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APPENDIX A

ELIMINATION OF THE ENVIRONMENTAL COORDINATES

From eq. (3.79), by omitting the subscript α on the environmental coordinate q_α, c_α, and m_α, we have the expression

$$\oint Dq(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^U\left(\frac{m}{2}\left(\dot{q}^2+\omega^2q^2\right)-cqx\right)d\tau\right\}, \tag{A.1}$$

where $\oint Dq(\tau)=\int_{-\infty}^{\infty}dq'\int_{q(0)=q'}^{q(U)=q'}Dq(\tau)$.

To evaluate the sum over all close paths in eq. (A.1), we first consider

$$\int_{q(0)=q'}^{q(U)=q'}Dq(\tau)\exp\left\{\int_0^U\left(\frac{m}{2}\left(\dot{q}^2+\omega^2q^2\right)-cqx\right)d\tau\right\}. \tag{A.2}$$

Eq. (A.2) is the standard form of the path integration of forced harmonic oscillator. From refs. [2] or [3] , eq. (A.2) is equal to

$$\left(\frac{m\omega}{2\pi\hbar\sinh(\omega U)}\right)^{1/2}\exp\left(-\frac{1}{\hbar}\Phi^E\left[x(\tau),q'\right]\right), \tag{A.3}$$

where

$$\begin{aligned}\Phi^E\left[x(\tau),q'\right]=&\frac{m\omega}{\sinh(\omega U)}\left[\cosh(\omega U)-1\right]q'^2\\&-\frac{cq'}{\sinh(\omega U)}\left[\int_0^U\left(\sinh\left[\omega(U-\tau)\right]+\sinh(\omega\tau)\right)x(\tau)d\tau\right]\\&-\frac{c^2}{m\omega\sinh(\omega U)}\int_0^U\int_0^\tau\sinh\left[\omega(U-\tau)\right]\sinh\omega\tau' x(\tau)x(\tau')d\tau'd\tau\end{aligned} \tag{A.4}$$

From eqs. (A.1) –(A.3), it is clear that

$$\oint Dq(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^U\left(\frac{m}{2}\left(\dot{q}^2+\omega^2q^2\right)-cqx\right)d\tau\right\}=\left(\frac{m\omega}{2\pi\hbar\sinh(\omega U)}\right)^{1/2}\int_{-\infty}^{\infty}e^{-\frac{1}{\hbar}\Phi^E\left[x(\tau),q'\right]}dq'. \tag{A.5}$$

By using eq. (A.4) and the formulae $\int_{-\infty}^{\infty}\exp\left(ax^2+bx+c\right)dx=\sqrt{\pi/-a}\exp\left(c-b^2/4a\right)$ and $\cosh(\omega U)-1=2\sinh^2(\omega U/2)$, eq. (A.5) can be written in the form

$$\oint Dq(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^U\left(\frac{m}{2}\left(\dot{q}^2+\omega^2 q^2\right)-cqx\right)d\tau\right\}=$$

$$\frac{1}{2\sinh\left(\frac{\omega U}{2}\right)}\exp\left\{\frac{c^2}{4m\omega\hbar\sinh(\omega U)[\cosh(\omega U)-1]}\right.$$

$$\times\int_0^U\int_0^U\{\sinh[\omega(U-\tau)]+\sinh(\omega\tau)\}\{\sinh[\omega(U-\tau')]+\sinh(\omega\tau')\}x(\tau)x(\tau')d\tau' d\tau$$

$$\left.+\frac{c^2}{m\omega\hbar\sinh(\omega U)}\int_0^U\int_0^\tau\sinh[\omega(U-\tau)]\sinh(\omega\tau')x(\tau)x(\tau')d\tau' d\tau\right\}.$$

(A.6)

