ผลเฉลยซูเปอร์ซิมเมทริกของเกจซูเปอร์กราวิตี 7 มิติและดูออลิตี AdS_7/CFT_6

นายภัทรดนัย นุชจิโน

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรดุษฎีบัณฑิต
สาขาวิชาฟิสิกส์ ภาควิชาฟิสิกส์
คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย
ปีการศึกษา 2562

SUPERSYMMETRIC SOLUTIONS OF SEVEN-DIMENSIONAL GAUGED SUPERGRAVITIES AND AdS_7/CFT_6 DUALITY

Mr. Patharadanai Nuchino

A Dissertation Submitted in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy Program in Physics
Department of Physics
Faculty of Science
Chulalongkorn University
Academic Year 2019

Thesis Title	SUPERSYMMETRIC SOLUTIONS OF SEVEN-DIMENSIONAL GAUGED SUPERGRAVITIES AND AdS_7/CFT_6 DUALITY
By	Mr. Patharadanai Nuchino
Field of Study	Physics
Thesis Advisor	Associate Professor Parinya Karndumri, Ph.D.

Accepted by the Faculty of Science, Chulalongkorn University in Partial Fulfillment of the Requirements for the Doctoral Degree

. . . . Dean of the Faculty of Science
(Professor Polkit Sangvanich, Ph.D.)

THESIS COMMITTEE

. . . . Chairman
(Assistant Professor Boonchoat Paosawatyanyong, Ph.D.)

. . . . Thesis Advisor
(Associate Professor Parinya Karndumri, Ph.D.)

. . . . Examiner
(Assistant Professor Auttakit Chatrabhuti, Ph.D.)

. . . . Examiner
(Assistant Professor Pawin Ittisamai, Ph.D.)

. . . . External Examiner
(Assistant Professor Pitayuth Wongjun, Ph.D.)

ภัทรดนัย นุชจิโน : ผลเฉลยซูเปอร์ซิมเมทริกของเกจซูเปอร์กราวิตี 7 มิติ และดูออลิตี AdS_7/CFT_6. (SUPERSYMMETRIC SOLUTIONS OF SEVEN-DIMENSIONAL GAUGED SUPERGRAVITIES AND AdS_7/CFT_6 DUALITY) อ.ที่ปรึกษาวิทยานิพนธ์หลัก : รศ.ดร.ปริญญา การดำริห์, 226 หน้า.

การอธิบายทฤษฎีสนามควอนตัมโดยใช้ดูออลิตี AdS/CFT สามารถแผ่ขยายการประยุกต์ใช้ทฤษฎีสตริงในบริบทของหลักโฮโลกราฟีกได้ งานวิจัยชิ้นนี้ทำการศึกษาผลเฉลยซูเปอร์ซิมเมทริกของเกจซูเปอร์กราวิตีที่คู่ควบกับสสารและเกจด้วยสมมาตรแบบ $SO(4)$ กับทฤษฎีสูงสุดในเจ็ดมิติซึ่งมีทฤษฎีสนามดูออลเป็นทฤษฎีสนามซูเปอร์คอนฟอร์มอลหกมิติ เราพบผลเฉลยโดเมนวอลล์หลายชนิดในทฤษฎีสูงสุดที่มีหลายเกจกรุป สำหรับเกจกรุป $SO(5)$ ที่มีสุญญากาศ AdS_7 ผลเฉลยที่ได้อธิบายการโฟลว์กรุปรีนอร์มอไลเซชัน (RG) โฮโลกราฟีกจากทฤษฎีสนามซูเปอร์คอนฟอร์มอล $N = (2,0)$ ไปยังทฤษฎีสนามไม่คอนฟอร์มอลในหกมิติ สำหรับเกจกรุปอื่นที่ไม่มีสุญญากาศ AdS_7 โดเมนวอลล์เหล่านี้คือสุญญากาศซูเปอร์ซิมเมทริกที่ดูออลกับทฤษฎีสนามไม่คอนฟอร์มอล $N = (2,0)$ หกมิติ เราพบผลเฉลยโดเมนวอลล์ที่ถูกประจุในทั้งสองทฤษฎีเกจซูเปอร์กราวิตีโดยการคู่ควบโดเมนวอลล์เข้ากับสนามทรีฟอร์ม ผลเฉลยที่ได้มีชั้นของ $AdS_3 \times S^3$ และสามารถตีความเป็นความบกพร่องพื้นผิวคอนฟอร์มอลในทฤษฎีสนามดูออลหกมิติได้ ยิ่งไปกว่านั้นเรายังพบผลเฉลยแบบบิดที่ตีความได้เป็นการโฟลว์ RG ข้ามมิติจากทฤษฎีสนามหกมิติไปยังทฤษฎีสนามซูเปอร์คอนฟอร์มอลในมิติที่ต่ำกว่า เราพิจารณาผลเฉลยของทฤษฎีที่คู่ควบกับสสารในกรณีที่มีสนามทรีฟอร์มและทำการขยายผลเฉลยแบบบิดของทฤษฎีสูงสุดให้ครอบคลุมซิงกูลาริตีในรูปแบบของโดเมนวอลล์โค้ง ในหลายกรณีซิงกูลาริตีเหล่านี้ยอมรับได้ในทางฟิสิกส์และสามารถตีความเป็นทฤษฎีสนามไม่คอนฟอร์มอลในมิติที่ต่ำกว่าได้ ผลเฉลยที่ได้จำนวนมากสามารถยกระดับขึ้นไปยังสิบและสิบเอ็ดมิติให้ผลลัพธ์เป็นผลเฉลยใหม่ในทฤษฎีสตริงและทฤษฎีเอ็มได้

ภาควิชา ฟิสิกส์ .... ลายมือชื่อนิสิต ....

สาขาวิชา ฟิสิกส์ .... ลายมือชื่อ อ.ที่ปรึกษาวิทยานิพนธ์หลัก ....

ปีการศึกษา 2562 ....

6072818623 : MAJOR PHYSICS
KEYWORDS : SUPERGRAVITY / ADS/CFT DUALITY / 7 DIMENSIONS

PATHARADANAI NUCHINO : SUPERSYMMETRIC SOLUTIONS OF SEVEN-DIMENSIONAL GAUGED SUPERGRAVITIES AND AdS_7/CFT_6 DUALITY. ADVISOR : ASSOC. PROF. PARINYA KARNDUMRI, Ph.D., 226 pp.

Describing quantum field theory using the AdS/CFT duality can broaden string theory applications in the context of the holographic principle. This research study supersymmetric solutions of matter-coupled $SO(4)$ and maximal gauged supergravities in seven dimensions of which the dual field theories are six-dimensional superconformal field theories (SCFTs). We find a large class of domain wall (DW) solutions in the maximal theory with various gauge groups. For $SO(5)$ gauge group admitting an AdS_7 vacuum, the solutions describe holographic renormalization group (RG) flows from an $N = (2, 0)$ SCFT to non-conformal field theories (SQFTs) in six dimensions. For other gauge groups without AdS_7 vacua, these DWs are supersymmetric vacua dual to six-dimensional $N = (2, 0)$ SQFTs. By coupling DWs to three-form fields, we find charged DW solutions in both theories. The solutions with $AdS_3 \times S^3$ slices are interpreted as conformal surface defects within the dual field theories in six dimensions. We also find twisted solutions describing holographic RG flows across dimensions from six-dimensional field theories to SCFTs in lower dimensions. We consider solutions of the matter-coupled theory in the presence of a three-form field and extend twisted solutions of the maximal theory to include singularities in the form of curved DWs. In many cases, these singularities are physically acceptable and can be interpreted as SQFTs in lower dimensions. Many solutions can be uplifted to ten and eleven dimensions resulting in new classes of solutions in string and M-theories.

Department : Physics .... Student's Signature ....

Field of Study : Physics .... Advisor's Signature ....

Academic Year : 2019 ....

ACKNOWLEDGEMENTS

The essential acknowledgments belong to Assoc. Prof. Dr. Parinya Karndumri, my supervisor, who has introduced me to the most super fantastic field of research in high-energy theoretical physics. Without him, there would be no publications and also this dissertation. I desire to appreciate his supervision, his care, and his patient along the way of my postgraduate education.

I am very grateful to my dissertation committee: Asst. Prof. Dr. Boonchoat Paosawatyanyong, Asst. Prof. Dr. Auttakit Chatrabhuti, Asst. Prof. Dr. Pawin Ittisamai, and Asst. Prof. Dr. Pitayuth Wongjun. All of their cooperation, suggestions, and comments they have bestowed are valuable.

I want to thank all people, both academic and administrative, in the Department of Physics, Chulalongkorn University. I much appreciate financial supports and excellent opportunities from the Science Achievement Scholarship of Thailand (SAST).

My special thanks go to Dr. Apichaya Kapanya, my fiancee. In the world without supersymmetry, she is my superpartner who always gives me tremendous morale and motivation to deal with every trouble.

Last but not least, I would like to thank Mr. Singha Nuchino, Mrs. Kannikar Kosin, and all of my family for their unconditional love and undoubted supports since 28 July 1991.

CONTENTS

page

CHAPTER **page**

CHAPTER **page**

LIST OF TABLES

Table **page**

LIST OF FIGURES

Figure **page**

Figure **page**

Figure **page**

Figure **page**

Figure **page**

Figure **page**

Figure **page**

Figure **page**

Figure **page**

LIST OF ABBREVIATIONS

Abbreviation	Definition
AdS	anti-de Sitter
BPS	Bogomol'nyi-Prasad-Sommerfield
CFT	conformal (quantum) field theory
DW	domain wall
GR	general relativity
IR	infared
NS	Neveu Schwarz
QFT	quantum field theory
R-R	Ramond-Ramond
RG	renormalization group
SCFT	supersymmetric conformal (quantum) field theory
SM	symplectic-Majorana
SQFT	supersymmetric (non-conformal) quantum field theory
SUSY	supersymmetry
UV	ultraviolet

CHAPTER I

INTRODUCTION

Finding a theory of quantum gravity is one of the significant issues attracting many experimental and theoretical physicists for a long time. While three from four fundamental forces, electromagnetic, weak, and strong, can be described through quantum field theory (QFT) in the context of gauge theory, gravity is still isolated and classically expressed by Einstein's general relativity (GR). These are recognized as the two distinguished principal theories of theoretical physics in the twentieth century. Achieving quantum gravity will give us a huge step closer to the final theory of everything unifying all four fundamental forces and matter particles. Unfortunately, despite eighty years of active research, a consistent and complete quantum theory of gravity has not yet been formulated. The most significant problem is that we do not know a proper way to quantize gravity [1,2].

For almost five decades, string theory [3–8] has been explored in various features as a promising candidate not only for quantum gravity but also for the theory of all interactions. Unlike theories of point-like particles, the theory contains one-dimensional fundamental objects, called strings, together with other p-dimensionally extended ones, called p-branes, where their illustration is shown in Figure 1.1. While ordinary QFT does not allow gravity to exist, string theory requires it since there always exists a massless spin-two particle called graviton, the quantum of gravitation, in the spectrum. However, ten-dimensional spacetime is needed in order for string theory to be a consistent Lorentz invariant quantum theory. To get down to four-dimensional flat spacetime $\mathbb{R}^{1,3}$, one can consider a compactification of the theory on a product geometry $\mathbb{R}^{1,3} \times M^6$ where M^n denotes an n-dimensional compact space. Lower-dimensional interactions are also determined by symmetry of the compact space.

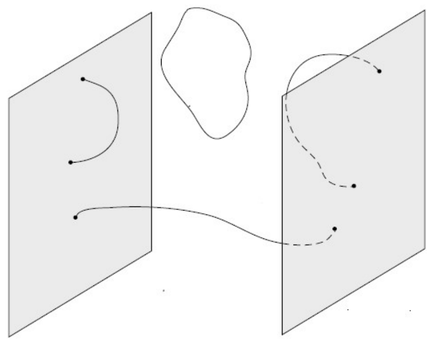


Figure 1.1: Illustration of a closed string, Dirichlet p-branes (Dp-branes), and open strings ending on them.[1]

Instead of lower-dimensional flat spacetime, a remarkable result was discovered when we consider string theory on a maximally symmetric spacetime with negative curvature, called anti-de Sitter (AdS) space. String theory on $AdS_{10-n} \times M^n$ space is dual to a supersymmetric conformal field theory (SCFT) living on flat spacetime $\mathbb{R}^{1,9-n}$, the boundary of the AdS_{10-n} space, in the way that there exist one-to-one maps, called dictionary, between local fields ϕ_i in the AdS bulk and operators $\mathcal{O}_i$ in the dual SCFT on the boundary. By the dictionary, correlation functions governing quantum interactions of the dual SCFT can be calculated from string theory. The first example was proposed in the late 1990s [9–11]. In this case, type IIB string theory on $AdS_5 \times S^5$ spacetime, in which S^5 is a five-dimensional sphere, is dual to $N = 4$ Super-Yang-Mills gauge theory living on flat spacetime $\mathbb{R}^{1,3}$. This duality is referred to as the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence, whose illustration is displayed in Figure 1.2, and has been widely tested and confirmed by a large number of impressive results over the past twenty years.

Although the duality is fascinating, the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence is too complicated to perform explicit calculations for generic values of parameters. Therefore, we need to reduce the strength of the correspondence by taking the 't Hooft limit [12]. In this limit, there is only one free parameter on both sides:

[1] Katrin Becker, Melanie Becker, and John H. Schwarz, **String theory and M-theory: A modern introduction**, (New York: Cambridge University Press, 2007), p.194.

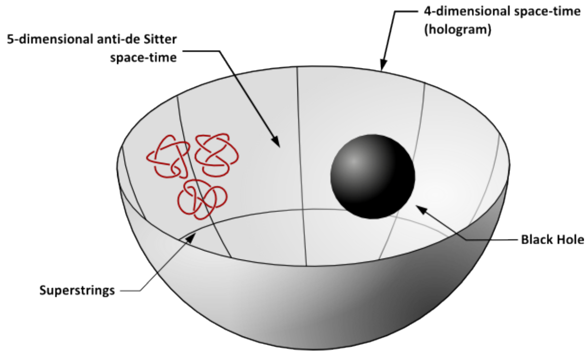


Figure 1.2: Illustration of the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence relating type IIB string theory on $AdS_5 \times S^5$ spacetime to $N = 4$ Super-Yang-Mills gauge theory on the boundary of the AdS_5.[2]

the 't Hooft coupling λ on the field theory side and the radius of curvature L/l_s, with l_s being the string length, on the string theory side. The two parameters are related through the correspondence by $(L/l_s)^4 = 2\lambda$. From this point, the most useful duality can be obtained by taking the limit $\lambda \to \infty$ on the field theory side, indicating that the field theories are strongly-coupled. On the string theory side, this limit corresponds to the low-energy limit where the radius of spacetime curvature is much bigger than the string scale, $l_s/L \to 0$. In this limit, string theory reduces to supergravity, and the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence accordingly relates strongly-coupled $N = 4$ Super-Yang-Mills to type IIB supergravity on weakly-curved $AdS_5 \times S^5$ space. This strong/weak duality is known as the weak form of the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence. Apart from this first example, there also exits the AdS/CFT correspondence relating string theory or supergravity in the low-energy limit on AdS_D background to $D-1$ dimensional SCFT with $D = 2, .., 7$.

One of the most astonishing implications of the strong/weak duality is the application for problems of condensed matter physics in our lower dimensions. Many systems in condensed matter physics are strongly-coupled so that standard perturbative calculations do not work. The AdS/CFT correspondence allows us to map these strongly-coupled behaviors to the general covariance of gravity

[2]**ads-cft** [Online]. Available from: http:// quantum-bits.org/ wp-content/ uploads/ 2015/09/ads-cft.png [2017, October 12]

theory that can be applied by using the standard computation of GR. In this way, unusual behaviors of condensed matter, such as strange metals or unconventional superconductors, can be examined in deeper detail, see [13–21] for an incomplete list.

Apart from the above AdS_5/CFT_4 duality, one of the interesting cases proposed in [9] to describe the dynamics of M5-branes in M-theory is the AdS_7/CFT_6 correspondence. Among five versions of string theory: type I, type IIA, type IIB, heterotic $SO(32)$, and heterotic $E_8 \times E_8$, M-theory is an eleven-dimensional non-perturbative theory connecting them through a web of dualities in Figure 1.3 [7]. The theory describes supersymmetric two- and five-dimensionally extended objects respectively called M2- and M5-branes. In the low-energy limit, M-theory is approximated by eleven-dimensional supergravity. However, a complete formulation of M-theory is still unclear. Studying these supersymmetric M-branes via the AdS/CFT correspondence has also played a crucial role in the development of M-theory.

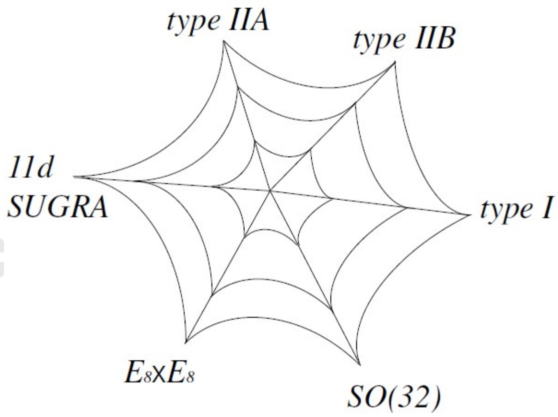


Figure 1.3: Five consistent string theories and eleven-dimensional supergravity are connected through a web of dualities.[3]

Since we do not know much about M-theory, studying the AdS/CFT correspondence from eleven-dimensional supergravity is a significant benefit. There exists a unique supergravity in eleven-dimensional spacetime containing the

[3]Katrin Becker, Melanie Becker, and John H. Schwarz, **String theory and M-theory: A modern introduction**, (New York: Cambridge University Press, 2007), p.12.

graviton, a gravitino (a supersymmetric partner of the graviton), and a three-form potential [22]. In general, p-branes are charged under $(p+1)$-form potentials, in the same way as in electromagnetism that a zero-brane (particle) can be charged under a one-form potential. In this case, M2- and M5-branes are charged under the three-form potential and their magnetic duality, a six-from potential. On the other hand, these extended objects can be approximately described by gauged supergravities in four and seven dimensions obtained from consistent truncations of eleven-dimensional supergravity on seven- and four-dimensional spheres, S^7 and S^4, respectively [23–27].

Therefore, the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence can be efficiently investigated by using gauged supergravities in seven dimensions so that $AdS_7 \times S^4$ geometry of M-theory, dual to $N = (2,0)$ SCFT in six dimensions, can be described by a vacuum solution of seven-dimensional $N = 4$, $SO(5)$ gauged supergravity. Besides, AdS_7 vacua of $N = 2$ gauged supergravity in seven dimensions is dual to six-dimensional $N = (1,0)$ SCFTs in the case of half-maximal supersymmetry [28,29]. On the CFT side, six is the maximum possible spacetime dimensions for SCFT [30]. However, there is no Lagrangian description for these six-dimensional SCFTs [31]. Studying the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence from gauged supergravities also gives advantages in describing the dynamics of these six-dimensional field theories.

The first version of the maximal $SO(5)$ gauged supergravity has been constructed in [32, 33]. The theory admits two AdS_7 vacua, but only one of them is supersymmetric and plays an important role in the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence. For the half-maximal $N = 2$ supergravity gauged by $SO(3)$ gauge group [34,35], there is no AdS_7 vacuum. In order for a supersymmetric AdS_7 vacuum to exist, an additional mass deformation for the three-form field is needed. The half-maximal $SO(3)$ gauged theory with this deformation is given in [36, 37]. Furthermore, $SO(3)$ gauge group in this $N = 2$ theory can be enlarged by coupling to n vector multiplets. The resulting matter-coupled theory allows many viable gauge groups being a subgroup of the global symmetry $\mathbb{R}^+ \times SO(3,n)$ [38]. In [39, 40], their supersymmetric AdS_7 vacua have been studied. A remarkable matter-coupled theory is the $SO(4)$ gauged theory, first constructed in [41]. This theory, obtained from coupling the minimal gauged supergravity to three vector multiplets, mediates between the maximal $SO(5)$ and the minimal $SO(3)$ gauged supergravities.

In addition to the rigid AdS_7 vacua, supergravity solutions being AdS_7 near the boundary but differ in the interior are also attractive. These solutions take the form of domain walls (DWs) interpolating between AdS_7 vacua and singularities. Via the $\mathrm{AdS}_7/\mathrm{CFT}_6$ duality, the solutions are dual to connections between different supersymmetric conformal fixed points of field theories known as holographic renormalization group (RG) flows [42–44]. On the CFT side, conformal symmetry on the boundary is broken by non-vanishing one-point functions $\langle \mathcal{O} \rangle$ with $\mathcal{O}$ being the corresponding dual operators. These one-point functions perturb the SCFT and induce RG flows to another SCFT or, in some cases, to a supersymmetric non-conformal (quantum) field theory (SQFT) dual to a singular geometry. The latter is of remarkable interest in the DW/QFT correspondence [45–47], a generalization of the AdS/CFT duality.

Not only the same dimensions but also field theories in different dimensions can be associated through RG flows across dimensions. In general, twisted solutions of D-dimensional supergravity on $AdS_{D-n} \times M^n$ geometry are AdS/CFT dual to RG flows across dimensions from an SCFT in $D-1$ dimensions to a $(D-1-n)$-dimensional one. This type of RG flows allows us to explore the structure and dynamics of less known SCFTs in higher, especially five and six, dimensions using the well-understood lower-dimensional SCFTs.

For the maximal $SO(5)$ gauged supergravity, these supersymmetric solutions have been extensively studied in [48–52] for DW and [53–59] for twisted solutions. The solutions for the matted-coupled $N = 2$ gauged supergravity have been discussed in [39, 40, 60]. For the minimal $SO(3)$ gauged supergravity, twisted solutions relating six-dimensional $N = (1, 0)$ SCFT to lower-dimensional SCFTs have been considered in [61], while DW has been reviewed in [62] where supersymmetric solutions with all bosonic fields non-vanishing have been found.

The latter type of solutions is called charged DW interpreted as two-dimensional conformal defects in the six-dimensional $N = (1,0)$ SCFT by the AdS/CFT correspondence.

