, conclusion and discussion of this study will be made.