In the exponent of eq. (A.6), we can change $\int_0^U d\tau\int_0^U d\tau'$ in the first term to $2\int_0^U d\tau\int_0^\tau d\tau'$ since the integrand is invariant under the interchange between τ and τ'. After this change, by using some hyperbolic function properties (it depends on one's experiences), one can show that

$$\oint Dq(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^U\left(\frac{m}{2}\left(\dot{q}^2+\omega^2 q^2\right)-cqx\right)d\tau\right\}=\frac{1}{2\sinh(\omega_\alpha U/2)}\times$$

$$\exp\left\{\frac{c^2}{2m\omega\hbar}\int_0^U\int_0^\tau\frac{\cosh(\omega[(\tau-\tau')-U/2])}{\sinh\left(\frac{\omega U}{2}\right)}x(\tau)x(\tau')d\tau' d\tau\right\} \quad \text{(A.7)}$$

Since, from eq. (A.7), the double integration $\int_0^U d\tau\int_0^\tau d\tau'$ in the exponent implies that $\tau>\tau'$, $(\tau-\tau')$ can be replaced by $|\tau-\tau'|$. After this replacement, the integrand of the exponent of eq. (A.7) is now invariant under the interchange between τ and τ' so the double integration $\int_0^U d\tau\int_0^\tau d\tau'$ can be changed to $\frac{1}{2}\int_0^U d\tau\int_0^U d\tau'$. Now, eq. (A.7) becomes (recalling the subscript α on $q_\alpha c_\alpha$, and m_α)

$$\oint Dq(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^U\left(\frac{m_\alpha}{2}\left(\dot{q}_\alpha^2+\omega_\alpha^2 q_\alpha^2\right)-c_\alpha q_\alpha x\right)d\tau\right\}=\frac{1}{2\sinh(\omega_\alpha U/2)}\times$$

$$\exp\left\{\frac{c_\alpha^2}{4m_\alpha\omega_\alpha\hbar}\int_0^U\int_0^U\frac{\cosh(\omega_\alpha[|\tau-\tau'|-U/2])}{\sinh(\omega_\alpha U/2)}x(\tau)x(\tau')d\tau' d\tau\right\}$$

(A.8)

APPENDIX B

ELIMINATION OF THE COORDINATE x OF A VORTEX

Since $\gamma_n = \gamma_{-n}, x_{-n} = x_n^*$ (since $x(\tau)$ must be real), and $\nu_0 = 0$, eq. (4.31) can be written as

$$\begin{aligned}
\frac{-U}{2\hbar}\sum_{n=-\infty}^{\infty}\gamma_n|x_n|^2 - \frac{UM\Omega}{\hbar}\sum_{n=-\infty}^{\infty}\nu_n y_n x_{-n} &= -\frac{U}{2\hbar}\gamma_0 x_0^2 - \frac{U}{\hbar}\sum_{n=1}^{\infty}\gamma_n|x_n|^2 - \frac{UM\Omega}{\hbar}\sum_{n=1}^{\infty}\nu_n y_n x_{-n} \\
&\quad - \frac{UM\Omega}{\hbar}\sum_{n=1}^{\infty}\nu_{-n} y_{-n} x_n \\
&= -\frac{Uk_x}{2\hbar}x_0^2 - \frac{U}{\hbar}\sum_{n=1}^{\infty}\gamma_n|x_n|^2 \\
&\quad - \frac{UM\Omega}{\hbar}\sum_{n=1}^{\infty}\left(\nu_n y_n x_{-n} + \nu_{-n} y_{-n} x_n\right).
\end{aligned} \tag{B.1}$$