Nonetheless, many supersymmetric solutions are still missing. There is no charged DW found in the matter-coupled and the maximal gauged theories, while none of the prior supersymmetric solutions from the matter-coupled theory involve the non-vanishing three-form field. Moreover, apart from $SO(5)$, there are many possible gauge groups for the maximal $N = 4$ gauged supergravity given by the embedding tensor formalism in [63]. Maximal gauged supergravities with these additional gauge groups can be obtained from consistent truncations of eleven-dimensional and type IIB supergravities see [64] and [65]. There is no systematic analysis for supersymmetric solutions for these gauge groups so far. In this dissertation, we want to find supersymmetric solutions that shade the light to these missing corners. The new supersymmetric solutions will give more advantages in the study of the $\text{AdS}_7/\text{CFT}_6$ duality and the more general DW_7/QFT_6 correspondence as well.

The dissertation is organized as follows. In Chapter 2, we review the relevant seven-dimensional supergravities, the matter-coupled $N = 2$ and the maximal $N = 4$ gauged theories. In Chapter 3, we study new supersymmetric solutions of the matter-coupled gauged theory, especially for $SO(4)$ gauge group. They are charged DWs and twisted solutions with non-vanishing three-form potential. Supersymmetric solutions of the maximal $N = 4$ theory with various gauge groups are considered in Chapter 4. We finish the dissertation with some general conclusions and discussions in Chapter 5. In Appendices A and B, general notations for GR and symplectic-Majorana (SM) spinor used in this dissertation are summarized, respectively. Consistent truncations of eleven-dimensional and type IIA supergravities giving rise to seven-dimensional gauged supergravities are reviewed in Appendix C.

CHAPTER II

SEVEN-DIMENSIONAL GAUGED SUPERGRAVITIES

Supergravity is GR accompanied by supersymmetry (SUSY) [66–68], the symmetry between bosonic and fermionic fields with integer and half-integer spins, respectively. Denoted by $\mathcal{B}(x)$ and $\mathcal{F}(x)$, bosonic and fermionic fields transform to each other through the following general form of local SUSY transformations

$$\delta\mathcal{B}(x) = \bar{\epsilon}(x) f_1(\mathcal{F}(x)) \quad \text{and} \quad \delta\mathcal{F}(x) = f_2(\mathcal{B}(x))\epsilon(x) \tag{2.1}$$

where $\epsilon(x)$ are SUSY spinor parameters depending on spacetime coordinate x and $\bar{\epsilon}(x)$ refer to their Dirac conjugations. The functions $f_1(\mathcal{F})$ and $f_2(\mathcal{B})$ usually include Dirac gamma matrices and spacetime derivatives. Supergravity actions are invariant under these SUSY transformations. For example, the simplest supergravity in four-dimensional spacetime was formulated in 1976 with one SUSY ($N = 1$) describing interactions between the graviton together with a fermionic field called gravitino with spin 3/2 [69, 70].

Simple supergravity can be extended by adding more SUSY. The more SUSY, the more fields of different spins in supergravity multiplet transform into each other within SUSY transformations (2.1). In four-dimensional spacetime, $N = 8$ is the maximally extended theory in which supergravity multiplet contains the graviton, eight gravitini, 28 vectors, 56 spinors, and 70 scalar fields with 256 degrees of freedom divided into 128 bosonic and also 128 fermionic states [71, 72]. Beyond $N = 8$, superalgebra representations inevitably contain massless fields with spin > 2, for which no consistent interactions exist. Note here that the degrees of freedom for maximal supergravity are identically 256 in any dimensions.

Not only in four-dimensional spacetime, but supergravity can also be formulated in various spacetime dimensions up to eleven [30]. Ten- and eleven-dimensional supergravities are remarkable since they respectively appear as the low-energy effective theories of string and M-theories. Consequently, the dynamics of the lowest-energy modes and vacua of these fundamental theories can be described by classical solutions of supergravity.

Although fermions play an important role in determining the structure of supergravity, they do not appear in classical backgrounds. Thus, classical solutions of supergravity are bosonic configurations satisfying Euler-Lagrange field equations with all fermionic fields vanishing. To obtain supersymmetric solutions, we need a non-trivial configuration of $\epsilon(x)$ for which both $\delta\mathcal{B}(x)$ and $\delta\mathcal{F}(x)$ vanish. Since fermions disappear classically, the condition $\delta\mathcal{B}(x) = 0$ is trivially satisfied, so we need to consider only the fermionic variations

$$\delta\mathcal{F}(x) = f_2(\mathcal{B}(x))\epsilon(x) = 0. \tag{2.2}$$

This condition gives us a set of Bogomol'nyi-Prasad-Sommerfield (BPS) equations that are first-order differential equations involving bosonic fields and the spinors $\epsilon(x)$. Solving BPS equations typically requires the existence of Killing spinors with n_Q real degrees of freedom where $0 \leq n_Q \leq n_{Q_0}$ and n_{Q_0} is a total amount of supercharges for $\epsilon(x)$. The resulting bosonic configurations are BPS solutions preserving n_Q supercharges or (n_Q/n_{Q_0})-SUSY. If $n_Q = n_{Q_0}$, BPS solutions are maximally supersymmetric, while $n_Q = 0$ means the solutions completely break SUSY.

Although possible, finding BPS solutions directly from supergravity in ten or eleven dimensions is extremely complicated due to substantial free parameters. Instead, we can examine lower-dimensional gauged supergravity, the theories with gauged R-symmetry or any subgroup thereof, which is a consistent truncation of ten- or eleven-dimensional supergravities. By consistent truncations, solutions to lower-dimensional gauged supergravity are also solutions to the ten- or eleven-dimensional theories. Some notable cases are maximal $SO(5)$ ($SO(8)$) gauged supergravity in seven (four) dimensions arising from a consistent truncation of eleven-dimensional supergravity on S^4 [25–27] (S^7 [23, 24]) and five-dimensional $SO(6)$ gauged supergravity obtained from a truncation of type IIB theory on S^5 [73]. Finding supersymmetric solutions from this approach is more convenient and manageable.

In this chapter, relevant formulae involving bosonic Lagrangian and SUSY transformations of fermions will be presented in order to find BPS solutions from gauged supergravities in seven dimensions. We will start with an introduction of matter-coupled $N = 2$ gauged supergravity in which the minimal $N = 2$ theory is coupled to an arbitrary number of vector multiplets by following conventions and notations used in [38, 39, 60]. After that, maximal gauged supergravity in the embedding tensor formalism [63] will be reviewed. For readers who are unfamiliar with general notations of GR and SM spinors, these building blocks are introduced in Appendices A and B.

2.1 Matter-Coupled $N = 2$ Gauged Supergravity

There are two supergravity theories in seven-dimensional spacetime called maximal ($N = 4$) and minimal ($N = 2$) supergravities that can be respectively gauged by $SO(5)$ [32, 33] and $SO(3)$ [34, 35]. Matter-coupled $N = 2$ gauged supergravity can be achieved by coupling the minimal theory to an arbitrary number n of vector multiplets [38]. Among many viable gaugings, matter-coupled $SO(4)$ gauged supergravity [41] is particular interested since it mediates the minimal $SO(3)$ and the maximal $SO(5)$ gauged theories. A truncation procedure of the $SO(5)$ gauged supergravity in order to get the $SO(4)$ gauged theory is given in [74] and reconsidered in Appendix C.1. Besides, vanishing of all vector-multiplet fields gives rise to the minimal $SO(3)$ gauged supergravity.

As a bridge delivering techniques of finding supersymmetric solutions from the minimal $SO(3)$ gauged theory to the maximal one, matter-coupled $SO(4)$ gauged supergravity is our main interest. Starting from an introduction of matter-coupled $N = 2$ gauged supergravity, we finish this section with $SO(4)$ gauging and the corresponding supersymmetric AdS_7 critical points. These critical points are, according to AdS/CFT correspondence, dual to $N = (1,0)$ SCFTs in six dimensions.

2.1.1 Bosonic Lagrangian and Fermionic SUSY Transformations

We first review the minimal $SO(3)$ gauged supergravity in seven dimensions [34,35] whose field content is given by

$$(e^{\hat{\mu}}_{\mu}, \psi^{\alpha}_{\mu}, A^{i}_{\mu}, \chi^{\alpha}, B_{\mu\nu}, \sigma). \tag{2.3}$$

They are the graviton $e^{\hat{\mu}}_{\mu}$, two gravitini ψ^{α}_{μ}, three vectors A^{i}_{μ}, two spin-$\frac{1}{2}$ fields χ^{α}, a two-form field $B_{\mu\nu}$, and a scalar field σ called the dilaton in the supergravity multiplet. Curved and flat seven-dimensional spacetime indices are denoted by μ, ν and $\hat{\mu}$, $\hat{\nu}$ respectively. The indices $i, j = 1, 2, 3$ and $\alpha, \beta = 1, 2$ label triplets and doublet of $SO(3)_R$ symmetry where the latter will be suppressed for simplicity. The two-form field will be dualized to a three-form $H_{\mu\nu\rho}$, which admits a topological mass term leading to a massive deformation of the $N = 2$ supergravity. Note that this additional deformation is important for the gauged supergravity to admit AdS_7 vacua [36, 37].

Matter-coupled $N = 2$ gauged supergravity [38] is formulated by coupling the minimal supergravity (2.3) to an arbitrary number n of vector multiplets:

$$(A_{\mu}, \lambda^{\alpha}, \phi^{i})^{r} \tag{2.4}$$

in which $r, s = 1, .., n$. Each vector multiplet contains a vector field A_{μ}, two gaugini λ^{α}, and three scalar fields ϕ^{i}. From both supergravity and vector multiplets, there are in total $(3 + n)$ vector fields denoted collectively by $A^{I}_{\mu} = (A^{i}_{\mu}, A^{r}_{\mu})$ where $I, J = 1, .., (3 + n)$ are $SO(3, n)$ fundamental indices raised and lowered by the $SO(3, n)$ invariant tensor $\eta_{IJ} = \text{diag}(- - - + ...+)$.

Totally, there are $3n$ scalar fields ϕ^{ir} in the vector multiplets parameterizing $SO(3, n)/SO(3) \times SO(n)$ coset manifold. They can be described through the following coset representative

$$L = (L_I{}^i, L_I{}^r) \tag{2.5}$$

together with its inverse

$$L^{-1} = (L^I{}_i, L^I{}_r). \tag{2.6}$$

The $SO(3)$ and $SO(n)$ indices, i, j and r, s, are raised and lowered by δ_{ij} and δ_{rs}, respectively. With these conventions, the following relations can be derived

$$L_I{}^i L^I{}_j = -\delta^i_j, \;\; L_I{}^r L^I{}_s = \delta^r_s, \tag{2.7}$$

and

$$\eta_{IJ} = -L_I{}^i L_J{}^i + L_I{}^r L_J{}^r. \tag{2.8}$$

Gaugings of matter-coupled $N = 2$ supergravity can be obtained by promoting a subgroup G_0 of the global symmetry $\mathbb{R}^+ \times SO(3,n)$ to be local. If the gauging does not involve the $\mathbb{R}^+$ factor, the embedding of G_0 in $SO(3,n)$ is represented by the $SO(3,n)$ tensor $f_{IJ}{}^K$ identified with the structure constants of G_0 through the gauge algebra

$$[T_I, T_J] = f_{IJ}{}^K T_K \tag{2.9}$$

where T_I denote the gauge generators. In the embedding tensor formalism, $f_{IJ}{}^K$ is a component of the full embedding tensor, see [75] for more detail. For a consistent gauging, preserving all of the original SUSY, $f_{IJ}{}^K$ must satisfy the conditions

$$f_{IJK} = \eta_{KL} f_{IJ}{}^L = f_{[IJK]} \qquad \text{and} \qquad f_{[IJ}{}^L f_{K]L}{}^M = 0\,. \tag{2.10}$$

Bosonic Lagrangian for matter-coupled $N = 2$ gauged supergravity is given in differential form language (see Appendix A.1 for a brief introduction) by

$$\begin{aligned}\mathcal{L} = &\frac{1}{2} R * \mathbf{1} - \frac{1}{2} e^{\sigma} a_{IJ} * F^I_{(2)} \wedge F^J_{(2)} - \frac{1}{2} e^{-2\sigma} * G_{(4)} \wedge G_{(4)} - \frac{5}{8} * d\sigma \wedge d\sigma \\ &- \frac{1}{2} * P^{ir} \wedge P^{ir} + \frac{1}{\sqrt{2}} G_{(4)} \wedge \omega_{(3)} - 4h G_{(4)} \wedge H_{(3)} - \mathbf{V} * \mathbf{1}.\end{aligned} \tag{2.11}$$

With $G_{(4)} = dH_{(3)}$, the constant h describes the topological mass term for the three-form $H_{(3)}$. The associated two-form field strength is defined as

$$F^I_{(2)} = dA^I_{(2)} + \frac{1}{2} f_{JK}{}^I A^J_{(1)} \wedge A^K_{(1)}. \tag{2.12}$$

The scalar matrix a_{IJ} appearing in the kinetic term of vector fields is

$$a_{IJ} = {L_I}^i L_{Ji} + {L_I}^r L_{Jr}. \tag{2.13}$$

The Chern-Simons three-form satisfying $d\omega_{(3)} = F^I_{(2)} \wedge F^I_{(2)}$ is given by

$$\omega_{(3)} = F^I_{(2)} \wedge A^I_{(1)} - \frac{1}{6} {f_{IJ}}^K A^I_{(1)} \wedge A^J_{(1)} \wedge A_{(1)K}. \tag{2.14}$$

The scalar potential is

$$\mathbf{V} = \frac{1}{4} e^{-\sigma} \left(C^{ir} C_{ir} - \frac{1}{9} C^2 \right) + 16 h^2 e^{4\sigma} - \frac{4\sqrt{2}}{3} h e^{\frac{3\sigma}{2}} C \tag{2.15}$$

where C-functions are defined as

$$\begin{aligned} C &= -\frac{1}{\sqrt{2}} {f_{IJ}}^K {L^I}_i {L^J}_j L_{Kk} \varepsilon^{ijk}, \\ C^{ir} &= \frac{1}{\sqrt{2}} {f_{IJ}}^K {L^I}_j {L^J}_k {L_K}^r \varepsilon^{ijk}, \\ C_{rsi} &= {f_{IJ}}^K {L^I}_r {L^J}_s L_{Ki}. \end{aligned} \tag{2.16}$$

The kinetic term of the scalar fields ϕ^{ir} is written in term of the following vielbein

$$P^{ir}_\mu = L^{Ir} \left(\delta^K_I \partial_\mu + {f_{IJ}}^K A^J_\mu \right) {L_K}^i. \tag{2.17}$$

The following field equations are derived from the bosonic Lagrangian (2.11)

$$0 = d(e^{-2\sigma} * G_{(4)}) + 8h G_{(4)} - \frac{1}{\sqrt{2}} F^I_{(2)} \wedge F^I_{(2)}, \tag{2.18}$$

$$0 = D(e^\sigma a_{IJ} * F^I_{(2)}) - \sqrt{2} G_{(4)} \wedge F^J_{(2)} + * P^{ir} {f_{IJ}}^K {L^I}_r L_{Ki}, \tag{2.19}$$

$$\begin{aligned} 0 = {} & D(*P^{ir}) - 2 e^\sigma {L_I}^i {L_J}^r * F^I_{(2)} \wedge F^J_{(2)} \\ & - \left(\frac{1}{\sqrt{2}} e^{-\sigma} C^{js} C_{rsk} \varepsilon^{ijk} + 4\sqrt{2} h e^{\frac{3\sigma}{2}} C^{ir} \right) * 1, \end{aligned} \tag{2.20}$$

$$\begin{aligned} 0 = {} & \frac{5}{4} d(*d\sigma) - \frac{1}{2} e^\sigma a_{IJ} * F^I_{(2)} \wedge F^J_{(2)} + e^{-2\sigma} * G_{(4)} \wedge G_{(4)} \\ & + \left[\frac{1}{4} e^{-\sigma} \left(C^{ir} C_{ir} - \frac{1}{9} C^2 \right) + 2\sqrt{2} h e^{\frac{3\sigma}{2}} C - 64 h^2 e^{4\sigma} \right] * 1, \end{aligned} \tag{2.21}$$

$$\begin{aligned} 0 = {} & R_{\mu\nu} - \frac{5}{4} \partial_\mu \sigma \partial_\nu \sigma - a_{IJ} e^\sigma \left(F^I_{\mu\rho} F^{J\rho}_\nu - \frac{1}{10} g_{\mu\nu} F^I_{\rho\sigma} F^{J\ \rho\sigma} \right) \\ & - P^{ir}_\mu P^{ir}_\nu - \frac{2}{5} g_{\mu\nu} \mathbf{V} - \frac{1}{6} e^{-2\sigma} \left(G_{\mu\rho\sigma\lambda} {G_\nu}^{\rho\sigma\lambda} - \frac{3}{20} g_{\mu\nu} G_{\rho\sigma\lambda\tau} G^{\rho\sigma\lambda\tau} \right). \end{aligned} \tag{2.22}$$

The last ingredients relevant for finding supersymmetric solutions are SUSY transformations of fermions

$$
\begin{aligned}
\delta\psi_\mu &= 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon - \frac{i}{20}e^{\frac{\sigma}{2}}F^i_{\rho\sigma}\sigma^i(3\gamma_\mu\gamma^{\rho\sigma} - 5\gamma^{\rho\sigma}\gamma_\mu)\epsilon \\
&\quad -\frac{1}{240\sqrt{2}}e^{-\sigma}G_{\rho\sigma\lambda\tau}(\gamma_\mu\gamma^{\rho\sigma\lambda\tau} + 5\gamma^{\rho\sigma\lambda\tau}\gamma_\mu)\epsilon, \qquad (2.23)\\
\delta\chi &= -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon + \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\epsilon - \frac{16}{5}e^{2\sigma}h\epsilon - \frac{i}{10}e^{\frac{\sigma}{2}}F^i_{\mu\nu}\sigma^i\gamma^{\mu\nu}\epsilon \\
&\quad -\frac{1}{60\sqrt{2}}e^{-\sigma}G_{\mu\nu\rho\sigma}\gamma^{\mu\nu\rho\sigma}\epsilon, \qquad (2.24)\\
\delta\lambda^r &= i\gamma^\mu P^{ir}_\mu\sigma^i\epsilon - \frac{1}{2}e^{\frac{\sigma}{2}}F^r_{\mu\nu}\gamma^{\mu\nu}\epsilon - \frac{i}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\sigma^i\epsilon \qquad (2.25)
\end{aligned}
$$

where σ^i are the usual Pauli matrices (B.3). Spacetime gamma matrices are related to the Dirac gamma matrices $\gamma^{\hat{\mu}}$, whose explicit forms are given in (B.2), by $\gamma^\mu = e^\mu_{\hat{\mu}}\gamma^{\hat{\mu}}$. The two $SO(3)$ and $SO(n)$ two-form field strengths can be written through the relations $F^i_{(2)} = L_I{}^iF^I_{(2)}$ and $F^r_{(2)} = L_I{}^rF^I_{(2)}$. The covariant derivative of the SUSY spinor parameter ϵ is given by

$$D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{4}\omega_\mu{}^{\hat{\nu}\hat{\rho}}\gamma_{\hat{\nu}\hat{\rho}}\epsilon + \frac{1}{2\sqrt{2}}Q^i_\mu\sigma^i\epsilon \qquad (2.26)$$

where $Q^i_\mu = \frac{i}{\sqrt{2}}\varepsilon^{ijk}Q^{jk}_\mu$ is defined in terms of the composite connection

$$Q^{ij}_\mu = L^{Ij}\left(\delta^K_I\partial_\mu + f_{IJ}{}^KA^J_\mu\right)L_K{}^i. \qquad (2.27)$$

2.1.2 $SO(4)$ Gauging and Supersymmetric AdS_7 Critical Points

Matter-coupled $SO(4)$ gauged supergravity is obtained when the minimal $N = 2$ supergravity in seven dimensions is coupling to three vector multiplets. In this case, $SO(4)$ gauge group is equivalent to a direct product between two $SO(3)$ symmetries, i.e. $SO(4) \sim SO(3)_R \times SO(3)$. The first $SO(3)$ factor is the R-symmetry identified by $SU(2)_R \sim SO(3)_R$, while the other one is the symmetry under which the three vector multiplets transform. The corresponding structure constants are separately given by

$$f_{IJK} = (g_1\varepsilon_{ijk}, -g_2\varepsilon_{rst}) \qquad (2.28)$$

in which g_1 and g_2 are $SO(3)_R$ and $SO(3)$ gauge coupling constants, respectively.

In [39], two supersymmetric AdS_7 vacua are discovered in matter-coupled $SO(4)$ gauged theory. These vacua are BPS solutions preserving full symmetry and are the critical points of the scalar potential. Instead of the nine scalars in $SO(3,3)/SO(3)_R \times SO(3)$ coset manifold, we can find the critical points from their subsets that are invariant under some subgroup H_0 of the full gauge symmetry $SO(4)$, as introduced in [76]. These subsets consist of all scalars that are singlet under the unbroken subgroup H_0. All critical points found from this approach are essentially critical points of the potential for the full scalars.

The metric on AdS_7 is given by

$$ds^2_{AdS_7} = e^{2r/L_{AdS_7}} dx^2_{1,5} + dr^2 \tag{2.29}$$

where L_{AdS_7} is a constant AdS_7 radius and $dx^2_{1,5} = \eta_{mn} dx^m dx^n$, $m, n = 0, 1, .., 5$ is the flat metric on six-dimensional spacetime. In the limit $r \to \infty$, there exists the conformal boundary, a six-dimensional Minkowski flat spacetime on which the isometry group of the AdS_7 acts as the conformal group. For $r \to -\infty$, there is a coordinate singularity called the Poincaré horizon, as shown in Figure 2.1. Using the vielbein formalism introduced in Appendix A.2, one can find the Ricci scalar $R = -42/L^2_{AdS_7}$ corresponding to the negative curvature of the AdS_7.

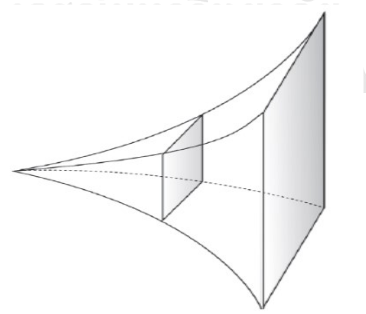


Figure 2.1: Illustration of anti-de Sitter space given in the metric coordinate (2.29) where the conformal boundary and the Poincaré horizon are located at $r \to \infty$ and $r \to -\infty$, respectively.[1]

[1] Martin Ammon and Johanna Erdmenger, **Gauge/Gravity Duality Foundations and Applications**, (United Kingdom; Cambridge University Press., 2015) p. 74.