Now, let us define $x_n' = \operatorname{Re} x_n, y_n' = \operatorname{Re} y_n, x_n'' = \operatorname{Im} x_n$, and $y_n'' = \operatorname{Im} y_n$. By these definitions and the properties $\nu_{-n} = -\nu_n$, $x_{-n} = x_n^*$, and $y_{-n} = y_n^*$, eq. (B.1) can be expressed, after completing the square, in the form

$$\begin{aligned}
-\frac{U}{2\hbar}\sum_{n=-\infty}^{\infty}\gamma_n|x_n|^2 - \frac{UM\Omega}{\hbar}\sum_{n=-\infty}^{\infty}\nu_n y_n x_{-n} &= -\frac{U}{2\hbar}k_x x_0^2 + \sum_{n=1}^{\infty}\left(-\frac{U\gamma_n}{\hbar}\right)\left(x_n' + \frac{iM\Omega\nu_n y_n''}{\gamma_n}\right)^2 \\
&\quad + \sum_{n=1}^{\infty}\left(-\frac{U\gamma_n}{\hbar}\right)\left(x_n'' - \frac{iM\Omega\nu_n y_n'}{\gamma_n}\right)^2 + \sum_{n=1}^{\infty}\left(-\frac{UM^2\Omega^2}{\hbar}\right)\frac{\nu_n^2}{\gamma_n}|y_n|^2.
\end{aligned} \tag{B.2}$$

Substituting eq. (B.2) for the exponent of the integrand of eq. (4.24) and using eq. (4.26) we obtain

$$\begin{aligned}
\oint Dx(\tau)\exp\left(-S_{X,1}^{E}[y,x]/\hbar\right)F^{E}[x] &= \Bigg\{\sqrt{\frac{M}{2\pi\hbar U}}\int_{-\infty}^{\infty}\exp\left[-\left(\frac{Uk_x}{2\hbar}\right)x_0^2\right]dx_0\cdot\frac{MU}{\pi\hbar}\times \\
&\quad \left(\prod_{n=1}^{\infty}\nu_n\int_{-\infty}^{\infty}\exp\left[-\frac{U\gamma_n}{\hbar}\left(x_n' + \frac{iM\Omega\nu_n y_n''}{\gamma_n}\right)^2\right]dx_n'\right)\times \\
&\quad \left(\prod_{n=1}^{\infty}\nu_n\int_{-\infty}^{\infty}\exp\left[-\frac{U\gamma_n}{\hbar}\left(x_n'' - \frac{iM\Omega\nu_n y_n'}{\gamma_n}\right)^2\right]dx_n''\right)\Bigg\} \\
&\quad \times\exp\left(-\frac{UM^2\Omega^2}{\hbar}\sum_{n=1}^{\infty}\frac{\nu_n^2}{\gamma_n}|y_n|^2\right).
\end{aligned} \tag{B.3}$$

After evaluating the usual Gaussian integrals in eq. (B.3) and recalling the definition of γ_n in eq. (4.31), eq. (B.3) can be written as

$$\oint Dx(\tau)\exp\left(-S_{X,I}^{E}[y,x]/\hbar\right)F^{E}[x] = \left[\frac{1}{U\omega_x}\prod_{n=1}^{\infty}\left(\frac{M\nu_n^2}{M\nu_n^2 + M\omega_x^2 + \xi_n}\right)\right]\times$$
$$\exp\left(-\frac{UM^2\Omega^2}{\hbar}\sum_{n=1}^{\infty}\frac{\nu_n^2}{\gamma_n}|y_n|^2\right) \qquad \text{(B.4)}$$

where $\omega_x^2 = k_x/M$.

APPENDIX C

EVALUATION OF THE SEMICLASSICAL EFFECTIVE ACTION

About $y = 0$, the potential $V(y)$ can be written, by Taylor series, as

$$V(y) = \frac{V''(0)}{2} y^2 + O(y^3) \quad \text{(since } V(0) = 0 = V'(0)\text{)}. \tag{C.1}$$

Similarly, about $y = y_b$, we have

$$V(y) = V_b + \frac{V''(y_b)}{2}(y - y_b)^2 + O((y - y_b)^3) \quad \text{(since } V'(y_b) = 0\text{)}. \tag{C.2}$$