The two AdS_7 critical points have different symmetries; the full $SO(4)$ one and the diagonal subgroup $SO(3)_{\text{diag}} \subset SO(3)_R \times SO(3)$. Thus, they can be simultaneously expressed by using an $SO(3)_{\text{diag}}$ singlet scalar ϕ corresponding to the non-compact generator

$$Y_S = Y_{11} + Y_{22} + Y_{33} \tag{2.30}$$

in which non-compact generators of $SO(3,3)$ are

$$Y_{ir} = e_{i,r+3} + e_{r+3,i}. \tag{2.31}$$

Here, e_{IJ} are $GL(6,\mathbb{R})$ matrices defined by

$$(e_{IJ})_{KL} = \delta_{IK}\delta_{JL}. \tag{2.32}$$

Accordingly, the coset representative is

$$L = e^{\phi Y_s}. \tag{2.33}$$

The scalar potential for the dilaton σ and the $SO(3)_{\text{diag}}$ singlet scalar ϕ is directly computed as

$$\begin{aligned}\mathbf{V} = \frac{1}{32}e^{-\sigma}\Big[&(g_1^2+g_2^2)\left(\cosh(6\phi) - 9\cosh(2\phi)\right) + 8g_1g_2\sinh^3(2\phi)\\ &+ 8\left[g_2^2 - g_1^2 + 64h^2e^{5\sigma} - 32e^{\frac{5\sigma}{2}}h(g_1\cosh^3\phi + g_2\sinh^3\phi)\right]\Big].\end{aligned} \tag{2.34}$$

The two supersymmetric AdS_7 critical points derived from this potential using $\frac{\partial \mathbf{V}}{\partial\sigma} = \frac{\partial \mathbf{V}}{\partial\phi} = 0$ condition. They are

(1) $SO(4)$ critical point:

$$\sigma = \phi = 0, \qquad \mathbf{V}_0 = -240h^2. \tag{2.35}$$

(2) $SO(3)_{\text{diag}}$ critical point:

$$\begin{aligned}\sigma &= \frac{1}{5}\ln\left[\frac{g_2^2}{g_2^2 - 256h^2}\right], \quad \phi = \frac{1}{2}\ln\left[\frac{g_2 - 16h}{g_2 + 16h}\right],\\ \mathbf{V}_0 &= -\frac{240g_2^{8/5}h^2}{(g^2 - 256h^2)^{4/5}}.\end{aligned} \tag{2.36}$$

We have chosen $g_1 = 16h$ to make the dilaton vanish at the $SO(4)$ critical point. $\mathbf{V}_0$ is the value of the scalar potential at the critical point. These critical points are maximally supersymmetric with $L_{AdS_7} = \sqrt{\frac{-15}{\mathbf{V}_0}}$, and respectively correspond to $N = (1,0)$ SCFTs in six dimensions with $SO(4)$ and $SO(3)$ symmetries.

An RG flow solution interpolating between these two critical points has already been studied in [39]. Instead of the AdS_7 space (2.29), in this case, the metric takes the more general form of a flat DW ansatz

$$ds^2_{DW} = e^{2U(r)}dx^2_{1,5} + dr^2 \tag{2.37}$$

in which $U(r)$ is a warp factor depending on the radial coordinate r. Moreover, the dilaton σ and the $SO(3)_{\text{diag}}$ singlet scalar ϕ also depend on r, in this case, with all other fields still vanishing. Imposing the projection condition

$$\gamma^r\epsilon = \epsilon, \tag{2.38}$$

we can derive the following BPS equations from SUSY transformations of fermions (2.23) to (2.25) satisfying $\delta\psi_m = 0$, $\delta\chi = 0$, and $\delta\lambda^r = 0$ conditions

$$U' = \frac{1}{40}e^{-\frac{\sigma}{2}-3\phi}\left[g_2(e^{2\phi}-1)^3 - g_1(e^{2\phi}+1)^3\right] + \frac{4}{5}he^{2\sigma}, \tag{2.39}$$

$$\sigma' = \frac{1}{20}\left[e^{-\frac{\sigma}{2}-3\phi}\left(g_2(e^{2\phi}-1)^3 - g_1(e^{2\phi}+1)^3\right) - 128he^{2\sigma}\right], \tag{2.40}$$

$$\phi' = \frac{1}{8}e^{-\frac{\sigma}{2}-3\phi}(e^{4\phi}-1)(g_1+g_2+e^{2\phi}g_1-e^{2\phi}g_2) \tag{2.41}$$

where $'$ denote r-derivatives. Besides, the condition $\delta\psi_r = 0$ provides the usual solution for the Killing spinors

$$\epsilon = e^{\frac{U}{2}}\epsilon_0 \tag{2.42}$$

with constant spinors ϵ_0 satisfying $\gamma^r\epsilon_0 = \epsilon_0$. By defining a new radial coordinate $\tilde{r}$ with $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$, the above BPS equations can be solved to obtain

$$U = \frac{1}{8}\left[2\phi - \sigma - 2\ln(2-2e^{4\phi}) + 2\ln\left(g_1+g_2+(g_1-g_2)e^{2\phi}\right)\right], \tag{2.43}$$

$$\sigma = \frac{2}{5}\ln\left[-\frac{g_1g_2e^{\phi}}{8h\left(g_1+g_2+(g_2-g_1)e^{2\phi}\right)}\right], \tag{2.44}$$

$$\frac{g_2\tilde{r}}{2} = \tan^{-1}e^{\phi} + \sqrt{\frac{g_2^2-g_1^2}{g_1^2}}\tanh^{-1}\left[e^{\phi}\sqrt{\frac{g_2-g_1}{g_2+g_1}}\right] + \frac{g_2}{2g_1}\ln\left[\frac{1-e^{\phi}}{1+e^{\phi}}\right]. \tag{2.45}$$

This is the RG flow connecting the $SO(4)$ critical point (2.35) in the UV to another $SO(3)_{\text{diag}}$ one (2.36) in the IR. As $\tilde{r} \to +\infty$, we find $\sigma \to 0$ and $\phi \to 0$ with an asymptotic behavior

$$\sigma \sim \phi \sim e^{-16hr} \sim e^{\frac{-4r}{L_{AdS_7}}} \quad \text{and} \quad U \sim \frac{r}{L_{AdS_7}}, \quad \text{with} \quad L_{AdS_7} = \frac{1}{4h}, \tag{2.46}$$

since $\tilde{r} \sim r$ near $\sigma \sim 0$. For $\tilde{r} \to -\infty$, the solution behaves as

$$\sigma \sim e^{\frac{-4r}{L_{AdS_7}}}, \phi \sim e^{\frac{4r}{L_{AdS_7}}} \quad \text{and} \quad U \sim \frac{r}{L_{AdS_7}}, \quad \text{with} \quad L_{AdS_7} = \frac{(g_2^2 - 256h^2)^{\frac{2}{5}}}{4hg_2^{\frac{4}{5}}}. \tag{2.47}$$

These critical points preserve AdS_7 isometry as well as all SUSY, while the whole RG flow relating them breaks AdS_7 isometry and preserves only $\frac{1}{2}$-SUSY due to the r-dependence of the scalar fields and the projector (2.38), respectively. Apart from this example, non-supersymmetric AdS_7 critical points and other RG flows connecting the $SO(4)$ critical point (2.35) to singularities in the IR are also given in [39]. According to the usual holographic interpretation, these flows to singularities should be dual to RG flows to SQFTs in six dimensions.

2.2 Maximal Gauged Supergravity

In this section, seven-dimensional $N = 4$ gauged supergravity is reviewed in the embedding tensor formalism. Apart from the well-known $SO(5)$ and non-compact, $SO(4,1)$ and $SO(3,2)$, gauged theories [32, 33], there additionally exist several gauge groups due to its irreducible embedding tensor in **15** and $\overline{\mathbf{40}}$ representations of $SL(5)$ global symmetry. After introducing all relevant formulae, a large class of possible gauge groups for the maximal gauged theory, together with their corresponding critical points, will be given. The complete construction of the maximal seven-dimensional gauged supergravity can be found in [63].

2.2.1 Bosonic Lagrangian and Fermionic SUSY Transformations

In seven dimensions, the only $N = 4$ supermultiplet is the supergravity multiplet with the following field content

$$(e_\mu^{\hat{\mu}}, \psi_\mu^a, A_\mu^{MN}, B_{\mu\nu M}, \chi^{abc}, \mathcal{V}_M{}^{ab}). \tag{2.48}$$

They are the graviton $e_\mu^{\hat{\mu}}$, four gravitini ψ_μ^a, ten vectors $A_\mu^{MN} = A_\mu^{[MN]}$, five two-form fields $B_{\mu\nu M}$, sixteen spin-$\frac{1}{2}$ fermions $\chi^{abc} = \chi^{[ab]c}$, and fourteen scalar fields parametrizing $SL(5)/USp(4)$ coset space $\mathcal{V}_M{}^{ab} = \mathcal{V}_M{}^{[ab]}$. Lower and upper $M, N = 1, ..., 5$ indices refer to the fundamental and anti-fundamental representations, **5** and $\overline{\mathbf{5}}$, of the global $SL(5)$ symmetry.

Under the local $USp(4)$ R-symmetry with fundamental indices $a, b = 1, ..., 4$, the gravitini transform as **4**, while the spin-$\frac{1}{2}$ fields χ^{abc} transform as **16** and satisfy the following conditions

$$\chi^{[abc]} = 0, \quad \Omega_{ab}\chi^{abc} = 0. \tag{2.49}$$

Here, $\Omega_{ab} = \Omega_{[ab]}$ is a $USp(4)$ symplectic form obeying the properties

$$(\Omega_{ab})^* = \Omega^{ab}, \quad \Omega_{ac}\Omega^{bc} = \delta_a^b. \tag{2.50}$$

Raising and lowering of the fundamental $USp(4)$ indices by Ω^{ab} and Ω_{ab} are associated to complex conjugation.

As seen from the field content (2.48), all bosonic fields of the theory come in representations of $SL(5)$, while all fermionic fields come in representations of $USp(4)$. The objects mediating between them are the scalar fields described through $SL(5)/USp(4)$ coset representative $\mathcal{V}_M{}^{ab}$ subject to the condition

$$\mathcal{V}_M{}^{ab}\Omega_{ab} = 0. \tag{2.51}$$

Besides, $\mathcal{V}_{ab}{}^M$ will denote the inverse of $\mathcal{V}_M{}^{ab}$. We then have the following relations

$$\mathcal{V}_M{}^{ab}\mathcal{V}_{ab}{}^N = \delta_M^N \quad \text{and} \quad \mathcal{V}_{ab}{}^M\mathcal{V}_M{}^{cd} = \delta_a^{[c}\delta_b^{d]} - \frac{1}{4}\Omega_{ab}\Omega^{cd}. \tag{2.52}$$

Since $USp(4) \sim SO(5)$, we can convert a pair of antisymmetric $USp(4)$ indices to an $SO(5)$ one, $A = 1, 2, .., 5$ raised and lowered by δ_{AB}, by using $SO(5)$ gamma matrices Γ_A that satisfy

$$(\Gamma_A)^{ab} = -(\Gamma_A)^{ba}, \qquad \Omega_{ab}(\Gamma_A)^{ab} = 0, \qquad ((\Gamma_A)^{ab})^* = \Omega_{ac}\Omega_{bd}(\Gamma_A)^{cd}, \tag{2.53}$$

as well as the Clifford algebra

$$\{\Gamma_A, \Gamma_B\} = 2\delta_{AB}\mathbf{1}_4. \tag{2.54}$$

The coset representative of the form ${\mathcal{V}_M}^{ab}$ and the inverse ${\mathcal{V}_{ab}}^M$ are then respectively related to the $SL(5)/SO(5)$ coset representative ${\mathcal{V}_M}^A$ in the fundamental representation of $SO(5)$ and the inverse ${\mathcal{V}_A}^M$ by the relations

$${\mathcal{V}_M}^{ab} = \frac{1}{2}{\mathcal{V}_M}^A(\Gamma_A)^{ab} \quad \text{and} \quad {\mathcal{V}_{ab}}^M = \frac{1}{2}{\mathcal{V}_A}^M(\Gamma^A)_{ab}. \tag{2.55}$$

In the embedding tensor formalism, the most general gaugings of the $N = 4$ supergravity are encoded in a real embedding tensor, ${\Theta_{MN,P}}^Q = {\Theta_{[MN],P}}^Q$. This tensor identifies gauge group generators $X_{MN} = X_{[MN]}$ for a gauge group G_0 among the $SL(5)$ generators ${t^M}_N$ satisfying ${t^M}_M = 0$ by

$$X_{MN} = {\Theta_{MN,P}}^Q {t^P}_Q. \tag{2.56}$$

SUSY restricts the embedding tensor to $\mathbf{15}$ and $\overline{\mathbf{40}}$ representations of the global $SL(5)$ symmetry. As a result, we can parameterize the embedding tensor in terms of a symmetric matrix Y_{MN} and a tensor $Z^{MN,P}$ with $Y_{MN} = Y_{(MN)}$, $Z^{MN,P} = Z^{[MN],P}$, and $Z^{[MN,P]} = 0$ so that

$${X_{MN,P}}^Q = {\Theta_{MN,P}}^Q = \delta^Q_{[M}Y_{N]P} - 2\epsilon_{MNPRS}Z^{RS,Q} \tag{2.57}$$

are the corresponding gauge group generators in the fundamental representation. Moreover, the embedding tensor needs to satisfy the so-called quadratic constraint to ensure that the gauge generators form a closed subalgebra of the $SL(5)$

$$Y_{MQ}Z^{QN,P} + 2\varepsilon_{MRSTU}Z^{RS,N}Z^{TU,P} = 0. \tag{2.58}$$

Therefore, admissible gauge groups can be classified by searching for the embedding tensor $\Theta_{MN,P}{}^{Q}$ in terms of Y_{MN} and $Z^{MN,P}$ that satisfy the quadratic constraint (2.58).

Unlike the ungauged supergravity in which all three-form fields can be dualized to two-forms, the field content of gauged supergravity can incorporate massive two- and three-form fields. As in Table 2.1, the degrees of freedom in the vector and tensor fields of the ungauged theory will be redistributed among massless and massive vector, two-form, and three-form fields after gauge fixing where $s \equiv \text{rank } Z$ and $t \equiv \text{rank } Y$. Therefore, different gaugings lead to different field contents in the resulting gauged supergravity. It should be noted that $t + s \leq 5$ by the quadratic constraint (2.58).

fields	#	# d.o.f
massless vectors	$10 - s$	5
massless two-forms	$5 - s - t$	10
massive two-forms	s	15
massive self-dual three-forms	t	10

Table 2.1: Distribution of the tensor fields' degrees of freedom after gauge fixing.

In order to describe every tensor field in Table 2.1 in a gauge covariant framework, the following modified two- and three-forms are defined

$$\mathcal{F}^{MN}_{(2)} = F^{MN}_{(2)} + gZ^{MN,P}B_{(2)P}, \qquad (2.59)$$

$$\begin{aligned}\mathcal{H}_{(3)M} &= gY_{MN}S^{N}_{(3)} + DB_{(2)M} \\ &\quad +\varepsilon_{MNPQR}A^{NP}_{(1)} \wedge (dA^{QR}_{(1)} + \frac{2}{3}gX_{ST,U}{}^{Q}A^{RU}_{(1)} \wedge A^{ST}_{(1)}). \qquad (2.60)\end{aligned}$$

The non-abelian two-form field strength is

$$F^{MN}_{(2)} = dA^{MN}_{(1)} + \frac{g}{2}(X_{PQ})_{RS}{}^{MN}A^{PQ}_{(1)} \wedge A^{RS}_{(1)} \qquad (2.61)$$

where g is a gauge coupling constant and $(X_{MN})_{PQ}{}^{RS} = 2X_{MN,[P}{}^{[R}\delta^{S]}_{Q]}$ are the gauge group generators in **10** representation. These modified two- and three-forms satisfy the following deformed Bianchi's identities

$$D\mathcal{F}^{MN}_{(2)} = gZ^{MN,P}\mathcal{H}_{(3)P}, \tag{2.62}$$

$$D\mathcal{H}_{(3)M} = \varepsilon_{MNPQR}\mathcal{F}^{NP}_{(2)} \wedge \mathcal{F}^{QR}_{(2)} + gY_{MN}\mathcal{G}^{N}_{(4)}, \tag{2.63}$$

with

$$\begin{aligned} Y_{MN}\mathcal{G}^{N}_{(4)} = Y_{MN} \Big[& DS^{N}_{(3)} + F^{NP}_{(2)} \wedge B_{(2)P} + \frac{g}{2}Z^{NP,Q}B_{(2)P} \wedge B_{(2)Q} \\ & + \frac{g}{6}\varepsilon_{PQRVW}X_{ST,U}{}^{V}A^{NP}_{(1)} \wedge A^{QR}_{(1)} \wedge A^{ST}_{(1)} \wedge A^{UW}_{(1)} \\ & + \frac{1}{3}\varepsilon_{PQRST}A^{NP}_{(1)} \wedge A^{QR}_{(1)} \wedge dA^{ST}_{(1)} \Big] \end{aligned} \tag{2.64}$$

being the covariant field strength of the three-form fields that always appear only under the projection with Y_{MN}.

In terms of the modified two- and three-forms, bosonic Lagrangian of the maximal gauged supergravity can be written as

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}R * 1 + \frac{1}{8} * D\mathcal{M}_{MN} \wedge D\mathcal{M}^{MN} - 2\mathcal{M}_{MP}\mathcal{M}_{NQ} * \mathcal{F}^{MN}_{(2)} \wedge \mathcal{F}^{PQ}_{(2)} \\ & - \mathcal{M}^{MN} * \mathcal{H}_{(3)M} \wedge \mathcal{H}_{(3)N} - \mathcal{L}_{VT} - \mathbf{V} * 1. \end{aligned} \tag{2.65}$$

Here, scalar fields are described by a unimodular symmetric matrix

$$\mathcal{M}_{MN} = \mathcal{V}_{M}{}^{ab}\mathcal{V}_{N}{}^{cd}\Omega_{ac}\Omega_{bd}, \tag{2.66}$$

together with its inverse

$$(\mathcal{M}_{MN})^{-1} = \mathcal{M}^{MN} = \mathcal{V}_{ab}{}^{M}\mathcal{V}_{cd}{}^{N}\Omega^{ac}\Omega^{bd}. \tag{2.67}$$

The explicit form of the vector-tensor Lagrangian $\mathcal{L}_{VT}$ can be found in [63] while the scalar potential is given by

$$\begin{aligned} \mathbf{V} = & \frac{g^2}{64}\left[2\mathcal{M}^{MN}Y_{NP}\mathcal{M}^{PQ}Y_{QM} - (\mathcal{M}^{MN}Y_{MN})^2\right] \\ & + g^2Z^{MN,P}Z^{QR,S}\left(\mathcal{M}_{MQ}\mathcal{M}_{NR}\mathcal{M}_{PS} - \mathcal{M}_{MQ}\mathcal{M}_{NP}\mathcal{M}_{RS}\right). \end{aligned} \tag{2.68}$$

From the Lagrangian (2.65), all bosonic Euler-Lagrange field equations can be derived

$$
\begin{aligned}
0 &= R_{\mu\nu} - \frac{1}{4}\mathcal{M}_{MP}\mathcal{M}_{NQ}(D_\mu\mathcal{M}^{MN})(D_\nu\mathcal{M}^{PQ}) - \frac{2}{5}g_{\mu\nu}\mathbf{V} \\
&\quad -4\mathcal{M}_{MP}\mathcal{M}_{NQ}\left(\mathcal{F}^{MN}_{\mu\rho}\mathcal{F}^{PQ}{}_{\nu}{}^{\rho} - \frac{1}{10}g_{\mu\nu}\mathcal{F}^{MN}_{\rho\sigma}\mathcal{F}^{PQ\rho\sigma}\right) \\
&\quad -\mathcal{M}^{MN}\left(\mathcal{H}_{\mu\rho\sigma M}\mathcal{H}_{\nu}{}^{\rho\sigma}{}_{N} - \frac{2}{15}g_{\mu\nu}\mathcal{H}_{\rho\sigma\lambda M}\mathcal{H}^{\rho\sigma\lambda}{}_{N}\right), \qquad (2.69) \\
0 &= D^\mu(\mathcal{M}_{MP}D_\mu\mathcal{M}^{PN}) - \frac{g^2}{8}\mathcal{M}^{PQ}\mathcal{M}^{RN}\left(2Y_{RQ}Y_{PM} - Y_{PQ}Y_{RM}\right) \\
&\quad -\frac{4}{6}\mathcal{M}^{PN}\mathcal{H}_{\mu\nu\rho M}\mathcal{H}^{\mu\nu\rho}{}_{P} - 8\mathcal{M}_{MP}\mathcal{M}_{QR}\mathcal{F}^{PQ}_{\mu\nu}\mathcal{F}^{RN\mu\nu} \\
&\quad +\frac{8}{5}\delta^N_M\left(\mathbf{V} + \mathcal{M}_{SP}\mathcal{M}_{QR}\mathcal{F}^{PQ}_{\mu\nu}\mathcal{F}^{RS\mu\nu} + \frac{1}{16}\mathcal{M}^{PQ}\mathcal{H}_{\mu\nu\rho P}\mathcal{H}^{\mu\nu\rho}{}_{Q}\right) \\
&\quad +4g^2 Z^{QT,P}Z^{NR,S}\mathcal{M}_{QM}(2\mathcal{M}_{TR}\mathcal{M}_{PS} - \mathcal{M}_{TP}\mathcal{M}_{RS}) \\
&\quad +4g^2 Z^{QT,P}Z^{RS,N}\mathcal{M}_{QS}(2\mathcal{M}_{TP}\mathcal{M}_{RM} - \mathcal{M}_{TR}\mathcal{M}_{PM}) \\
&\quad -4g^2\delta^N_M Z^{TU,P}Z^{QR,S}\mathcal{M}_{TQ}\left(\mathcal{M}_{UR}\mathcal{M}_{PS} - \mathcal{M}_{UP}\mathcal{M}_{RS}\right), \qquad (2.70) \\
0 &= 4D_\nu(\mathcal{M}_{MP}\mathcal{M}_{NQ}\mathcal{F}^{PQ\nu\mu}) - \frac{g}{2}X_{MNP}{}^{Q}\mathcal{M}_{QR}D^\mu\mathcal{M}^{PR} \\
&\quad -2\varepsilon_{MNPQR}\mathcal{M}^{PS}\mathcal{H}^{\mu\nu\rho}{}_{S}\mathcal{F}^{QR}_{\nu\rho} + \frac{1}{9}e^{-1}\epsilon^{\mu\nu\rho\lambda\sigma\tau\kappa}\mathcal{H}_{\nu\rho\lambda M}\mathcal{H}_{\sigma\tau\kappa N}, \qquad (2.71) \\
0 &= D_\rho\left(\mathcal{M}^{MN}\mathcal{H}^{\rho\mu\nu}{}_{N}\right) - 2gZ^{NP,M}\mathcal{M}_{NQ}\mathcal{M}_{PR}\mathcal{F}^{QR\mu\nu} \\
&\quad -\frac{1}{3}e^{-1}\epsilon^{\mu\nu\rho\lambda\sigma\tau\kappa}\mathcal{F}^{MN}_{\rho\lambda}\mathcal{H}_{\sigma\tau\kappa N}, \qquad (2.72) \\
0 &= \epsilon^{\mu\nu\rho\lambda\sigma\tau\kappa}Y_{MN}\mathcal{G}^{N}_{\lambda\sigma\tau\kappa} - 6Y_{MN}\mathcal{M}^{NP}\mathcal{H}^{\mu\nu\rho}{}_{P}. \qquad (2.73)
\end{aligned}
$$