Since we have to find the "semiclassical" effective action $S_{eff}^{E(0)}$ and $S_{eff}^{E(b)}$ through eq. (4.51), eqs. (C.1) and (C.2) can be approximately written as

$$V(y) = \frac{1}{2} M\omega_0^2 y^2 \quad \text{(since } V''(0) = M\omega_0^2\text{)}, \tag{C.3}$$

and

$$V(y) = V_b - \frac{1}{2} M\omega_b^2 (y - y_b)^2 \quad \text{(since } V''(y_b) = -M\omega_b^2\text{)}, \tag{C.4}$$

respectively. About $y = 0$, let us consider $\int_0^U V(y) d\tau$. From eq. (C.3), we have

$$\int_0^U V(y) d\tau = \frac{1}{2} M\omega_0^2 \int_0^U y^2(\tau) d\tau . \tag{C.5}$$

Substituting eq (4.30) for $y(\tau)$ in eq (C.5), we obtain

$$\int_0^U V(y) d\tau = \frac{1}{2} M\omega_0^2 U \sum_{n=-\infty}^{\infty} |y_n|^2 . \tag{C.6}$$

To arrive this result, we have used the orthogonality relation $\int_0^U e^{i\nu_n\tau} e^{-i\nu_m\tau} d\tau = U\delta_{n,m}$.

Similarly, about $y = y_b$, we have

$$\int_0^U V(y) d\tau = V_b U - \frac{1}{2} M\omega_b^2 \int_0^U (y(\tau) - y_b)^2 d\tau . \tag{C.7}$$

Let $y(\tau) = y_b + \sum_{n=-\infty}^{\infty} \zeta_n e^{i\nu_n\tau}$. It is clear from eq. (4.30) that

$$y_0 = y_b + \zeta_0 \quad \text{and} \quad y_n = \zeta_n \text{ for all } n \neq 0 . \tag{C.8}$$

Now, by using the orthogonality relation of $e^{i\nu_n\tau}$, eq. (C.7) becomes

$$\int_0^U V(y)d\tau = V_b U - \frac{1}{2}M\omega_b^2 U \sum_{n=-\infty}^{\infty}|\zeta_n|^2 . \quad \text{(C.9)}$$

Inserting eq. (C.6) into eq. (4.51), we get

$$S_{\text{eff}}^{\text{E(0)}}[y] = \frac{1}{2}mU \sum_{n=-\infty}^{\infty}\lambda_n|y_n|^2 + \frac{1}{2}M\omega_0^2 U \sum_{n=-\infty}^{\infty}|y_n|^2 .$$

By using the properties $y_{-n} = y_n^*$ and $\lambda_{-n} = \lambda_n$, the above equation can be written in the form

$$S_{\text{eff}}^{\text{E(0)}}[y] = \frac{1}{2}MU\lambda_0^{(0)}y_0^2 + MU\sum_{n=1}^{\infty}\lambda_n^{(0)}|y_n|^2 \;;\; \lambda_n^{(0)} = \nu_n^2 + \omega_0^2 + \nu_n\hat{\gamma}_M(\nu_n). \quad \text{(C.10)}$$

Similarly, when inserting eq. (C.9) into eq. (4.51), we obtain

$$S_{\text{eff}}^{\text{E(b)}}[y] = V_b U + \frac{1}{2}MU\sum_{n=-\infty}^{\infty}\lambda_n|y_n|^2 - \frac{1}{2}M\omega_b^2 U \sum_{n=-\infty}^{\infty}|\zeta_n|^2 . \quad \text{(C.11)}$$

By using eq. (C.8) and the properties $\lambda_{-n} = \lambda_n$, $y_{-n} = y_n^*$ and $\zeta_{-n} = \zeta_n^*$, eq. (C.11) becomes

$$S_{\text{eff}}^{\text{E(b)}}[y] = V_b U + \frac{1}{2}MU\lambda_0^{(b)}(y_0 - y_b)^2 + MU\sum_{n=1}^{\infty}\lambda_n^{(b)}|y_n|^2 \;;\lambda_n^{(b)} = \nu_n^2 - \omega_b^2 + \nu_n\hat{\gamma}_M(\nu_n) \quad \text{(C.12)}$$