The fermionic SUSY transformations that are essential for finding supersymmetric solutions read

$$
\begin{aligned}
\delta\psi^a_\mu &= D_\mu\epsilon^a - g\gamma_\mu A_1^{ab}\Omega_{bc}\epsilon^c + \frac{1}{5}\mathcal{F}^{MN}_{\nu\rho}(\gamma_\mu{}^{\nu\rho} - 8\delta^\nu_\mu\gamma^\rho)\mathcal{V}_M{}^{ad}\Omega_{de}\mathcal{V}_N{}^{eb}\Omega_{bc}\epsilon^c \\
&\quad +\frac{1}{15}\mathcal{H}_{\nu\rho\lambda M}(\gamma_\mu{}^{\nu\rho\lambda} - \frac{9}{2}\delta^\nu_\mu\gamma^{\rho\lambda})\Omega^{ab}\mathcal{V}_{bc}{}^{M}\epsilon^c, \qquad (2.74) \\
\delta\chi^{abc} &= 2\Omega^{cd}P_{\mu de}{}^{ab}\gamma^\mu\epsilon^e + gA_2^{d,abc}\Omega_{de}\epsilon^e \\
&\quad +2\mathcal{F}^{MN}_{\mu\nu}\gamma^{\mu\nu}\Omega_{de}\left[\mathcal{V}_M{}^{cd}\mathcal{V}_N{}^{e[a}\epsilon^{b]} - \frac{1}{5}(\Omega^{ab}\delta^c_g - \Omega^{c[a}\delta^{b]}_g)\mathcal{V}_M{}^{gf}\Omega_{fh}\mathcal{V}_N{}^{hd}\epsilon^e\right] \\
&\quad -\frac{1}{6}\mathcal{H}_{\mu\nu\rho M}\gamma^{\mu\nu\rho}\mathcal{V}_{fe}{}^{M}\left[\Omega^{af}\Omega^{be}\epsilon^c - \frac{1}{5}(\Omega^{ab}\Omega^{cf} + 4\Omega^{c[a}\Omega^{b]f})\epsilon^e\right]. \qquad (2.75)
\end{aligned}
$$

The covariant derivative of the SUSY parameters is defined by

$$D_\mu\epsilon^a = \partial_\mu\epsilon^a + \frac{1}{4}\omega_\mu{}^{\hat{\nu}\hat{\rho}}\gamma_{\hat{\nu}\hat{\rho}}\epsilon^a - Q_\mu{}_b{}^a\epsilon^b. \qquad (2.76)$$

The composite connection $Q_{\mu\, a}{}^{b}$ and the vielbein $P_{\mu ab}{}^{cd}$ on the $SL(5)/SO(5)$ coset are obtained from

$$P_{\mu ab}{}^{cd} + 2Q_{\mu[a}{}^{[c}\delta^{d]}_{b]} = \mathcal{V}_{ab}{}^{M}(\partial_\mu \mathcal{V}_{M}{}^{cd} - gA^{PQ}_{\mu}X_{PQ,M}{}^{N}\mathcal{V}_{N}{}^{cd}). \tag{2.77}$$

The fermion shift matrices, A_1 and A_2, are

$$A_1^{ab} = -\frac{1}{4\sqrt{2}}\left(\frac{1}{4}B\Omega^{ab} + \frac{1}{5}C^{ab}\right), \tag{2.78}$$

$$A_2^{d,abc} = \frac{1}{2\sqrt{2}}\left[\Omega^{ec}\Omega^{fd}(C^{ab}{}_{ef} - B^{ab}{}_{ef}) + \frac{1}{4}(C^{ab}\Omega^{cd} + \frac{1}{5}\Omega^{ab}C^{cd} + \frac{4}{5}\Omega^{c[a}C^{b]d})\right] \tag{2.79}$$

where B and C tensors are functions of the scalar fields

$$B = \frac{\sqrt{2}}{5}\Omega^{ac}\Omega^{bd}Y_{ab,cd}, \tag{2.80}$$

$$B^{ab}{}_{cd} = \sqrt{2}\left[\Omega^{ae}\Omega^{bf}\delta^{gh}_{cd} - \frac{1}{5}(\delta^{ab}_{cd} - \frac{1}{4}\Omega^{ab}\Omega_{cd})\Omega^{eg}\Omega^{fh}\right]Y_{ef,gh}, \tag{2.81}$$

$$C^{ab} = 8\Omega_{cd}Z^{(ac)[bd]}, \tag{2.82}$$

$$C^{ab}{}_{cd} = 8\left(-\Omega_{ce}\Omega_{df}\delta^{ab}_{gh} + \Omega_{g(c}\delta^{ab}_{d)e}\Omega_{fh}\right)Z^{(ef)[gh]} \tag{2.83}$$

in which "dressed" components of the embedding tensor are defined by

$$Y_{ab,cd} = \mathcal{V}_{ab}{}^{M}\mathcal{V}_{cd}{}^{N}Y_{MN}, \qquad Z^{(ac)[ef]} = \sqrt{2}\mathcal{V}_{M}{}^{ab}\mathcal{V}_{N}{}^{cd}\mathcal{V}_{P}{}^{ef}\Omega_{bd}Z^{MN,P}. \tag{2.84}$$

2.2.2 Gauge Fixing and Supersymmetric Critical Points

The maximal $N = 4$ supergravity in seven dimensions can be gauged by gauge groups of the form $CSO(p, q, 5-p-q)$ and $CSO(p, q, 4-p-q)$ corresponding to the embedding tensor in **15** and $\overline{\mathbf{40}}$ representations of the global symmetry $SL(5)$. These gauged supergravities can be embedded respectively in eleven-dimensional and type IIB supergravities. Besides, from the embedding tensor with both **15** and $\overline{\mathbf{40}}$ representations non-vanishing, non-semisimple $SO(2,1) \ltimes \mathbf{R}^4$ and $SO(2) \ltimes \mathbf{R}^4$ gauge groups are considered.

2.2.2.1 Gaugings in 15 Representation

We begin with the maximal theory gauged in **15** representation. In this case, $Z^{MN,P} = 0$ makes the quadratic constraint (2.58) identically satisfied. The $SL(5)$

symmetry can be applied to diagonalize Y_{MN} to be

$$Y_{MN} = \text{diag}(\underbrace{1,..,1}_{p},\underbrace{-1,..,-1}_{q},\underbrace{0,..,0}_{5-p-q}). \tag{2.85}$$

Therefore, the corresponding gauge group generators take the form

$$(X_{MN})_P{}^Q = \delta^Q_{[M}Y_{N]P}, \tag{2.86}$$

and give rise to the gauge group

$$G_0 = CSO(p,q,5-p-q) = SO(p,q) \ltimes \mathbf{R}^{(p+q)(5-p-q)}. \tag{2.87}$$

Among many possible gauge groups, $SO(5)$ and $CSO(4,0,1)$ gauged theories are remarkable since they are obtained respectively from consistent truncations of eleven-dimensional supergravity on S^4 [25–27] and type IIA theory on S^3 [77]. As shown in [64], using the framework of exceptional field theory, these $CSO(p,q,5-p-q)$ gauged theories can be obtained from a consistent truncation of eleven-dimensional supergravity on a non-compact manifold $H^{p,q} \circ T^{(5-p-q)}$. This manifold is a product of a $(p+q-1)$-dimensional hyperbolic space and a $(5-p-q)$-dimensional torus. Unfortunately, their complete truncation ansatze have not been constructed to date.

From (2.68), the scalar potential in this case reads

$$\mathbf{V} = \frac{g^2}{64}\left(2\mathcal{M}^{MN}Y_{NP}\mathcal{M}^{PQ}Y_{QM} - (\mathcal{M}^{MN}Y_{MN})^2\right). \tag{2.88}$$

This potential admits two AdS_7 critical points when the theory is gauged by $SO(5)$ gauge group [33]. However, only one of them preserves $N=4$ supersymmetry and should be AdS/CFT dual to $N=(2,0)$ SCFT in six dimensions. For $CSO(2,0,3)$ gauge group, the scalar potential (2.88) is vanishing; hence the theory admits a critical point related to a Minkowski vacuum, as pointed out in [63].

2.2.2.2 Gaugings in $\overline{\mathbf{40}}$ Representation

For gaugings in $\overline{\mathbf{40}}$ representation with $Y_{MN}=0$, the quadratic constraint (2.58) is reduced to

$$\varepsilon_{MRSTU}Z^{RS,N}Z^{TU,P} = 0. \tag{2.89}$$

This constraint can be solved by

$$Z^{MN,P} = v^{[M}w^{N]P} \tag{2.90}$$

where $w^{MN} = w^{(MN)}$ and v^M being a five-dimensional vector. We also use the $SL(5)$ symmetry to fix $v^M = \delta^M_5$, thus, the $SL(5)$ index splits as $M = (i, 5)$. By setting $w^{55} = w^{i5} = 0$, the remaining $SL(4)$ symmetry can be used to diagonalize w^{ij} to be

$$w^{ij} = \text{diag}(\underbrace{1, .., 1}_{p}, \underbrace{-1, .., -1}_{q}, \underbrace{0, .., 0}_{4-p-q}). \tag{2.91}$$

The resulting gauge group generators then take the form

$$(X_{ij})_k{}^l = 2\epsilon_{ijkm}w^{ml} \tag{2.92}$$

and generate $CSO(p, q, 4-p-q)$ gauge groups. In analogy to the discussion of the last section, $CSO(p, q, 4-p-q)$ gauged theory is related to a reduction of the type IIB theory over a non-compact manifold $H^{p,q} \circ T^{(4-p-q)}$. This procedure has been examined in [65] along with a partial result on the corresponding truncation ansatze.

To compute the scalar potential for these gaugings, we decompose the $SL(5)/SO(5)$ coset representative as

$$\mathcal{V} = e^{b_i t^i}\widetilde{\mathcal{V}}e^{\phi_0 t_0}. \tag{2.93}$$

Here, $\widetilde{\mathcal{V}}$ is the $SL(4)/SO(4)$ coset representative and t_0, t^i refer to $SO(1,1)$ and four nilpotent generators, respectively, in the decomposition $SL(5) \to SL(4) \times SO(1,1)$. For the unimodular matrix $\mathcal{M}_{MN}$, this yields a block decomposition

$$\mathcal{M}_{MN} = \begin{pmatrix} e^{-2\phi_0}\widetilde{\mathcal{M}}_{ij} + e^{8\phi_0} & e^{8\phi_0}b_i \\ e^{8\phi_0}b_j & e^{8\phi_0} \end{pmatrix} \tag{2.94}$$

with $\widetilde{\mathcal{M}}_{ij} = \widetilde{\mathcal{V}}\widetilde{\mathcal{V}}^T$ and $\widetilde{\mathcal{M}}^{ij}$ being the inverse of $\widetilde{\mathcal{M}}_{ij}$. From (2.68), we can compute the scalar potential of the form

$$\mathbf{V} = \frac{g^2}{4}e^{14\phi_0}b_i w^{ij}\widetilde{\mathcal{M}}_{jk}w^{kl}b_l + \frac{g^2}{4}e^{4\phi_0}\left(2\widetilde{\mathcal{M}}_{ij}w^{jk}\widetilde{\mathcal{M}}_{kl}w^{li} - (\widetilde{\mathcal{M}}_{ij}w^{ij})^2\right). \tag{2.95}$$

The presence of the dilaton prefactor e^{ϕ_0} shows that this potential does not admit any critical points with non-vanishing potential. In particular, the potential (2.95) of the $CSO(2,0,2)$ theory admits a critical point with vanishing potential. This critical point is also Minkowski vacuum as in the $CSO(2,0,3)$ theory from gaugings in **15** representation.

2.2.2.3 Gaugings in 15 and $\overline{\mathbf{40}}$ Representations

For non-vanishing components of the embedding tensor in both **15** and $\overline{\mathbf{40}}$ representations, we choose an appropriate basis such that the embedding tensor's components are given by

$$Y_{xy}, \qquad Z^{x\alpha,\beta} = Z^{x(\alpha,\beta)}, \qquad Z^{\alpha\beta,\gamma}, \tag{2.96}$$

in which $x = 1, ..., t$ and $\alpha = t+1, ..., 5$ for $t \equiv \text{rank}\, Y$. On the other hand, the $SL(5)$ index splits into $M = (x, \alpha)$. In terms of these components of the embedding tensor, the quadratic constraint (2.58) reads

$$Y_{xy}Z^{y\alpha,\beta} + 2\varepsilon_{xMNPQ}Z^{MN,\alpha}Z^{PQ,\beta} = 0. \tag{2.97}$$

With Y_{xy} choosen to be

$$Y_{xy} = \text{diag}(\underbrace{1, .., 1}_{p}, \underbrace{-1, .., -1}_{q}), \tag{2.98}$$

there exist real solutions for the embedding tensors that satisfy the quadratic constraint (2.97) for $SO(2,1) \ltimes \mathbf{R}^4$ and $SO(2) \ltimes \mathbf{R}^4$ gauge groups. Note that the later can be obtained from Scherk-Schwarz reduction [78] of the maximal gauged supergravity in eight dimensions [79].

(1) <u>$SO(2,1) \ltimes \mathbf{R}^4$ gauge group</u>

We begin with the $t = 3$ case in which $Y_{xy} = \text{diag}(1, 1, \pm 1)$. In this case, the component $Z^{\alpha\beta,\gamma}$ is not constrained by the quadratic constraint (2.97). Therefore, we can set it to zero, then (2.97) becomes

$$\varepsilon_{xyz}Z^{y\alpha,\gamma}\epsilon_{\gamma\delta}Z^{z\delta,\beta} = \frac{1}{8}Y_{xu}Z^{u\alpha,\beta} \tag{2.99}$$

which suggests that the (2×2) matrices $(\zeta^x)_\alpha{}^\beta = -16\varepsilon_{\alpha\gamma}Z^{x\gamma,\beta}$ satisfy the algebra

$$[\zeta^x,\zeta^y] = 2\varepsilon^{xyu}Y_{uz}\zeta^z. \tag{2.100}$$

In terms of ζ^x, the embedding tensor's component $Z^{x\alpha,\beta}$ takes the form

$$Z^{x\alpha,\beta} = -\frac{1}{16}\epsilon^{\alpha\gamma}(\zeta^x)_\gamma{}^\beta. \tag{2.101}$$

As pointed out in [63], there exists a real non-vanishing solution for $Z^{x\alpha,\beta}$ when Y_{xy} generates a non-compact $SO(2,1)$ gauge group. In this case, we use $Y_{xy} = \text{diag}(1,1-1)$ together with the following explicit form for ζ^x in terms of Pauli matrices,

$$\zeta^1 = \sigma_1, \quad \zeta^2 = \sigma_3, \quad \zeta^3 = i\sigma_2. \tag{2.102}$$

Therefore, the corresponding gauge generators are given by

$$X_M{}^N = \begin{pmatrix} \lambda^z(t^z)_x{}^y & Q_x^{(4)\beta} \\ 0_{2\times 3} & \frac{1}{2}\lambda^z(\zeta^z)_\alpha{}^\beta \end{pmatrix} \tag{2.103}$$

with $\lambda^z \in \mathbb{R}$. Note here that the $SO(2,1)$ subgroup is embedded diagonally in these gauge generators. The nilpotent generators $Q_x^{(4)\alpha}$ transform as **4** under $SO(2,1)$. Therefore, the resulting gauge group is $SO(2,1)\ltimes \mathbf{R}^4$. None of the theories in this sector possesses a critical point from its scalar potential.

(2) $SO(2)\ltimes \mathbf{R}^4$ gauge group

In this case, we consider $t = 2$ together with $Y_{xy} = \text{diag}(1,1)$. Only the non-vanishing component $Z^{\alpha\beta,\gamma}$ is allowed by the quadratic constraint. This component can be parametrized by a (3×3) traceless matrix $Z_\alpha{}^\beta$ with $Z_\alpha{}^\alpha = 0$,

$$Z^{\alpha\beta,\gamma} = \frac{1}{8}\varepsilon^{\alpha\beta\delta}Z_\delta{}^\gamma. \tag{2.104}$$

The corresponding gauge generators are then given by

$$X_M{}^N = \begin{pmatrix} \lambda t_x{}^y & Q_x{}^\beta \\ 0_{3\times 2} & \lambda Z_\alpha{}^\beta \end{pmatrix} \tag{2.105}$$

with $\lambda \in \mathbb{R}$. Here, $t_x{}^y = i\sigma_2$ generates the $SO(2)$ compact subgroup while $Q_x{}^\alpha \in \mathbb{R}$ generally generate six translations $\mathbf{R}^6$ resulting in $SO(2) \ltimes \mathbf{R}^6$ gauge group.* It can be checked that the scalar potential vanishes identically in this case. Furthermore, as pointed out in [63], the number of independent translations is reduced if there exist non-trivial solutions for Q satisfying

$$tQ - QZ = 0. \tag{2.106}$$

We are interested in the compact case with $\mathrm{Tr}Z^2 = -2$ in which half of the supersymmetry ($N = 2$) is preserved and the gauge group is reduced to $SO(2) \ltimes \mathbf{R}^4 \sim CSO(2,0,2)$. Unlike in $CSO(2,0,3)$ and $CSO(2,0,2)$ gauged theories separatly obtained from gaugings in **15** and $\overline{\mathbf{40}}$ representations, the Minkowski critical point in this case is half-supersymmetric.

We finish this chapter by listing a large class of admissible gauge groups of the maximal gauged supergravity that will be examined in this work together with their critical points in Table 2.2. Explicit forms of these critical points will be given in Section 4.1.1 where supersymmetric DW solutions of the maximal gauged supergravity are considered.

*By using $Y_{xy} = \mathrm{diag}(1, -1)$ corresponding to $t_x{}^y = \sigma_1$, $SO(1,1) \ltimes \mathbf{R}^6$ gauge group is also accepted. However, we are not interested in this case since the non-semisimple $SO(1,1) \ltimes \mathbf{R}^6$ admits no compact unbroken symmetry.

Y_{MN}	$Z^{\alpha\beta,\gamma}$	$Z^{x\alpha,\beta}$	gauge group	critical points	SUSY
$(+++++)$	-	-	$SO(5)$	2 AdS_7	4, 0
$(++++-)$	-	-	$SO(4,1)$	-	
$(+++--)$	-	-	$SO(3,2)$	-	
$(++++\ 0)$	-	-	$CSO(4,0,1)$	-	
$(+++-\ 0)$	-	-	$CSO(3,1,1)$	-	
$(++--\ 0)$	-	-	$CSO(2,2,1)$	-	
$(+++\ 0\ 0)$	-	-	$CSO(3,0,2)$	-	
$(++-\ 0\ 0)$	-	-	$CSO(2,1,2)$	-	
$(++-\ 0\ 0)$	-	$\frac{1}{16}\varepsilon^{\gamma\alpha}(\Sigma^x)_\gamma{}^\beta$	$SO(2,1)\ltimes\mathbb{R}^4$	-	
$(++\ 0\ 0\ 0)$	-	-	$CSO(2,0,3)$	Mkw_7	0
$(+-\ 0\ 0\ 0)$	-	-	$CSO(1,1,3)$	-	
$(++\ 0\ 0\ 0)$	$\frac{1}{8}\varepsilon^{\alpha\beta\delta}Z_\delta{}^\gamma$	-	$SO(2)\ltimes\mathbb{R}^4$	Mkw_7	2
$(+\ 0\ 0\ 0\ 0)$	-	-	$CSO(1,0,4)$	-	
$(\ 0\ 0\ 0\ 0\ 0\ )$	$v^{[\alpha}w^{\beta]\gamma}$	-	$CSO(p,q,r)$ $(p+q+r=4)$	Mkw_7 $(p=2=r)$	0

Table 2.2: Gaugings of the maximal gauged supergravity examined in this work.

CHAPTER III

SUPERSYMMETRIC SOLUTIONS OF MATTER-COUPLED $SO(4)$ GAUGED SUPERGRAVITY

In this chapter, we enlarge the study of the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence with sixteen supercharges. In this case, the AdS_7 background is dual to an $N = (1, 0)$ SCFT in six dimensions. In many aspects, six-dimensional SCFTs with $N = (1, 0)$ supersymmetry are interesting. It has been pointed out in [80] that the theories admit non-trivial RG fixed points. Moreover, the dynamics of these field theories also arises from string theory [81], see also a review in [82].

In the holographic study of the above $N = (1, 0)$ SCFTs, the half-maximal $N = 2$ gauged supergravity in seven dimensions coupled to vector multiplets, reviewed in Section 2.1, is examined. Especially in the remarkable case of $SO(4)$ gauge group, supersymmetric AdS_7 critical points with $SO(4)$ and $SO(3)_{\mathrm{diag}}$ symmetries together with analytic RG flows interpolating between them have been completely studied in [39]. Holographic description for twisted compactifications of six-dimensional $N = (1, 0)$ SCFTs on two- and three-manifolds, Σ^2 and Σ^3, has been considered in [60]. The corresponding supergravity solutions take the form of $AdS_5 \times \Sigma^2$ and $AdS_4 \times \Sigma^3$, respectively. These twisted solutions interpolating between the AdS_7 vacua and AdS_5 or AdS_4 geometries should describe RG flows across dimensions from six-dimensional $N = (1, 0)$ SCFTs to SCFTs in four and three dimensions.

With these studies, supersymmetric solutions of the $SO(4)$ gauged supergravity seem to be complete. However, the massive self-dual three-form $H_{(3)}$ does not involve in these solutions. In this chapter, the solutions studied in [39] and [60] are respectively generalized by including a non-vanishing three-form.

3.1 Charged Domain Wall Solutions

For the pure $N = 2$ gauged supergravity in seven dimensions, supersymmetric solutions with all bosonic fields, including the three-form and vector fields, non-vanishing have appeared recently in [62] along with their embedding in M-theory by using the result of [83]. Without the $SU(2)$ gauge fields, the solution has also been uplifted to massive type IIA theory in [84] in which the solution is interpreted as a two-dimensional conformal defect in a six-dimensional $N = (1, 0)$ SCFT. The principal method to include the massive self-dual three-form into their solutions is using the $AdS_3 \times S^3$-sliced DW ansatz instead of the usual flat one (2.37). In this geometry, dyonic profiles of the three-form potential are required in order to support non-vanishing curvature of the background.

In the following analysis, we will find supersymmetric solutions with non-vanishing self-dual three-form, called "charged" DWs, from the matter-coupled $SO(4)$ gauged supergravity. The solutions could be uplifted to eleven-dimensional supergravity by truncation ansatze constructed in [74].

3.1.1 Solutions Flowing between AdS_7 Vacua

We start with supersymmetric solutions involving only the seven-dimensional metric, the dilaton σ, the $SO(3)_{\text{diag}}$ singlet scalar ϕ, and the self-dual three-form $H_{(3)}$. In this case, the $SO(3)_{\text{diag}}$ singlet corresponding to the coset representative (2.33) is chosen in order to find charged DW solutions interpolating between the two AdS_7 vacua.

Following [62], we take the metric ansatz to be an $AdS_3 \times S^3$-sliced DW

$$ds^2 = e^{2U(r)} ds^2_{AdS_3} + e^{2V(r)} dr^2 + e^{2W(r)} ds^2_{S^3} \tag{3.1}$$

with the metrics on AdS_3 and S^3 given by

$$ds^2_{AdS_3} = \frac{1}{\tau^2}\left[(dx^1)^2 + \cosh^2 x^1 (dx^2)^2 - (dt - \sinh x^1 dx^2)^2\right], \tag{3.2}$$

$$ds^2_{S^3} = \frac{1}{\kappa^2}\left[(d\theta_2)^2 + \cos^2\theta_2 (d\theta_3)^2 + (d\theta_1 + \sin\theta_2 d\theta_3)^2\right] \tag{3.3}$$

where τ and κ are constants. The seven-dimensional spacetime coordinates are taken to be $x^\mu = (x^a, r, x^m)$ with $a = 0, 1, 2$ and $m = 4, 5, 6$. We always use $x^0 = t$, while $x^3 = r$ is chosen in this section. The S^3 part is described by Hopf coordinates $x^m = (\theta_1, \theta_2, \theta_3)$. The corresponding flat spacetime indices are denoted by $\hat{\mu} = (\hat{a}, \hat{3}, \hat{m})$. In the limit $\tau \to 0$ and $\kappa \to 0$, the AdS_3 and S^3 become Minkowski space Mkw_3 and flat space $\mathbb{R}^3$, respectively.

With the vielbeins on AdS_3 and S^3 of the form

$$e^{\hat{0}} = \frac{1}{\tau}(dt - \sinh x^1 dx^2), \tag{3.4}$$

$$e^{\hat{1}} = \frac{1}{\tau}(\cos t dx^1 - \sin t \cosh x^1 dx^2), \tag{3.5}$$

$$e^{\hat{2}} = \frac{1}{\tau}(\sin t dx^1 + \cos t \cosh x^1 dx^2) \tag{3.6}$$

and

$$e^{\hat{4}} = \frac{1}{\kappa}(d\theta_1 + \sin\theta_2 d\theta_3), \tag{3.7}$$

$$e^{\hat{5}} = \frac{1}{\kappa}(\cos\theta_1 d\theta_2 - \sin\theta_1 \cos\theta_2 d\theta_3), \tag{3.8}$$

$$e^{\hat{6}} = \frac{1}{\kappa}(\sin\theta_1 d\theta_2 + \cos\theta_1 \cos\theta_2 d\theta_3), \tag{3.9}$$

the corresponding spin connections take a simple form

$$\omega_{\hat{a}}{}^{\hat{a}}{}_{\hat{3}} = e^{-V}U', \qquad \omega_{\hat{a}\hat{b}\hat{c}} = \frac{\tau}{2}e^{-U}\varepsilon_{\hat{a}\hat{b}\hat{c}}, \tag{3.10}$$

$$\omega_{\hat{m}}{}^{\hat{m}}{}_{\hat{3}} = e^{-V}W', \qquad \omega_{\hat{m}\hat{n}\hat{p}} = \frac{\kappa}{2}e^{-W}\varepsilon_{\hat{m}\hat{n}\hat{p}} \tag{3.11}$$

with $\varepsilon_{\hat{0}\hat{1}\hat{2}} = -\varepsilon^{\hat{0}\hat{1}\hat{2}} = \varepsilon_{\hat{4}\hat{5}\hat{6}} = \varepsilon^{\hat{4}\hat{5}\hat{6}} = 1$.

As in the usual DW solutions, the scalar fields σ and ϕ are functions of only the radial coordinate r, while the ansatz for the three-form field is taken to be

$$H_{(3)} = k(r)\mathrm{Vol}_{AdS_3} + l(r)\mathrm{Vol}_{S^3}. \tag{3.12}$$

With the metrics given in (3.2) and (3.3), the volume forms on AdS_3 and S^3 are respectively given by

$$\mathrm{Vol}_{AdS_3} = \frac{1}{\tau^3}\cosh x^1 dx^1 \wedge dx^2 \wedge dx^3 \quad \text{and} \quad \mathrm{Vol}_{S^3} = \frac{1}{\kappa^3}\cos\theta_2 d\theta_1 \wedge d\theta_2 \wedge d\theta_3. \tag{3.13}$$

For Killing spinors, the ansatz corresponding to the unbroken SUSY takes the form of

$$\epsilon = Y(r)\left[\cos\theta(r)\mathbf{1}_8 + \sin\theta(r)\gamma^{\hat{0}\hat{1}\hat{2}}\right]\epsilon_0 \tag{3.14}$$

with the constant spinors ϵ_0 satisfying the projection condition

$$\gamma^{\hat{3}}\epsilon_0 = \epsilon_0\,. \tag{3.15}$$

$Y(r)$ and $\theta(r)$ are functions of r to be determined.

To find supersymmetric solutions, we consider BPS equations obtained from SUSY transformations of fermionic fields given in (2.23) to (2.25). Using the Killing spinors (3.14) and the projection (3.15), we find two equations from $\delta\lambda^r = 0$ conditions

$$P^{ir}_{\hat{3}}\cos 2\theta\sigma^i - \frac{1}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\sigma^i = 0, \tag{3.16}$$

$$P^{ir}_{\hat{3}}\sigma^i - \frac{1}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\cos 2\theta\sigma^i = 0\,. \tag{3.17}$$

Compatibility between these equations needs $\cos(2\theta) = \pm 1$ leading to $\sin\theta = 0$ or $\cos\theta = 0$. Up to a redefinition of ϵ_0 to $\tilde{\epsilon}_0 = \gamma^{\hat{0}\hat{1}\hat{2}}\epsilon_0$ and a sign change in the projection condition (3.15), the two choices give equivalent BPS equations. For definiteness, we will choose $\theta = 0$. This leads to a simpler form of the Killing spinors

$$\epsilon = Y(r)\epsilon_0, \tag{3.18}$$

and the BPS equation for general scalars from the vector multiples

$$P^{ir}_{\hat{3}}\sigma^i - \frac{1}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\sigma^i = 0. \tag{3.19}$$

This BPS equation is the same as that obtained in the usual flat DW solutions [39], which means the three-form field does not directly couple to scalars from the vector multiplets. Therefore, for more general cases, including more scalars invariant under smaller residual symmetries, similar solutions can be found in the same way as this $SO(3)_{\text{diag}}$ case. Using the coset representative (2.33), we can readily compute P^{ir}_μ and C^{ir} for the $SO(3)_{\text{diag}}$ singlet ϕ. The resulting BPS equation is given by

$$\phi' = -e^{V-\frac{\sigma}{2}}(g_1\cosh\phi + g_2\sinh\phi)\cosh\phi\sinh\phi\,. \tag{3.20}$$

We now consider $\delta\chi = 0$ equation. This condition involves a contribution from the three-form field of the form $G_{\mu\nu\rho\sigma}\gamma^{\mu\nu\rho\sigma}\epsilon$. Using the relation $\varepsilon_{\hat{a}\hat{b}\hat{c}}\gamma^{\hat{a}\hat{b}\hat{c}}\gamma^{\hat{3}} = -\varepsilon_{\hat{m}\hat{n}\hat{p}}\gamma^{\hat{m}\hat{n}\hat{p}}$ derived from the identity (B.6), we find

$$\frac{1}{4!}G_{\mu\nu\rho\sigma}\gamma^{\mu\nu\rho\sigma}\epsilon = (l'e^{-V-3W} - k'e^{-V-3U})\gamma^{\hat{0}\hat{1}\hat{2}}\epsilon\,. \tag{3.21}$$

Since there is no other term contributing $\gamma^{\hat{0}\hat{1}\hat{2}}$ matrix in the $\delta\chi$ variation, this term must vanish by itself. This can be achieved by setting

$$k'e^{-3U} = l'e^{-3W} \tag{3.22}$$

which leads to the following BPS equation for σ

$$\sigma' = \frac{2}{5}e^{V-\frac{\sigma}{2}}\left[g_1\cosh^3\phi + g_2\sinh^3\phi - 16he^{\frac{5}{2}\sigma}\right]. \tag{3.23}$$

We then move on to the BPS equations from $\delta\psi_\mu = 0$ conditions. After using the $\gamma^{\hat{3}}$ projection (3.15) and the three-form ansatz (3.12) in the conditions $\delta\psi_a = 0$ and $\delta\psi_m = 0$, we find two types of terms with $\gamma^{\hat{0}\hat{1}\hat{2}}$ and $\mathbf{1}_8$. The former gives the BPS equations for k and l

$$k' = \frac{\tau}{\sqrt{2}}e^{2U+\sigma+V}, \qquad l' = \frac{\kappa}{\sqrt{2}}e^{2W+\sigma+V} \tag{3.24}$$

while the latter gives rise to the corresponding BPS equations for U and W

$$U' = W' = \frac{1}{5}e^{V-\frac{\sigma}{2}}\left[4he^{\frac{5}{2}\sigma} + g_1\cosh^3\phi + g_2\sinh^3\phi\right]. \tag{3.25}$$

The last equation implies that $U = W + C_1$ for a constant C_1. In order to find solutions interpolating between AdS_7 vacua, we require that the solutions are asymptotically locally AdS_7 at which $U = W$. This condition requires $C_1 = 0$.

Imposing $U = W$ in equation (3.22), we obtain $k' = l'$ or $k = l + C_2$ for a constant C_2. This constant can be set to zero by a suitable redefinition of k and l. We will accordingly set $k = l$. With all these, the BPS equations in (3.24) give

$$\tau = \kappa\,. \tag{3.26}$$

We eventually end up with the BPS equations for U and k in the form of

$$U' = \frac{1}{5}e^{V-\frac{\sigma}{2}}\left[4he^{\frac{5}{2}\sigma} + g_1\cosh^3\phi + g_2\sinh^3\phi\right], \tag{3.27}$$

$$k' = \frac{\kappa}{\sqrt{2}}e^{2U+\sigma+V}\,. \tag{3.28}$$

It should be noted that the contribution from $H_{(3)}$ is canceled by the spin connections on AdS_3 and S^3. Therefore, for non-vanishing $H_{(3)}$ and $k = l$, there can be no background with Mkw_3 and $\mathbb{R}^3$. This result entirely agrees with the solution considered in [62] but without the scalar from vector multiplets. Moreover, it can also be easily checked that any solutions to the above BPS equations solve all the field equations (2.18) to (2.22).

We finally consider the equation from $\delta\psi_3 = 0$ condition. This gives the BPS equation for $Y(r)$

$$Y' = \frac{1}{2}YU' \tag{3.29}$$

which can be solved by a solution $Y \sim e^{\frac{U}{2}}$.

We are now ready to solve all of the BPS equations. To find an analytic solution, we first choose a function $V(r) = \frac{\sigma}{2}$. This is equivalent to changing to a new radial coordinate $\tilde{r}$ defined by the relation $\frac{d\tilde{r}}{dr} = e^{-\frac{\sigma}{2}}$ in [39]. After choosing $V(r) = \frac{\sigma}{2}$, we obtain the solution for (3.20)

$$g_1 g_2 r = \frac{(g_1 - g_2)^2}{g_1 + g_2}\ln[g_2 - g_1 - (g_1 + g_2)e^{2\phi}] - g_1\ln(1 + e^{2\phi}) - g_2\ln(1 - e^{2\phi}) \tag{3.30}$$

in which an integration constant has been neglected.

By treating U, σ, and k as functions of ϕ, we find the solution of equations (3.23), (3.27), and (3.28)

$$\begin{aligned}
\sigma &= \frac{2}{5}\ln\left[\frac{g_1 g_2}{16h(g_1\sinh\phi + g_2\cosh\phi)}\right], && (3.31)\\
U &= \frac{1}{4}\phi - \frac{1}{8}\sigma - \frac{1}{4}\ln(e^{4\phi} - 1) + \frac{1}{4}\ln[g_2 - g_1 - (g_1 + g_2)e^{2\phi}], && (3.32)\\
k &= -\frac{\tau}{4}\left[\frac{g_1}{g_2} + \frac{g_2}{g_1} + 2\coth(2\phi)\right] && (3.33)
\end{aligned}$$

in which irrelevant integration constants in U and k have been removed. However, the integration constant in σ is essential and has been chosen such that the solution for σ interpolates between the two supersymmetric AdS_7 critical points, see [39] for more detail.

As $r \to \pm\infty$, the solution is asymptotic to the AdS_7 critical points with

$$U \sim 4hr, \qquad \sigma \sim \phi \sim 0, \qquad G_{\hat{0}\hat{1}\hat{2}\hat{3}} \sim G_{\hat{3}\hat{4}\hat{5}\hat{6}} \sim 0 \tag{3.34}$$

for $r \to \infty$, and

$$U \sim 4h\left(\frac{g_2^2}{g_2^2-256h^2}\right)^{\frac{2}{5}} r, \qquad \sigma \sim \frac{1}{5}\ln\left[\frac{g_2^2}{g_2^2-256h^2}\right],$$
$$\phi \sim \frac{1}{2}\ln\left[\frac{g_2-16h}{g_2+16h}\right], \qquad G_{\hat{0}\hat{1}\hat{2}\hat{3}} \sim G_{\hat{3}\hat{4}\hat{5}\hat{6}} \sim 0 \tag{3.35}$$

for $r \to -\infty$. In these equations, we have set $g_1 = 16h$. It should be pointed out that the four-form field strength does not indeed vanish in these limits, as can be seen from the BPS equation (3.28). Moreover, the existence of $H_{(3)}$ is needed in order to support the AdS_3 and S^3 factors, as mentioned above. However, its effect in the limit $r \to \pm\infty$ is highly suppressed compared to the scalar potential. The solution is then asymptotically locally AdS_7 as $r \to \pm\infty$.

3.1.2 Solutions with Known Higher Dimensional Origin

For a particular case of $g_2 = g_1$, solutions of the $N = 2$, $SO(4)$ gauged supergravity can be uplifted to eleven dimensions. Setting $g_2 = g_1$, we find the BPS equations

$$\phi' = -e^{V-\frac{\sigma}{2}-\phi} g_1 \cosh\phi \sinh\phi, \tag{3.36}$$
$$\sigma' = \frac{2}{5}e^{V-\frac{\sigma}{2}}\left[g_1\cosh^3\phi - g_1\sinh^3\phi - 16he^{\frac{5}{2}\sigma}\right], \tag{3.37}$$
$$U' = \frac{1}{5}e^{V-\frac{\sigma}{2}}\left[g_1\cosh^3\phi - g_1\sinh^3\phi + 4he^{\frac{5}{2}\sigma}\right], \tag{3.38}$$
$$k' = \frac{\kappa}{\sqrt{2}}e^{2U+\sigma+V}. \tag{3.39}$$

As seen from the ϕ' equation, there is only one supersymmetric AdS_7 background at $\phi = 0$. The solutions interpolating between this AdS_7 and physically acceptable singular geometries dual to SQFTs in the case of $k = 0$ have already been studied in [74]. In this section, we will give a solution with a non-vanishing three-form. This solution can be found by the analysis similar to the previous section. The resulting solution is given by

$$g_1 r = 2\tanh^{-1} e^{\phi} - 2\tan^{-1} e^{\phi}, \tag{3.40}$$
$$\sigma = \frac{2}{5}\phi - \frac{2}{5}\ln\left[1 - 12C_0(e^{4\phi}-1)\right], \tag{3.41}$$
$$U = \frac{1}{5}\phi - \frac{1}{4}\ln(e^{4\phi}-1) + \frac{1}{20}\ln[1-12C_0(e^{4\phi}-1)], \tag{3.42}$$

and

$$k = \frac{\tau}{2h}\left(\frac{h^4}{2^9 g_1^4}\right)^{\frac{1}{10}}\sqrt{\frac{1-12C_0(e^{4\phi}-1)}{e^{4\phi}-1}}\,. \tag{3.43}$$

It can be seen from (3.40) that $\phi \to -\infty$ as $r \to 0$. Hence, the solution is singular at this point. The integration constant C_0 controls the behavior near the singularity, see [74] for more detail.

For $C_0 = 0$, the solution near $r = 0$ becomes

$$\begin{aligned} &\phi \sim -\ln(4hr), \qquad \sigma \sim -\frac{2}{5}\ln(4hr), \qquad k \sim e^{-2\phi} \sim (4hr)^2, \\ &ds_7^2 = (4hr)^2\left(ds^2_{AdS_3} + ds^2_{S^3}\right) + (4hr)^{-\frac{1}{5}}dr^2 \end{aligned} \tag{3.44}$$

in which we have set $g_1 = 16h$. For $C_0 \neq 0$, we find

$$\begin{aligned} &\phi \sim -\ln(4hr), \qquad \sigma \sim \frac{6}{5}\ln(4hr), \qquad k \sim \text{constant}, \\ &ds_7^2 = (4hr)^{\frac{3}{4}}\left(ds^2_{AdS_3} + ds^2_{S^3}\right) + (4hr)^{\frac{3}{5}}dr^2\,. \end{aligned} \tag{3.45}$$

As pointed out in [74], all of these singularities are curvature singularities that are physically acceptable since the scalar potential is bounded from above, $\mathbf{V} \to -\infty$, as required by the criterion in [85].*

In this case, the solution can be embedded in eleven dimensions by using the reduction ansatz given in Appendix C.1. The nine scalars from vector multiplets can be equivalently described by $SL(4,\mathbb{R})/SO(4)$ coset due to the isomorphism $SL(4,\mathbb{R}) \sim SO(3,3)$. For the $SO(3)_{\text{diag}}$ singlet scalar, we find the $SL(4,\mathbb{R})/SO(4)$ coset representative

$$\mathcal{V}_\alpha{}^R = \text{diag}(e^{\frac{\phi}{2}}, e^{\frac{\phi}{2}}, e^{\frac{\phi}{2}}, e^{-\frac{3\phi}{2}}) \tag{3.46}$$

which gives a symmetric (4×4) matrix with unit determinant

$$\tilde{T}_{\alpha\beta} = \text{diag}(e^{\phi}, e^{\phi}, e^{\phi}, e^{-3\phi}) = (\delta_{pq}e^{\phi}, e^{-3\phi}). \tag{3.47}$$

Here, we use $\hat{\mu}^p$ with $p, q = 1, 2, 3$ to denote coordinates on the internal S^2 obeying $\hat{\mu}^p\hat{\mu}^p = 1$ together with the S^3 coordinates $\mu^\alpha = (\cos\psi\hat{\mu}^p, \sin\psi)$ with $\alpha = 1, 2, 3, 4$

*In [85], it has been shown that in order for a curvature singularity in geometries of the DW to have an event horizon, the scalar potential should be bounded from above in the solution. This criterion is a useful rule for distinguishing good singularities from bad (unphysical) ones that break the cosmic censorship principle and cannot be applied to the holographic principle.

satisfying $\mu^\alpha \mu^\alpha = 1$.

With all these and the seven-dimensional fields given previously, we obtain the eleven-dimensional metric

$$\begin{aligned} d\hat{s}^2_{11} = {} & \Delta^{\frac{1}{3}}\left[e^{2U}\left(ds^2_{AdS_3} + ds^2_{S^3}\right) + e^{2V}dr^2\right] \\ & + \frac{1}{32h^2}\Delta^{-\frac{2}{3}}e^{-2\sigma}\left[\cos^2\xi + e^{\frac{5}{2}\sigma}\sin^2\xi(e^{-\phi}\cos^2\psi + e^{3\phi}\sin^2\psi)\right]d\xi^2 \\ & + \frac{1}{128h^2}\Delta^{-\frac{2}{3}}e^{\frac{\sigma}{2}}\cos^2\xi\left[(e^{3\phi}\cos^2\psi + e^{-\phi}\sin^2\xi)d\psi^2 + e^{-\phi}\cos^2\psi d\Omega^2_2\right] \\ & + \frac{1}{64h^2}\Delta^{-\frac{2}{3}}e^{\frac{\sigma}{2}}\sin\xi\sin\psi\cos\psi(e^{-\phi} - e^{3\phi})d\xi d\psi \end{aligned} \tag{3.48}$$

with the warped factor given by

$$\Delta = e^{-\frac{\sigma}{2}}\cos^2\xi(e^{\phi}\cos^2\psi + e^{-3\phi}\sin^2\psi) + e^{2\sigma}\sin^2\xi, \tag{3.49}$$

and the metric on a unit two-sphere can be written as $d\Omega^2_2 = d\hat{\mu}^p d\hat{\mu}^p$. Note that the S^2 in the internal S^3 is unchanged. Its isometry corresponds to the $SO(3)_{\text{diag}}$ unbroken symmetry.