Since, in our problem, $S_{\text{eff}}^{\text{E(b)}}[y]$ will be used to evaluate the reduced partition function $Z_{\text{d}}^{(\text{b})}$ in eq. (4.48) only, it is clear from the function measure (4.26) (here, x_n must be replaced by y_n) that we can write $(y_0 - y_b)$ in the second term of eq. (C.12) as y_0 without affecting the reduced partition function $Z_{\text{d}}^{(\text{b})}$. By this reason, eq. (C.12) can be written in the form

$$S_{\text{eff}}^{\text{E(b)}}[y] = V_b U + \frac{1}{2}MU\lambda_0^{(b)}y_0^{\,2} + mU\sum_{n=1}^{\infty}\lambda_n^{(b)}|y_n|^2 \;;\; \lambda_n^{(b)} = \nu_n^2 - \omega_b^2 + \nu_n\hat{\gamma}_M(\nu_n). \quad \text{(C.13)}$$

APPENDIX D
LINEARIZATION OF THE EQUATION OF MOTION

From eq. (4.44), we have the equation of motion

$$-M\ddot{y}_c(\tau)+V'(y_c)+2y_c(\tau)\int_0^U[g(\tau-\tau')-k(\tau-\tau')]d\tau'$$

$$-2\int_0^U[g(\tau-\tau')-k(\tau-\tau')]y_c(\tau')d\tau'=0. \qquad \text{(D.1)}$$

Inserting eq. (4.68) into the third term of eq. (D.1) and using the orthogonality relation $\int_0^U e^{i\nu_n\tau}e^{-i\nu_m\tau}d\tau=U\delta_{n,m}$ with the fact that $F_0=1$, eq. (D.1) can be written as

$$-M\ddot{y}_c(\tau)+V'(y_c)+M\Omega^2y_c(\tau)-2\int_0^U[g(\tau-\tau')-k(\tau-\tau')]y_c(\tau')d\tau'=0. \qquad \text{(D.2)}$$

Slightly below T_0, $y_c(\tau)$ can be replaced by $y_B(\tau)$ expressed in eq. (4.67) and U is now approximately replaced by $U_0=2\pi\ \ \beta_0\hbar=2\pi k_BT_0\ \ \hbar$. After this replacement, eq. (D.2) becomes

$$M\varepsilon\omega_R^2\cos(\omega_R\tau)+V'[y_b+\cos(\omega_R\tau)]+M\Omega^2y_b+M\Omega^2\varepsilon\cos(\omega_R\tau)$$

$$-2y_b\int_0^{U_0}[g(\tau-\tau')-k(\tau-\tau')]d\tau'-2\varepsilon\int_0^U[g(\tau-\tau')-k(\tau-\tau')]\cos(\omega_R\tau')d\tau'=0,$$

(D.3)

where $\omega_R=2\pi/\beta_0\hbar$. When inserting eq. (4.68) (U is now equal to U_0) in the fifth term of eq. (D.3) and using the orthogonality relation of $e^{i\nu_n\tau}$ (ν_n is now equal to $2n\pi/\beta_0\hbar$), one can see that the fifth term will cancel with the third term. In the second term of eq. (D.3), expanding $V'[y_b+\varepsilon\cos(\omega_R\tau)]$ about y_b and using the fact that $O(\varepsilon^2)=0$, we get $V'[y_b+\varepsilon\cos(\omega_R\tau)]=V''(y_b)\varepsilon\cos(\omega_R\tau)$ since $V'(y_b)=0$. By these reasons, eq. (D.3) can be written in the form

$$M\varepsilon\omega_R^2\cos(\omega_R\tau)+\varepsilon V''(y_b)\cos(\omega_R\tau)+M\Omega^2\varepsilon\cos(\omega_R\tau)$$

$$-2\varepsilon\int_0^{U_0}[g(\tau-\tau')-k(\tau-\tau')]\cos(\omega_R\tau')d\tau'=0. \qquad \text{(D.4)}$$