The four-form field strength of eleven-dimensional supergravity is given by

$$\begin{aligned} \hat{F}_{(4)} = {} & \sin\xi\sqrt{2}k'(dr\wedge \text{Vol}_{AdS_3} + dr\wedge\text{Vol}_{S^3}) \\ & - \frac{\sqrt{2}}{8h}e^{-2\sigma}\cos\xi k' e^{-V}(\text{Vol}_{AdS_3} + \text{Vol}_{S^3})\wedge d\xi \\ & + \frac{1}{(8h)^3}\Delta^{-2}\mathcal{U}\cos^3\xi\cos^2\psi d\xi\wedge d\psi\wedge\epsilon_{(2)} \\ & + \frac{1}{(8h)^3}\Delta^{-2}e^{\frac{3}{2}\sigma}\sin\xi\cos^4\xi\cos^2\psi\left[e^{\phi}\cos^2\psi\left(\phi' - \frac{5}{2}\sigma'\right)\right. \\ & \left. - e^{-3\phi}\sin^2\psi\left(\frac{5}{2}\sigma' + 3\phi'\right)\right]dr\wedge d\psi\wedge\epsilon_{(2)} \\ & - \frac{1}{(8h)^3}\Delta^{-2}\cos^2\xi\cos^3\psi\sin\psi\left[\left[4e^{-2\phi-\sigma}\cos^3\xi + e^{\frac{3}{2}\sigma}(e^{\phi} + 3e^{-3\phi})\right]\phi'\right. \\ & \left. - \frac{5}{2}\sin^2\xi e^{\frac{3}{2}\sigma}(e^{\phi} - e^{-3\phi})\sigma'\right]dr\wedge d\xi\wedge\epsilon_{(2)} \end{aligned} \tag{3.50}$$

where $\epsilon_{(2)} = \frac{1}{2}\varepsilon_{pqs}\hat{\mu}^p d\hat{\mu}^q\wedge d\hat{\mu}^s$ is the volume form on S^2. In this equation, we have also used $\varepsilon_{pqs4} = \varepsilon_{pqs}$. The scalar function $\mathcal{U}$ is given by

$$\begin{aligned} \mathcal{U} = {} & \sin^2\xi(e^{4\sigma} - 3e^{\phi+\frac{3}{2}\sigma} - e^{\frac{3}{2}\sigma-3\phi}) - \cos^2\xi\left[e^{\frac{3}{2}\sigma}(e^{\phi}\cos^2\psi + e^{-3\phi}\sin^2\psi)\right. \\ & \left. + e^{-\sigma}(e^{2\phi}\cos^2\psi + 3e^{2\phi}\sin^2\psi + e^{-2\phi}\cos^2\psi - e^{-6\phi}\sin^2\psi)\right]. \end{aligned} \tag{3.51}$$

Similar to the discussion in [62], we expect the uplifted solution to express eleven-dimensional configurations involving M2-M5-brane bound states due to the dyonic profile of $H_{(3)}$. It is also interesting to consider the (00)-component of the eleven-dimensional metric

$$\hat{g}_{00} = -\frac{1}{\kappa^2}\Delta^{\frac{1}{3}} e^{2U(r)} \,. \tag{3.52}$$

Near the singularity at $r = 0$, we find that

$$\hat{g}_{00} \sim (4hr)^{\frac{26}{15}} \to 0 \qquad \text{and} \qquad \hat{g}_{00} \sim (4hr)^{\frac{13}{60}} \to 0 \tag{3.53}$$

for $C_0 = 0$ and $C_0 \neq 0$, respectively. According to the criterion of [86], these singularities are physical in agreement with the seven-dimensional results obtained from the criterion of [85]. We then expect that the solution holographically describes a two-dimensional conformal defect in a six-dimensional $N = (1, 0)$ SCFT with known M-theory origin.

3.1.3 Domain Walls with Three-Form Potential and Vector Fields

In this section, we consider more general solutions with non-vanishing vector fields. We first determine an appropriate ansatz for the $SO(4) \sim SO(3) \times SO(3)$ gauge fields. As in [62], we will take this ansatz in the form of

$$A^I_{(1)} = A^I_i d\theta_i \tag{3.54}$$

in which the components A^I_i will be functions of only the radial coordinate r. These components are given by

$$A^i_j = \frac{e^{-W}\kappa}{2} A(r)\delta^i_j \quad \text{and} \quad A^r_i = \frac{e^{-W}\kappa}{2} B(r)\delta^r_i \,. \tag{3.55}$$

It is now straightforward to compute the field strength tensors $F^i_{(2)} = {L_I}^i F^I_{(2)}$ and $F^r_{(2)} = {L_I}^r F^I_{(2)}$. Non-vanishing components of these tensors are given by

$$F^i_{3j} = \mathbf{f}\,\delta^i_j, \qquad F^i_{jk} = \mathbf{g}\,\varepsilon_{ijk}, \qquad F^r_{3i} = \bar{\mathbf{f}}\,\delta^r_i, \qquad F^r_{jk} = \bar{\mathbf{g}}\,\delta^r_i\varepsilon_{ijk} \tag{3.56}$$

where

$$\mathbf{f} = e^{-V-W}\frac{\kappa}{2}\left[A'\cosh\phi + B'\sinh\phi\right], \tag{3.57}$$

$$\bar{\mathbf{f}} = e^{-V-W}\frac{\kappa}{2}\left[A'\sinh\phi + B'\cosh\phi\right], \tag{3.58}$$

$$\mathbf{g} = e^{-2W}\frac{\kappa^2}{4}\left[A(2-g_1A)\cosh\phi + B(2-g_2B)\sinh\phi\right], \tag{3.59}$$

$$\bar{\mathbf{g}} = e^{-2W}\frac{\kappa^2}{4}\left[A(2-g_1A)\sinh\phi + B(2-g_2B)\cosh\phi\right]. \tag{3.60}$$

To implement $SO(3)_{\text{diag}}$, we set

$$g_2B = g_1A\,. \tag{3.61}$$

We still use the ansatz for the Killing spinors (3.14) together with the projection (3.15). Besides, the following additional projectors are needed due to the extra contributions from non-vanishing gauge fields,

$$\gamma^{\hat{4}\hat{5}}\epsilon = -i\sigma^3\epsilon \quad \text{and} \quad \gamma^{\hat{5}\hat{6}}\epsilon = -i\sigma^1\epsilon. \tag{3.62}$$

Therefore, the BPS solutions (if exist) will preserve only two supercharges or $\frac{1}{8}$-BPS after imposing the projection (3.91).

With all these, we can now set up the BPS equations. By the relation (3.61), the composite connection along S^3 takes a very simple form

$$Q_{ijk} = \omega_{i+3,j+3,k+3} \tag{3.63}$$

in which $\omega_{i+3,j+3,k+3}$ is the spin connection given in (3.11). Using the same procedure as in the previous section, we find the following set of BPS equations

$$\begin{aligned} U' = {} & \frac{e^V}{30\cos 2\theta}\Big[\big(12he^{2\sigma} + e^{-\frac{\sigma}{2}}C\big)(3\cos 4\theta - 1) + 9e^{\frac{\sigma}{2}}\mathbf{g}(\cos 4\theta - 3) \\ & + 12e^{-U}\tau\sin 2\theta + 9e^{-W}\kappa(g_1A - 1)\sin 4\theta\Big], \end{aligned} \tag{3.64}$$

$$\begin{aligned} W' = {} & -\frac{e^V}{15\cos 2\theta}\Big[\big(12he^{2\sigma} + e^{-\frac{\sigma}{2}}C\big)(\cos 4\theta - 2) + 3e^{\frac{\sigma}{2}}\mathbf{g}(\cos 4\theta - 8) \\ & + 9e^{-U}\tau\sin 2\theta + 3e^{-W}\kappa(g_1A - 1)\sin 4\theta\Big], \end{aligned} \tag{3.65}$$

$$\begin{aligned} Y' = {} & \frac{e^VY}{60\cos 2\theta}\Big[\big(12he^{2\sigma} + e^{-\frac{\sigma}{2}}C\big)(3\cos 4\theta - 1) + 9e^{\frac{\sigma}{2}}\mathbf{g}(\cos 4\theta - 3) \\ & + 12e^{-U}\tau\sin 2\theta + 9e^{-W}\kappa(g_1A - 1)\sin 4\theta\Big], \end{aligned} \tag{3.66}$$

$$\begin{aligned}
\theta' &= \frac{e^V}{2}\left[-\left(4he^{2\sigma}+e^{-\frac{\sigma}{2}}C\right)\sin 2\theta - 3e^{\frac{\sigma}{2}}\mathbf{g}\sin 2\theta + 3e^{-U}\tau\right.\\
&\quad \left.+3e^{-W}\kappa(g_1A-1)\cos 2\theta\right], \qquad (3.67)\\
k' &= \frac{e^{3U+V}e^{\sigma}}{3\sqrt{2}}\left[2\left[12he^{2\sigma}+e^{-\frac{\sigma}{2}}C\right]\tan 2\theta + 18e^{\frac{\sigma}{2}}\mathbf{g}\tan 2\theta\right.\\
&\quad \left.-6e^{-U}\tau\sec 2\theta - 9e^{-W}\kappa(g_1A-1)\right], \qquad (3.68)\\
l' &= \frac{1}{\sqrt{2}}e^{3W+V}e^{\sigma}\left[e^{-U}\tau - 8he^{2\sigma}\sin 2\theta\right], \qquad (3.69)\\
\sigma' &= \frac{e^V}{15\cos 2\theta}\left[Ce^{-\frac{\sigma}{2}}(3\cos 4\theta - 1) - 24he^{2\sigma}(\cos 4\theta + 3) + 12e^{-U}\tau\sin 2\theta\right.\\
&\quad \left.+9e^{\frac{\sigma}{2}}\mathbf{g}(\cos 4\theta - 3) + 9e^{-W}\kappa(g_1A-1)\sin 4\theta\right], \qquad (3.70)\\
\phi' &= -e^V\left[e^{-\frac{\sigma}{2}}\mathcal{C} + e^{\frac{\sigma}{2}}\bar{\mathbf{g}}\right]\cos 2\theta, \qquad (3.71)\\
A' &= -\frac{2g_2e^{V+W-\frac{\sigma}{2}}}{3\kappa(g_1\sinh\phi - g_2\cosh\phi)}\left[Ce^{-\frac{\sigma}{2}}\sin 2\theta + 3e^{\frac{\sigma}{2}}\mathbf{g}\sin 2\theta\right.\\
&\quad \left.-3\left[e^{-U}\tau + e^{-W}\kappa(g_1A-1)\cos 2\theta\right]\right] \qquad (3.72)
\end{aligned}$$

where the quantities C and $\mathcal{C}$ are defined by

$$\begin{aligned}
C &= 3\left(g_1\cosh^3\phi + g_2\sinh^3\phi\right), \qquad (3.73)\\
\mathcal{C} &= \frac{1}{2}\sinh(2\phi)\left(g_1\cosh\phi + g_2\sinh\phi\right). \qquad (3.74)
\end{aligned}$$

In addition to these BPS equations, there also exists an algebraic constraint arising from the fact that the SUSY transformations from the gravity multiplet ($\delta\psi_\mu$ and $\delta\chi$) and those from the vector multiplets ($\delta\lambda^r$) lead to different BPS equations for A. Consistency between these two equations results in a constraint

$$\begin{aligned}
0 &= e^{\frac{\sigma}{2}}\sin 2\theta\left[\left(\frac{e^{-\sigma}}{3}C + \mathbf{g}\right) + \frac{g_1\sinh\phi + g_2\cosh\phi}{g_1\cosh\phi + g_2\sinh\phi}\left(e^{-\sigma}\mathcal{C} + \bar{\mathbf{g}}\right)\right]\\
&\quad +e^{-W}\kappa(1-g_1A)\cos 2\theta - e^{-U}\tau. \qquad (3.75)
\end{aligned}$$

This means supersymmetric solutions must satisfy the above BPS equations as well as the constraint (3.75) in order for the Killing spinors to exist. We have explicitly verified that the BPS equations (3.64) to (3.72), together with the constraint (3.75), imply all of the field equations.

However, it turns out that the constraint (3.75) is not compatible with the BPS equations (3.64) to (3.72). This can be readily checked by differentiating equation (3.75) and substituting the BPS equations (3.64) to (3.72). The result is given by

$$0 = e^{-2U-\frac{\sigma}{2}}k'\mathbf{g} - \frac{2e^{U+\frac{\sigma}{2}}(g_2^2 - g_1^2)\phi'\mathbf{f}}{(g_1\cosh\phi + g_2\sinh\phi)(g_1\sinh\phi + g_2\cosh\phi)}. \tag{3.76}$$

This equation implies that ϕ and k cannot flow independently but relate to each other. Note that this relation is trivially satisfied for $A = 0$ in which $\mathbf{f} = \mathbf{g} = 0$. This case has already been considered in the previous section. Another possibility is to set $k' = 0$ and $g_2 = g_1$, but this also leads to $l' = 0$. The three-form field then has vanishing field strength. In this case, the gauge fields are either zero or constant. The former is the usual flat DW solutions in [39], while the latter has already been studied in [60] in the context of twisted compactifications. Therefore, we conclude that there are no supersymmetric solutions with non-vanishing $SO(3)_{\text{diag}}$ gauge fields and non-trivial three-form field for matter-coupled $SO(4)$ gauged supergravity in seven dimensions.

3.2 Twisted Solutions with Three-Form Potential

In this section, we are interested in supersymmetric $AdS_3 \times M^4$ solutions. We will consider a four-manifold M^4 with constant curvature of two types, a product of two Riemann surfaces $\Sigma^2 \times \Sigma^2$ and a Kahler four-cycle K^4. In general, non-vanishing spin connections on these curved internal spaces are the significant impediments breaking all SUSY. This problem can be seen explicitly by looking at the covariant derivative of the SUSY parameter (2.26) in the SUSY transformations of gravitini (2.23) for the component along the internal space

$$D_\alpha\epsilon = \partial_\alpha\epsilon + \frac{1}{4}\omega_\alpha{}^{\hat{\nu}\hat{\rho}}\gamma_{\hat{\nu}\hat{\rho}}\epsilon + \frac{1}{2\sqrt{2}}Q^i_\alpha\sigma^i\epsilon \tag{3.77}$$

in which α is a spacetime index on M^4. In the second term, the spin connections are generally non-vanishing and depending on the coordinates of the internal space. Without gauge fields, only $\epsilon = 0$ can solve BPS equations in this case since no other terms depend on the M^4 coordinates. To preserve some amount of SUSY, we need to include the third term on the right-hand side of (3.77) by turning on some non-vanishing gauge fields proportional to the spin connections. Then, additional projection conditions relating $\sigma^i\epsilon$ to $\gamma^{\hat{\nu}\hat{\rho}}\epsilon$ are needed in order to twist the Killing spinor in the way that the contributions from the curvature of the internal space orient along the direction identified by the gauge fields. With some particular conditions on magnetic charges of the gauge fields called twist conditions, the second and third terms in (3.77) can be arranged to eliminate each other.

In the first case, we perform the twists by using $SO(2)_R \subset SO(3)_R$ gauge fields. The resulting supersymmetric twisted solutions will have $SO(2) \times SO(2)$, $SO(2)_{\text{diag}}$, and $SO(2)_R$ symmetries. For a Kahler four-cycle, since K^4 has a $U(2) \sim SU(2) \times U(1)$ spin connection, the twists can be performed by turning on either $SO(2)_R \subset SO(3)_R$ or $SO(3)_R$ gauge fields to cancel the $U(1) \sim SO(2)$ or the $SU(2) \sim SO(3)$ parts of the spin connection, respectively. Nevertheless, a twist by canceling the full $U(2)$ spin connection is impossible since the $SO(3)_R$ R-symmetry of the $N = 2$, $SO(4)$ gauged supergravity is not large enough, i.e. $U(2) \not\subset SO(4)$. It should also be noted that these gauge fields always give a non-vanishing $F^I_{(2)} \wedge F^I_{(2)}$ term in the field equation of the three-form field, as can be seen from (2.18). This term is the main reason why non-vanishing $H_{(3)}$ is the key to obtain $AdS_3 \times M^4$ solutions.

For a particular case with equal $SO(3)$ coupling constants in the $SO(4)$ gauge group, the resulting twisted solutions can be embedded in eleven-dimensional supergravity via a truncation on S^4 [74]. As a result, these solutions will provide several new two-dimensional SCFTs with known M-theory dual.

3.2.1 Supersymmetric $AdS_3 \times \Sigma^2 \times \Sigma^2$ Solutions

We first look for supersymmetric solutions of the form $AdS_3 \times \Sigma^2_{k_1} \times \Sigma^2_{k_2}$ with $\Sigma^2_{k_i}$ for $i = 1, 2$ being two-dimensional Riemann surfaces. The constants k_i describe the curvature of $\Sigma^2_{k_i}$ with values $k_i = 1, 0, -1$ corresponding to a two-dimensional sphere S^2, a flat space $\mathbb{R}^2$, or a hyperbolic space H^2, respectively.

We choose the following ansatz for the seven-dimensional metric

$$ds^2_7 = e^{2U(r)} dx^2_{1,1} + dr^2 + e^{2V(r)} ds^2_{\Sigma^2_{k_1}} + e^{2W(r)} ds^2_{\Sigma^2_{k_2}} \tag{3.78}$$

where $dx_{1,1}^2 = \eta_{ab}dx^a dx^b$ with $a, b = 0, 1$ is the flat metric on the two-dimensional spacetime. The explicit form of the metric on $\Sigma^2_{k_i}$ can be written as

$$ds^2_{\Sigma^2_{k_i}} = d\theta_i^2 + f_{k_i}(\theta_i)^2 d\varphi_i^2 \,. \tag{3.79}$$

The functions $f_{k_i}(\theta_i)$ are defined as

$$f_{k_i}(\theta_i) = \begin{cases} \sin\theta_i, & k_i = 1 \\ \theta_i, & k_i = 0 \\ \sinh\theta_i, & k_i = -1 \end{cases} \,. \tag{3.80}$$

By using an obvious choice of vielbein

$$\begin{aligned} e^{\hat{a}} &= e^U dx^a, \qquad e^{\hat{2}} = dr, \qquad e^{\hat{3}} = e^V d\theta_1, \\ e^{\hat{4}} &= e^V f_{k_1}(\theta_1) d\varphi_1, \qquad e^{\hat{5}} = e^W d\theta_2, \qquad e^{\hat{6}} = e^W f_{k_2}(\theta_2) d\varphi_2, \end{aligned} \tag{3.81}$$

we can compute non-vanishing components of the spin connection of the form

$$\begin{aligned} &\omega^{\hat{a}}{}_{\hat{2}} = U' e^{\hat{a}}, \qquad \omega^{\hat{3}}{}_{\hat{2}} = V' e^{\hat{3}}, \qquad \omega^{\hat{4}}{}_{\hat{2}} = V' e^{\hat{4}}, \qquad \omega^{\hat{5}}{}_{\hat{2}} = W' e^{\hat{5}}, \\ &\omega^{\hat{6}}{}_{\hat{2}} = W' e^{\hat{6}}, \qquad \omega^{\hat{4}}{}_{\hat{3}} = e^{-V} \frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)} e^{\hat{4}}, \qquad \omega^{\hat{6}}{}_{\hat{5}} = e^{-W} \frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)} e^{\hat{6}} \,. \end{aligned} \tag{3.82}$$

Apart from r-derivative, we also use primes to denote derivatives of a function with respect to its explicit argument, for example, $f'_{k_i}(\theta_i) = df_{k_i}(\theta_i)/d\theta_i$.

To find supersymmetric $AdS_3 \times \Sigma^2_{k_1} \times \Sigma^2_{k_2}$ solutions with non-vanishing Killing spinors, we perform a twist using gauge fields along $\Sigma^2_{k_1} \times \Sigma^2_{k_2}$. In the following analyses, we will examine various possible twists with different unbroken symmetries.

3.2.1.1 AdS_3 Vacua with $SO(2) \times SO(2)$ Symmetry

For the solutions with $SO(2) \times SO(2)$ symmetry, we turn on the following $SO(2) \times SO(2)$ gauge fields on $\Sigma^2_{k_1} \times \Sigma^2_{k_2}$ to perform the twist

$$A^3_{(1)} = -\frac{p_{11}}{k_1} e^{-V} \frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)} e^{\hat{4}} - \frac{p_{12}}{k_2} e^{-W} \frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)} e^{\hat{6}}, \tag{3.83}$$

$$A^6_{(1)} = -\frac{p_{21}}{k_1} e^{-V} \frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)} e^{\hat{4}} - \frac{p_{22}}{k_2} e^{-W} \frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)} e^{\hat{6}} \tag{3.84}$$

where p_{ij} are constant magnetic charges.

There is only one $SO(2)\times SO(2)$ singlet scalar from $SO(3,3)/SO(3)\times SO(3)$ coset corresponding to the non-compact generator Y_{33}. We then parametrize the coset representative by

$$L = e^{\phi Y_{33}} \qquad (3.85)$$

with ϕ depending only on the radial coordinate. By computing the composite connection Q^{ij}_μ along $\Sigma^2_{k_1} \times \Sigma^2_{k_2}$, we can cancel the spin connection (3.82) by imposing the twist conditions

$$g_1 p_{11} = k_1 \qquad \text{and} \qquad g_1 p_{12} = k_2 \qquad (3.86)$$

together with the following projection conditions on the Killing spinors (2.42)

$$\gamma^{\hat{3}\hat{4}}\epsilon = \gamma^{\hat{5}\hat{6}}\epsilon = i\sigma^3\epsilon\,. \qquad (3.87)$$

It should be noted that only the gauge field $A^3_{(1)}$ enters the twist procedure since $A^3_{(1)}$ is the gauge field of $SO(2)_R \subset SO(3)_R$ under which SUSY parameters and the gravitini are charged.

From the $SO(2) \times SO(2)$ gauge fields given in (3.83) and (3.84), we can compute the corresponding two-form field strengths of the form

$$F^3_{(2)} = e^{-2V} p_{11} e^{\hat{3}} \wedge e^{\hat{4}} + e^{-2W} p_{12} e^{\hat{5}} \wedge e^{\hat{6}}, \qquad (3.88)$$

$$F^6_{(2)} = e^{-2V} p_{21} e^{\hat{3}} \wedge e^{\hat{4}} + e^{-2W} p_{22} e^{\hat{5}} \wedge e^{\hat{6}}\,. \qquad (3.89)$$

We also need to turn on the three-form field associated with the four-form field strength

$$G_{(4)} = \frac{1}{8\sqrt{2}h} e^{-2(V+W)}(p_{21}p_{22} - p_{11}p_{12}) e^{\hat{3}} \wedge e^{\hat{4}} \wedge e^{\hat{5}} \wedge e^{\hat{6}}. \qquad (3.90)$$

This is very similar to the solutions of maximal $SO(5)$ gauged supergravity considered in [58].

By imposing an additional projector

$$\gamma^{\hat{2}}\epsilon = \epsilon \qquad (3.91)$$

required by $\delta\chi = 0$ and $\delta\lambda^r = 0$ conditions, we find the following BPS equations

$$U' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[\left(g_1 e^{-\sigma}\cosh\phi + 4he^{\frac{3\sigma}{2}}\right) + \frac{3}{8h}e^{-\frac{3\sigma}{2}-2(V+W)}(p_{11}p_{12} - p_{21}p_{22}) - e^{-2V}(p_{11}\cosh\phi + p_{21}\sinh\phi) - e^{-2W}(p_{12}\cosh\phi + p_{22}\sinh\phi)\right], \quad (3.92)$$

$$V' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[\left(g_1 e^{-\sigma}\cosh\phi + 4he^{\frac{3\sigma}{2}}\right) - \frac{1}{4h}e^{-\frac{3\sigma}{2}-2(V+W)}(p_{11}p_{12} - p_{21}p_{22}) + 4e^{-2V}(p_{11}\cosh\phi + p_{21}\sinh\phi) - e^{-2W}(p_{12}\cosh\phi + p_{22}\sinh\phi)\right], \quad (3.93)$$

$$W' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[\left(g_1 e^{-\sigma}\cosh\phi + 4he^{\frac{3\sigma}{2}}\right) - \frac{1}{4h}e^{-\frac{3\sigma}{2}-2(V+W)}(p_{11}p_{12} - p_{21}p_{22}) - e^{-2V}(p_{11}\cosh\phi + p_{21}\sinh\phi) + 4e^{-2W}(p_{12}\cosh\phi + p_{22}\sinh\phi)\right], \quad (3.94)$$

$$\sigma' = \frac{2}{5}e^{\frac{\sigma}{2}}\left[\left(g_1 e^{-\sigma}\cosh\phi - 16he^{\frac{3\sigma}{2}}\right) - \frac{1}{4h}e^{-\frac{3\sigma}{2}-2(V+W)}(p_{11}p_{12} - p_{21}p_{22}) - e^{-2V}(p_{11}\cosh\phi + p_{21}\sinh\phi) - e^{-2W}(p_{12}\cosh\phi + p_{22}\sinh\phi)\right], \quad (3.95)$$

$$\phi' = -e^{\frac{\sigma}{2}}\left[e^{-2V}(p_{11}\sinh\phi + p_{21}\cosh\phi) + e^{-2W}(p_{12}\sinh\phi + p_{22}\cosh\phi)\right] - g_1 e^{-\frac{\sigma}{2}}\sinh\phi\,. \quad (3.96)$$

It can be checked that these BPS equations satisfy all the field equations. At large r, we have $U \sim V \sim W \sim r$ and $\phi \sim \sigma \sim e^{-\frac{4r}{L_{AdS_7}}}$ with the AdS_7 radius given by $L_{AdS_7} = \frac{1}{4h}$, and the terms involving gauge fields and the three-form field are highly suppressed. In this limit, we find the $SO(4)$ AdS_7 critical point from the BPS equations. The solutions are then asymptotically locally AdS_7 as $r \to \infty$.

We now look for supersymmetric AdS_3 solutions satisfying $V' = W' = \sigma' = \phi' = 0$ and $U' = \frac{1}{L_{AdS_3}}$ as $r \to -\infty$. We find a class of AdS_3 fixed point solutions

$$e^{\frac{5}{2}\sigma} = \frac{g_1 Z e^{\phi}}{4h(p_{21}(p_{12} - 3p_{22}) + p_{11}(p_{12} + p_{22}))}, \quad (3.97)$$

$$e^{\phi} = \sqrt{\frac{p_{21}(p_{12} - 3p_{22}) + p_{11}(p_{12} + p_{22})}{p_{11}(p_{12} - p_{22}) - p_{21}(p_{12} + 3p_{22})}}, \quad (3.98)$$

$$e^{2V} = \frac{p_{21} - p_{11} - (p_{11} + p_{21})e^{2\phi}}{8he^{\phi + \frac{3}{2}\sigma}}, \quad (3.99)$$

$$e^{2W} = \frac{p_{22} - p_{12} - (p_{12} + p_{22})e^{2\phi}}{8he^{\phi + \frac{3}{2}\sigma}}, \quad (3.100)$$

$$L_{AdS_3} = \frac{8he^{\sigma + 2V + 2W}}{p_{11}p_{12} - p_{21}p_{22} + 32h^2 e^{2V + 2W + 3\sigma}} \quad (3.101)$$

where

$$Z = \frac{(p_{12}(p_{11}^2 + p_{21}^2) - 2p_{11}p_{21}p_{22})(-2p_{12}p_{21}p_{22} + p_{11}(p_{12}^2 + p_{22}^2))}{(p_{11}^2(3p_{12}^2 + p_{22}^2) + p_{21}^2(p_{12}^2 + 3p_{22}^2) - 8p_{11}p_{12}p_{21}p_{22})}. \qquad (3.102)$$

To achieve real solutions, $e^{2V} > 0$, $e^{2W} > 0$, $e^{\sigma} > 0$, and $e^{\phi} > 0$ are required. These conditions are possible if and only if one of the two k_i is equal to -1. Besides, there are only two parameters p_{21} and p_{22} characterizing the solutions since the charges p_{11} and p_{12} are fixed by the twist conditions (3.86). Regions in the parameter space (p_{21}, p_{22}) for good AdS_3 vacua are shown in Figure 3.1 for $g_1 = 16h$ and $h = 1$. It should be noted that these regions are the same as those given in [58] for supersymmetric $AdS_3 \times \Sigma^2 \times \Sigma^2$ solutions of maximal seven-dimensional $SO(5)$ gauged supergravity.

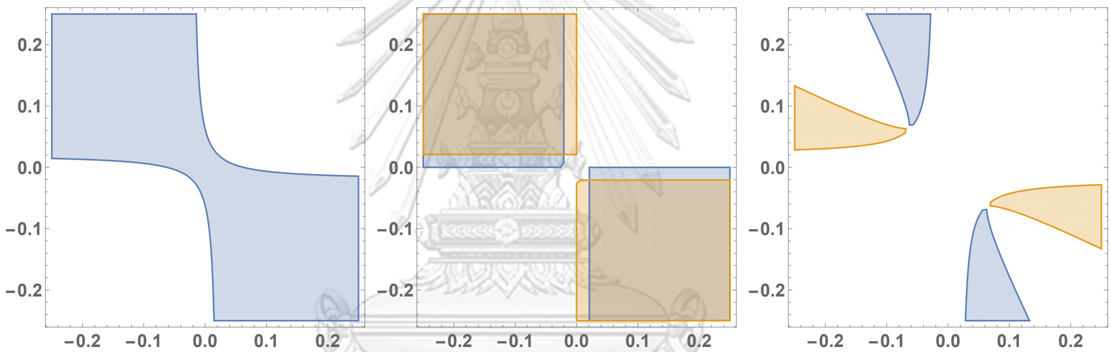
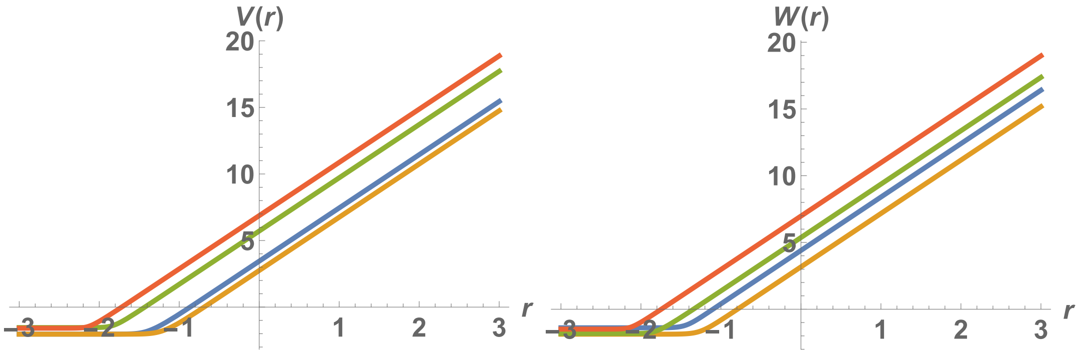
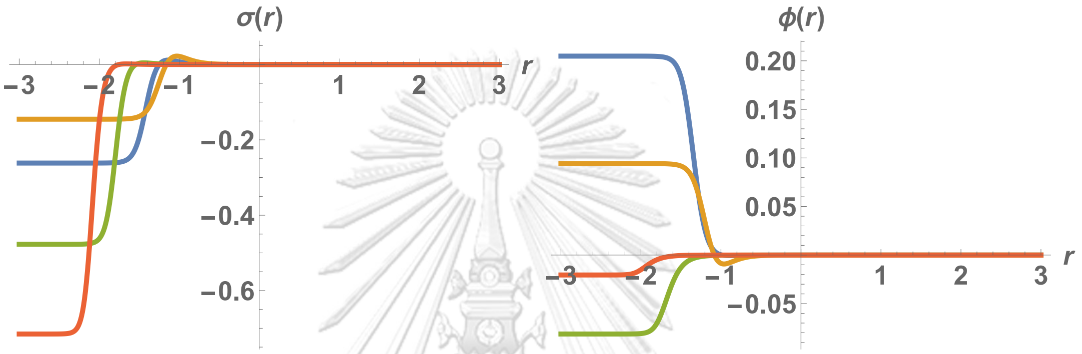


Figure 3.1: Regions (blue) in the parameter space (p_{21}, p_{22}) where good AdS_3 vacua exist for $g_1 = 16h$ and $h = 1$. From left to right, these are the cases of $(k_1 = k_2 = -1)$, $(k_1 = -1, k_2 = 0)$, and $(k_1 = -k_2 = -1)$, respectively. The orange regions correspond to exchanging k_1 and k_2.

These AdS_3 fixed points preserve four supercharges due to the two projectors in (3.87) and correspond to $N = (2, 0)$ SCFTs in two dimensions with $SO(2) \times SO(2)$ symmetry. On the other hand, the entire RG flow solutions, interpolating between the $SO(4)$ AdS_7 critical point and these AdS_3 geometries, preserve only two supercharges due to the extra projector (3.91). Examples of these RG flows from the AdS_7 critical point to $AdS_3 \times H^2 \times H^2$, $AdS_3 \times H^2 \times \mathbb{R}^2$, and $AdS_3 \times H^2 \times S^2$ fixed points with $g_1 = 16h$, $h = 1$, and different values of p_{21} and p_{22} are shown in Figures 3.2, 3.3, and 3.4, respectively.



(a) V solution (b) W solution



(c) σ solution (d) ϕ solution

Figure 3.2: Numerical solutions from the $SO(4)$ AdS_7 vacuum in UV as $r \to 3$ to $AdS_3 \times H^2 \times H^2$ fixed points in IR as $r \to -3$ for $g_1 = 16h$, $h = 1$, and $(p_{21}, p_{22}) = (\frac{1}{12}, -\frac{1}{2}), (\frac{1}{12}, -\frac{1}{7}), (\frac{1}{3}, -\frac{1}{7}), (-\frac{1}{4}, \frac{1}{3})$ (blue, yellow, green, and red).

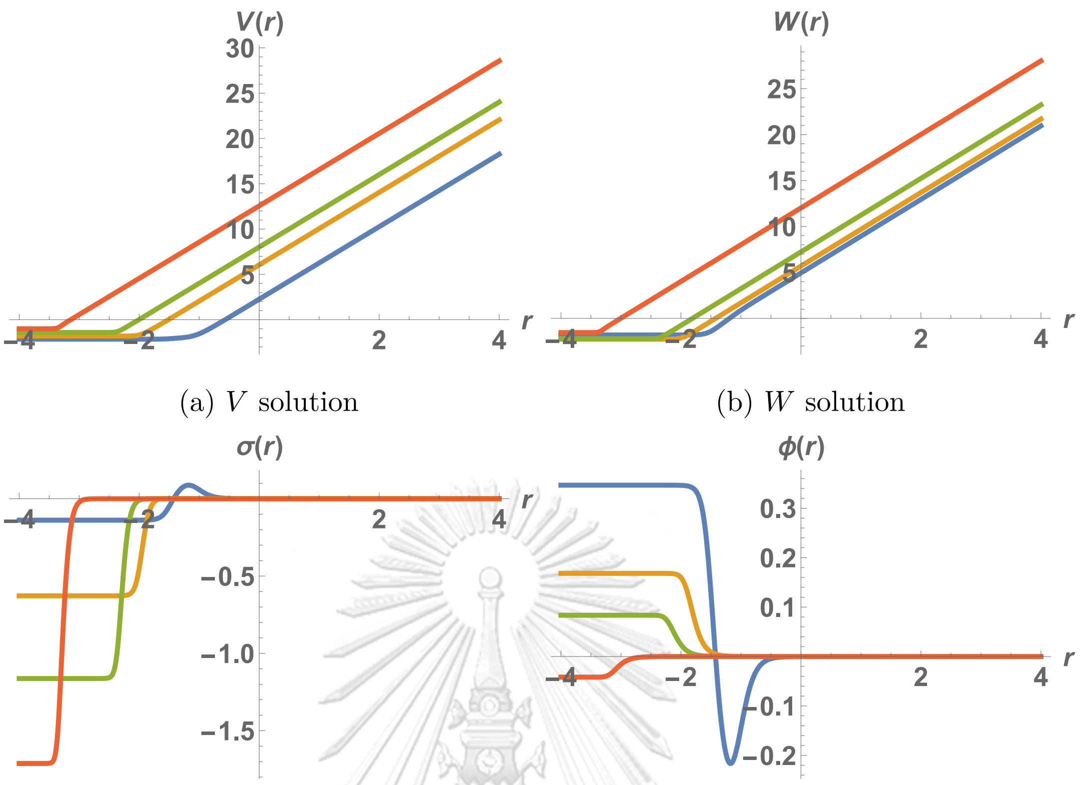
As seen in the above solutions, the coupling constant g_2 does not appear so that the solutions can be uplifted to eleven dimensions by setting $g_2 = g_1$. Since the solutions involve all seven-dimensional fields, the eleven-dimensional four-form field strength is very complicated. For brevity, we omit an explicit form of the four-form and give only the uplifted eleven-dimensional metric.

Using the S^3 coordinates

$$\mu^\alpha = (\cos\tilde{\psi}\cos\tilde{\alpha}, \cos\tilde{\psi}\sin\tilde{\alpha}, \sin\tilde{\psi}\cos\tilde{\beta}, \sin\tilde{\psi}\sin\tilde{\beta}), \tag{3.103}$$

and the $SL(4,\mathbb{R})/SO(4)$ matrix of the form

$$\tilde{T}^{-1}_{\alpha\beta} = \mathrm{diag}(e^{\phi}, e^{\phi}, e^{-\phi}, e^{-\phi}), \tag{3.104}$$



(a) V solution

(b) W solution

(c) σ solution

(d) ϕ solution

Figure 3.3: Numerical solutions from the $SO(4)$ AdS_7 vacuum in UV as $r \to 4$ to $AdS_3 \times H^2 \times \mathbb{R}^2$ fixed points in IR as $r \to -4$ for $g_1 = 16h$, $h = 1$, and $(p_{21}, p_{22}) = (\frac{1}{16}, -\frac{1}{4}), (\frac{1}{8}, -\frac{1}{10}), (\frac{1}{4}, -\frac{1}{10}), (-\frac{1}{2}, \frac{1}{3})$ (blue, yellow, green, and red).

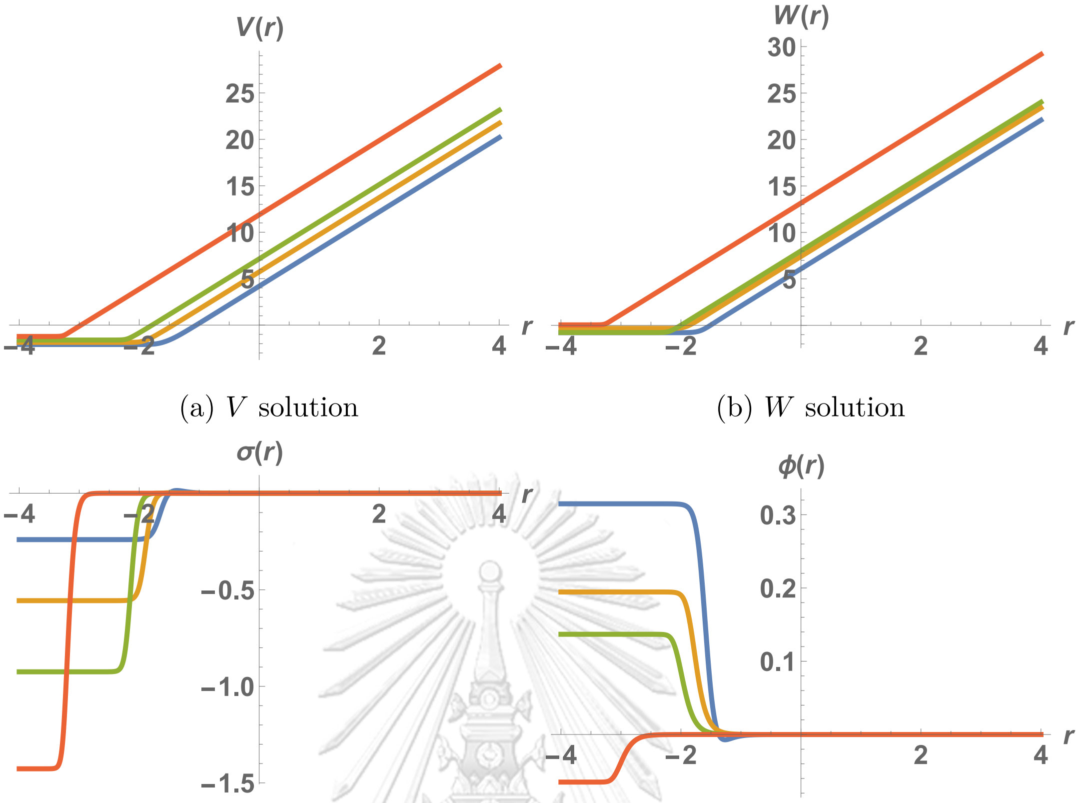
we find the eleven-dimensional metric

$$\begin{aligned} d\hat{s}^2_{11} &= \Delta^{\frac{1}{3}}\left[e^{2U}dx^2_{1,1} + dr^2 + e^{2V}ds^2_{\Sigma^2_{k_1}} + e^{2W}ds^2_{\Sigma^2_{k_2}}\right] \\ &\quad + \frac{2}{g^2}\Delta^{-\frac{2}{3}}\left[e^{-2\sigma}\cos^2\xi + e^{\frac{\sigma}{2}}\sin^2\xi(e^{\phi}\cos^2\tilde{\psi} + e^{-\phi}\sin^2\tilde{\psi})\right]d\xi^2 \\ &\quad + \frac{1}{2g^2}\Delta^{-\frac{2}{3}}e^{\frac{\sigma}{2}}\cos^2\xi\left[(e^{\phi}\sin^2\tilde{\psi} + e^{-\phi}\cos^2\tilde{\psi})d\tilde{\psi}^2\right. \\ &\quad \left. + e^{\phi}\cos^2\tilde{\psi}(d\tilde{\alpha} - gA^{12})^2 + e^{-\phi}\sin^2\tilde{\psi}(d\tilde{\beta} - gA^{34})^2\right] \end{aligned} \quad (3.105)$$

with $A^{12} = A^3_{(1)} + A^6_{(1)}$, $A^{34} = A^3_{(1)} - A^6_{(1)}$, and

$$\Delta = e^{2\sigma}\sin^2\xi + e^{-\frac{\sigma}{2}}\cos^2\xi\left(e^{-\phi}\cos^2\tilde{\psi} + e^{\phi}\sin^2\tilde{\psi}\right). \quad (3.106)$$

From this metric, we notice that the $SO(2)\times SO(2)$ residual symmetry corresponds to the isometry along the $\tilde{\alpha}$ and $\tilde{\beta}$ directions.



(a) V solution (b) W solution (c) σ solution (d) ϕ solution

Figure 3.4: Numerical solutions from the $SO(4)$ AdS_7 vacuum in UV as $r \to 4$ to $AdS_3 \times H^2 \times S^2$ fixed points in IR as $r \to -4$ for $g_1 = 16h$, $h = 1$, and $(p_{21}, p_{22}) = (\frac{1}{14}, -2), (\frac{1}{9}, -5), (\frac{1}{6}, -2), (-\frac{1}{3}, 9)$ (blue, yellow, green, and red).