Now, let us consider

$$\int_0^{U_0}[g(\tau-\tau')-k(\tau-\tau')]\cos(\omega_R\tau')d\tau'=\mathrm{Re}\left\{\int_0^{U_0}[g(\tau-\tau')-k(\tau-\tau')]e^{i\omega_R\tau'}d\tau'\right\}. \qquad \text{(D.5)}$$

By using eq. (4.68) and the orthogonality relation of $e^{i\nu_n\tau}$ with the fact that $\omega_R = 2\pi / \beta_0\hbar = \nu_1$, one can show that

$$\int_0^{U_0}\left[g(\tau-\tau')-k(\tau-\tau')\right]e^{i\omega_R\tau'}d\tau' = \frac{M\Omega^2}{2}F_1 e^{i\omega_R\tau}. \tag{D.6}$$

Inserting eq. (D.6) into eq. (D.5) and recalling the definition of F_n form eq. (4.68), we obtain

$$\int_0^{U_0}\left[g(\tau-\tau')-k(\tau-\tau')\right]\cos(\omega_R\tau')d\tau' = \frac{M\Omega^2}{2}\left[\frac{M\omega_x^2+\xi_1}{M\omega_R^2+M\omega_x^2+\xi_1}-\frac{\xi_1}{M\Omega^2}\right]\cos(\omega_R\tau). \tag{D.7}$$

Inserting eq. (D.7) into eq. (D.4) and using the relation in eq. (4.50) with the fact that $\nu_1 = \omega_R$, we obtain

$$\left\{M\omega_R^2+V''(y_b)+M\Omega^2-M\Omega^2\left[\frac{\omega_x^2+\omega_R\hat{\gamma}(\omega_R)}{\omega_R^2+\omega_x^2+\omega_R\hat{\gamma}(\omega_R)}-\frac{\omega_R\hat{\gamma}(\omega_R)}{\Omega^2}\right]\right\}\varepsilon\cos(\omega_R\tau)=0 \tag{D.8}$$

APPENDIX E

PROOF OF $\eta_0 > \eta_c$

We have already known that η_c obeys the equation

$$\frac{\Omega^2}{1+\frac{2\eta_c}{n\pi}\int_0^\infty \frac{j(\omega)}{\omega^3}d\omega} = \omega_b^2 \tag{E.1}$$

From eq. (4.74) (with $\omega_x = 0$) and the definition of $\hat{\gamma}_\eta$ in eq. (4.75), it is clear that η_0 obeys the equation

$$\frac{2\Omega^2}{1+\frac{2\eta_0}{n\pi}\int_0^\infty \frac{j(\omega)}{\omega(\omega^2+\omega_R^2)}d\omega} = \omega_b^2 \tag{E.2}$$

Equating the left hand side of eq.(E.1) with the left hand side of eq.(E.2), we obtain

$$\begin{aligned}\frac{2\eta_0}{n\pi}\int_0^\infty \frac{j(\omega)}{\omega(\omega^2+\omega_R^2)}d\omega &= 1-\frac{4\eta_c}{n\pi}\int_0^\infty \frac{j(\omega)}{\omega^3}d\omega \\ &> \frac{2\eta_c}{n\pi}\int_0^\infty \frac{j(\omega)}{\omega^3}d\omega \\ &\geq \frac{2\eta_c}{n\pi}\int_0^\infty \frac{j(\omega)}{\omega(\omega^2+\omega_R^2)}d\omega\end{aligned} \tag{E.3}$$

Now, it is clear from (E.3) that $\eta_0 > \eta_c$.

CURRICULUM VITAE

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