3.2.1.2 AdS_3 Vacua with $SO(2)_{\text{diag}}$ Symmetry

We now consider AdS_3 solutions with $SO(2)_{\text{diag}} \subset SO(2) \times SO(2) \subset SO(3) \times SO(3)$ symmetry. In this case, there are three scalars invariant under $SO(2)_{\text{diag}}$ corresponding to non-compact generators

$$\hat{Y}_1 = Y_{11} + Y_{22}, \qquad \hat{Y}_2 = Y_{33}, \qquad \hat{Y}_3 = Y_{12} - Y_{21}\,. \tag{3.107}$$

The coset representative takes the form of

$$L = e^{\phi_1 \hat{Y}_1} e^{\phi_2 \hat{Y}_2} e^{\phi_3 \hat{Y}_3}\,. \tag{3.108}$$

The ansatz for $SO(2)_{\text{diag}}$ gauge fields is obtained from the $SO(2) \times SO(2)$ ones in (3.83) and (3.84) by setting $g_2 A^6 = g_1 A^3$ or, equivalently,

$$g_2 p_{21} = g_1 p_{11} \qquad \text{and} \qquad g_2 p_{22} = g_1 p_{12} \,. \tag{3.109}$$

We also simplify the notation by redefining the magnetic charges $p_1 = p_{11}$ and $p_2 = p_{12}$. In this case, the four-form field strength is given by

$$G_{(4)} = \frac{p_1 p_2}{8\sqrt{2} h g_2^2} e^{-2(V+W)} (g_1^2 - g_2^2) e^{\hat{3}} \wedge e^{\hat{4}} \wedge e^{\hat{5}} \wedge e^{\hat{6}}, \tag{3.110}$$

and the twist conditions read

$$g_1 p_1 = k_1 \qquad \text{and} \qquad g_1 p_2 = k_2 \,. \tag{3.111}$$

Using the projection conditions (3.87) and (3.91), we find the corresponding BPS equations. It turns out that compatibility between these BPS equations and the field equations requires either $\phi_1 = 0$ or $\phi_3 = 0$. Furthermore, setting $\phi_3 = 0$ gives the same BPS equations as setting $\phi_1 = 0$ with ϕ_3 and ϕ_1 interchanged. We will then consider only the $\phi_3 = 0$ case with the following BPS equations

$$\begin{aligned} U' = \frac{1}{10} e^{\frac{\sigma}{2}} \Big[& \cosh 2\phi_1 (g_1 e^{-\sigma} \cosh\phi_2 + g_2 e^{-\sigma} \sinh\phi_2) + 8 h e^{\frac{3\sigma}{2}} \\ & -2 p_1 e^{-2V} \left(\cosh\phi_2 + \frac{g_1}{g_2} \sinh\phi_2 \right) - 2 p_2 e^{-2W} \left(\cosh\phi_2 + \frac{g_1}{g_2} \sinh\phi_2 \right) \\ & + g_1 e^{-\sigma} \cosh\phi_2 - g_2 e^{-\sigma} \sinh\phi_2 - \frac{3}{4 h g_2^2} e^{-\frac{3\sigma}{2} - 2(V+W)} (g_1^2 - g_2^2) p_1 p_2 \Big], \end{aligned} \tag{3.112}$$

$$\begin{aligned} V' = \frac{1}{10} e^{\frac{\sigma}{2}} \Big[& \cosh 2\phi_1 (g_1 e^{-\sigma} \cosh\phi_2 + g_2 e^{-\sigma} \sinh\phi_2) + 8 h e^{\frac{3\sigma}{2}} \\ & +8 p_1 e^{-2V} \left(\cosh\phi_2 + \frac{g_1}{g_2} \sinh\phi_2 \right) - 2 p_2 e^{-2W} \left(\cosh\phi_2 + \frac{g_1}{g_2} \sinh\phi_2 \right) \\ & + g_1 e^{-\sigma} \cosh\phi_2 - g_2 e^{-\sigma} \sinh\phi_2 + \frac{1}{2 h g_2^2} e^{-\frac{3\sigma}{2} - 2(V+W)} (g_1^2 - g_2^2) p_1 p_2 \Big], \end{aligned} \tag{3.113}$$

$$\begin{aligned} W' = \frac{1}{10} e^{\frac{\sigma}{2}} \Big[& \cosh 2\phi_1 (g_1 e^{-\sigma} \cosh\phi_2 + g_2 e^{-\sigma} \sinh\phi_2) + 8 h e^{\frac{3\sigma}{2}} \\ & -2 p_1 e^{-2V} \left(\cosh\phi_2 + \frac{g_1}{g_2} \sinh\phi_2 \right) + 8 p_2 e^{-2W} \left(\cosh\phi_2 + \frac{g_1}{g_2} \sinh\phi_2 \right) \\ & + g_1 e^{-\sigma} \cosh\phi_2 - g_2 e^{-\sigma} \sinh\phi_2 + \frac{1}{2 h g_2^2} e^{-\frac{3\sigma}{2} - 2(V+W)} (g_1^2 - g_2^2) p_1 p_2 \Big], \end{aligned} \tag{3.114}$$

$$\sigma' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[\cosh 2\phi_1(g_1e^{-\sigma}\cosh\phi_2 + g_2e^{-\sigma}\sinh\phi_2) - 32he^{\frac{3\sigma}{2}}\right.$$
$$-2p_1e^{-2V}\left(\cosh\phi_2 + \frac{g_1}{g_2}\sinh\phi_2\right) - 2p_2e^{-2W}\left(\cosh\phi_2 + \frac{g_1}{g_2}\sinh\phi_2\right)$$
$$\left.+g_1e^{-\sigma}\cosh\phi_2 - g_2e^{-\sigma}\sinh\phi_2 + \frac{1}{2hg_2^2}e^{-\frac{3\sigma}{2}-2(V+W)}(g_1^2-g_2^2)p_1p_2\right], \quad (3.115)$$
$$\phi_1' = -\frac{1}{2}e^{-\frac{\sigma}{2}}\sinh 2\phi_1(g_1\cosh\phi_2 + g_2\sinh\phi_2), \quad (3.116)$$
$$\phi_2' = e^{\frac{\sigma}{2}}\left[\frac{e^{-\sigma}}{2}\left[g_2\cosh\phi_2 - g_1\sinh\phi_2 - \cosh 2\phi_1(g_2\cosh\phi_2 + g_1\sinh\phi_2)\right]\right.$$
$$\left.-p_1e^{-2V}\left(\sinh\phi_2 + \frac{g_1}{g_2}\cosh\phi_2\right) - p_2e^{-2W}\left(\sinh\phi_2 + \frac{g_1}{g_2}\cosh\phi_2\right)\right]. \quad (3.117)$$

In this case, the BPS solutions are asymptotic to the two supersymmetric AdS_7 vacua with $SO(4)$ and $SO(3)_{\text{diag}}$ symmetries at large r.

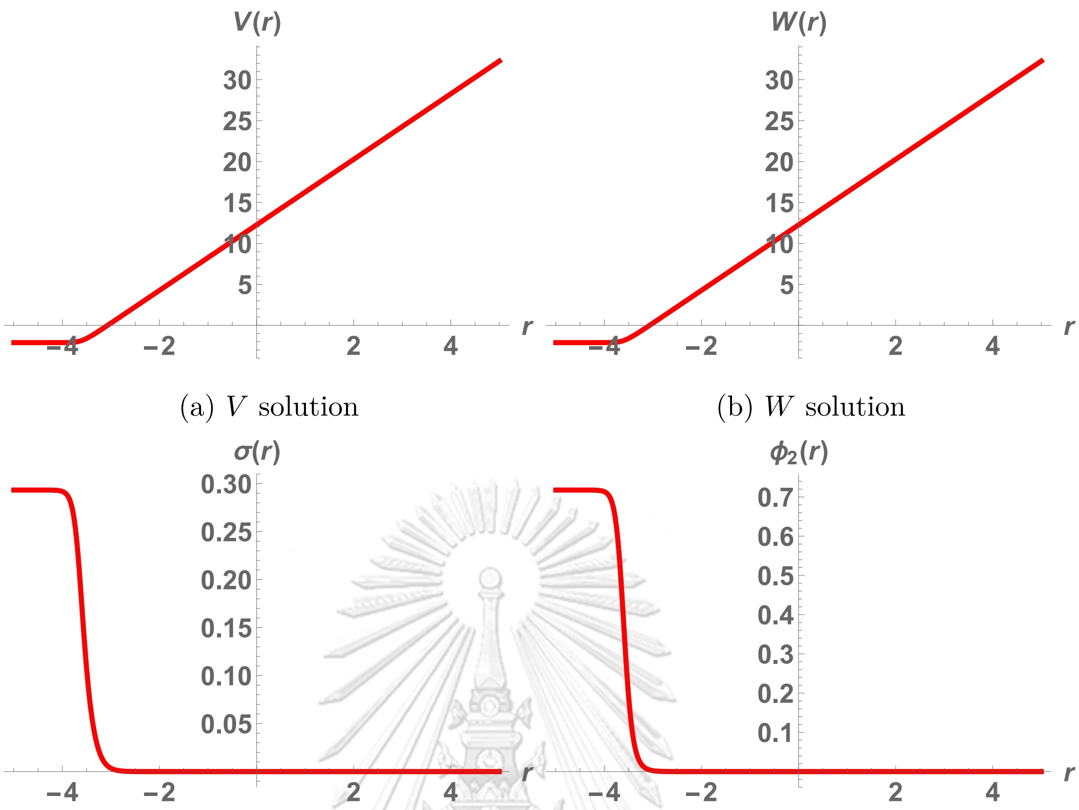
There exist only $AdS_3 \times H^2 \times H^2$ fixed point solutions preserving four supercharges and corresponding to $N = (2,0)$ SCFTs with $SO(2)_{\text{diag}}$ symmetry in two dimensions. We begin with a class of AdS_3 fixed points for $\phi_1 = 0$

$$\sigma = \frac{2}{5}\phi_2 + \frac{2}{5}\ln\left[\frac{g_1g_2^2}{12h(g_2^2+2g_1g_2-3g_1^2)}\right], \quad (3.118)$$
$$\phi_2 = \frac{1}{2}\ln\left[\frac{3g_1^2-2g_1g_2-g_2^2}{3g_1^2+2g_1g_2-g_2^2}\right], \quad (3.119)$$
$$V = W = \frac{1}{10}\ln\left[\frac{27(g_1-g_2)^4(g_1+g_2)^4}{16h^2g_1^8g_2^6(g_2^2-9g_1^2)}\right], \quad (3.120)$$
$$L_{AdS_3} = \left[\frac{8(9g_1^4g_2 - 10g_1^2g_2^3 + g_2^5)^2}{3hg_1^4(g_2^2-3g_1^2)^5}\right]^{\frac{1}{5}} \quad (3.121)$$

with $g_2 > 3g_1$ or $g_2 < -3g_1$ for AdS_3 vacua to exist. An example of RG flows from the $SO(4)$ AdS_7 critical point to this fixed point for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$ is shown in Figure 3.5 with ϕ_1 set to zero along the flow.

Another class of $AdS_3 \times H^2 \times H^2$ solutions with $\phi_1 \neq 0$ is given by

$$\sigma = \frac{2}{5}\ln\left[\frac{g_1g_2}{12h\sqrt{(g_2+g_1)(g_2-g_1)}}\right], \quad (3.122)$$
$$\phi_1 = \phi_2 = \frac{1}{2}\ln\left[\frac{g_2-g_1}{g_2+g_1}\right],$$
$$V = W = \frac{1}{10}\ln\left[\frac{27(g_1^2-g_2^2)^4}{16h^2g_1^8g_2^8}\right], \quad (3.123)$$
$$L_{AdS_3} = \left[\frac{8(g_1^2-g_2^2)^2}{3hg_1^4g_2^4}\right]^{\frac{1}{5}} \quad (3.124)$$



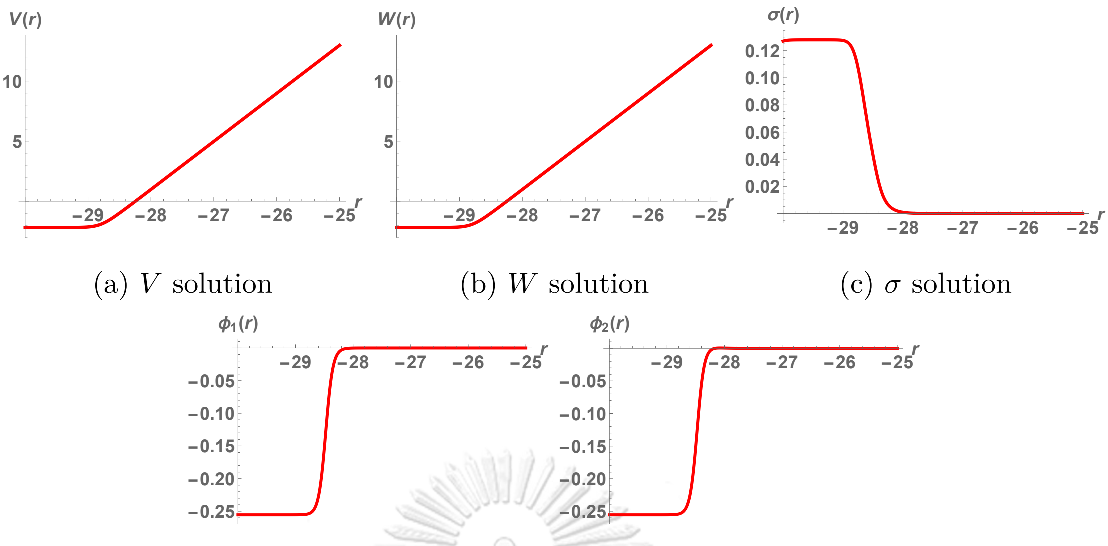
(a) V solution (b) W solution

(c) σ solution (d) ϕ_2 solution

Figure 3.5: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 4$ to an $AdS_3 \times H^2 \times H^2$ fixed point with $SO(2)_{\text{diag}}$ symmetry in IR as $r \to -4$ for $g_2 = 4g_1$, $g_1 = 16h$, $h = 1$, and $\phi_1 = 0$ along the flow.

with the condition $g_2 > g_1$ for good AdS_3 vacua. Examples of RG flows from the $SO(4)$ and $SO(3)_{\text{diag}}$ AdS_7 critical points to this $AdS_3 \times H^2 \times H^2$ fixed point are respectively shown in Figures 3.6 and 3.7 for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$.

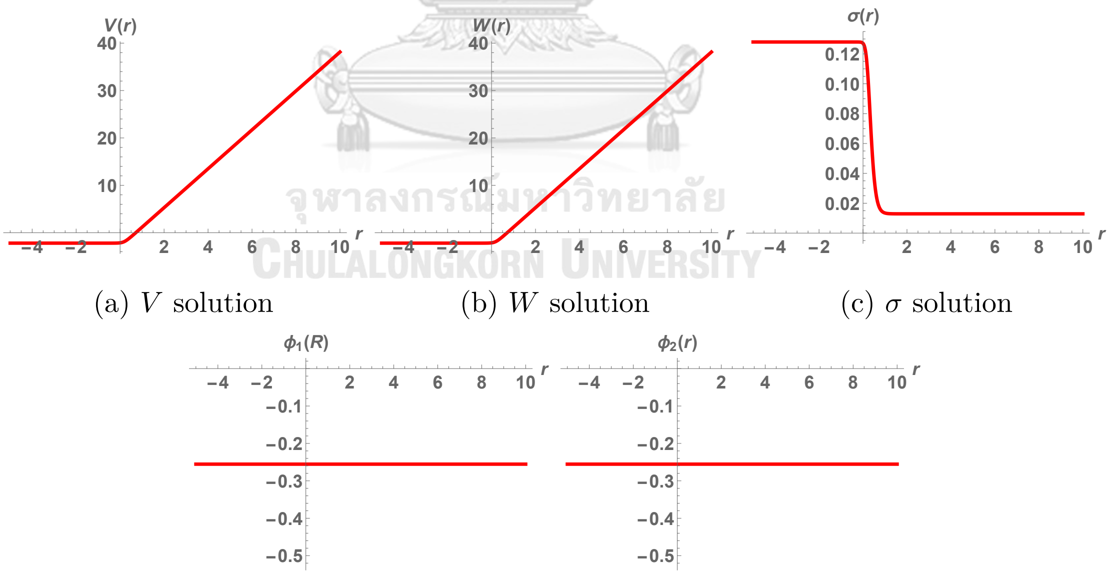
Moreover, with a suitable set of boundary conditions, there also exists an RG flow from the $SO(4)$ AdS_7 to the $SO(3)_{\text{diag}}$ AdS_7 critical points and then to this $AdS_3 \times H^2 \times H^2$ fixed point as shown in Figure 3.8. Unfortunately, all AdS_3 vacua and RG flows in this case with $g_1 \neq g_2$ cannot be uplifted to eleven dimensions.



(a) V solution (b) W solution (c) σ solution

(d) ϕ_1 solution (e) ϕ_2 solution

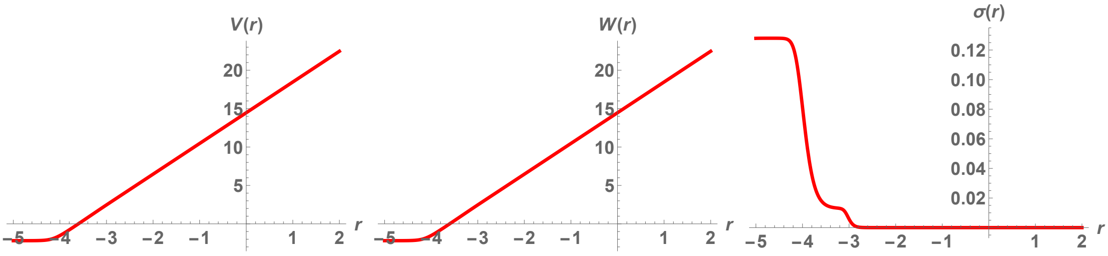
Figure 3.6: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 25$ to an $AdS_3 \times H^2 \times H^2$ fixed point with $SO(2)_{\text{diag}}$ symmetry in IR as $r \to -30$ for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$.



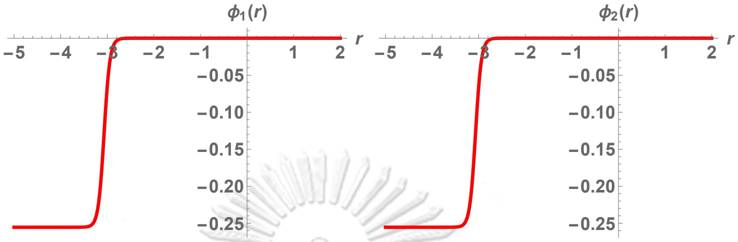
(a) V solution (b) W solution (c) σ solution

(d) ϕ_1 solution (e) ϕ_2 solution

Figure 3.7: A numerical solution from the $SO(3)_{\text{diag}}$ AdS_7 vacuum in UV as $r \to 10$ to an $AdS_3 \times H^2 \times H^2$ fixed point with $SO(2)_{\text{diag}}$ symmetry in IR as $r \to -4$ for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$.



(a) V solution (b) W solution (c) σ solution



(d) ϕ_1 solution (e) ϕ_2 solution

Figure 3.8: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 2$ to the $SO(3)_{\text{diag}}$ AdS_7 critical point and then to an $AdS_3 \times H^2 \times H^2$ fixed point with $SO(2)_{\text{diag}}$ symmetry in IR as $r \to -5$ for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$.

3.2.1.3 AdS_3 Vacua with $SO(2)_R$ Symmetry

We now move on to AdS_3 solutions with $SO(2)_R \subset SO(3)_R$ symmetry. There are three $SO(2)_R$ singlet scalars corresponding to non-compact generators Y_{31}, Y_{32}, and Y_{33}. Therefore, the coset representative can be written as

$$L = e^{\phi_1 Y_{31}} e^{\phi_2 Y_{32}} e^{\phi_3 Y_{33}} \,. \tag{3.125}$$

To perform the twist, we use the following $SO(2)_R$ gauge field

$$A^3_{(1)} = -\frac{p_1}{k_1} e^{-V} \frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)} e^{\hat{4}} - \frac{p_2}{k_2} e^{-W} \frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)} e^{\hat{6}}. \tag{3.126}$$

The four-form field strength in this case is given by

$$G_{(4)} = -\frac{1}{8\sqrt{2}h} e^{-2(V+W)} p_1 p_2 e^{\hat{3}} \wedge e^{\hat{4}} \wedge e^{\hat{5}} \wedge e^{\hat{6}} \,. \tag{3.127}$$

We can now repeat the same procedure to find the corresponding BPS equations. In this case, it turns out that compatibility between the BPS equations and the field equations allows only one of the ϕ_i, $i = 1, 2, 3$, to be non-vanishing.

We have verified that any of the ϕ_i leads to the same set of BPS equations. We will choose $\phi_1 = \phi_2 = 0$ and $\phi_3 \neq 0$ for definiteness so that the BPS equations are

$$U' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[g_1 e^{-\sigma} + 4he^{\frac{3\sigma}{2}} - e^{-2V}p_1 - e^{-2W}p_2 + \frac{3}{8h}e^{-2(V+W)}p_1p_2\right], \quad (3.128)$$

$$V' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[g_1 e^{-\sigma} + 4he^{\frac{3\sigma}{2}} + 4e^{-2V}p_1 - e^{-2W}p_2 - \frac{1}{4h}e^{-2(V+W)}p_1p_2\right], \quad (3.129)$$

$$W' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[g_1 e^{-\sigma} + 4he^{\frac{3\sigma}{2}} - e^{-2V}p_1 + 4e^{-2W}p_2 - \frac{1}{4h}e^{-2(V+W)}p_1p_2\right], \quad (3.130)$$

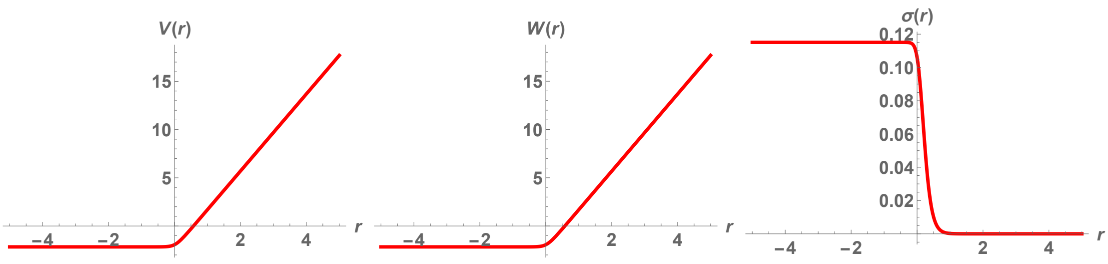
$$\sigma' = \frac{2}{5}e^{\frac{\sigma}{2}}\left[g_1 e^{-\sigma} - 16he^{\frac{3\sigma}{2}} - e^{-2V}p_1 - e^{-2W}p_2 - \frac{1}{4h}e^{-2(V+W)}p_1p_2\right], \quad (3.131)$$

$$\phi_3' = -e^{-\frac{\sigma}{2}}\left[g_1 + e^{\sigma}(e^{-2V}p_1 + e^{-2W}p_2)\right]\sinh\phi_3\,. \quad (3.132)$$

From these BPS equations, there exist AdS_3 fixed points only for $k_1 = k_2 = -1$. The resulting $AdS_3 \times H^2 \times H^2$ solution is given by

$$\phi_3 = 0, \qquad \sigma = \frac{2}{5}\ln\left[\frac{g_1}{12h}\right],$$
$$V = W = \frac{1}{10}\ln\left[\frac{27}{16h^2g_1^8}\right], \qquad L_{AdS_3} = \left[\frac{8}{3hg_1^4}\right]^{\frac{1}{5}}. \quad (3.133)$$

This solution preserves four supercharges and corresponds to a two-dimensional $N = (2, 0)$ SCFT with $SO(2)_R$ symmetry. An example of RG flow solutions from the $SO(4)$ AdS_7 vacuum to this fixed point for $g_1 = 16h$, $h = 1$, and $\phi_3 = 0$ is shown in Figure 3.9. Note that this $AdS_3 \times H^2 \times H^2$ fixed point and the RG flow are also solutions of pure $N = 2$ gauged supergravity with $SO(3)$ gauge group.



(a) V solution (b) W solution (c) σ solution

Figure 3.9: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 4$ to an $AdS_3 \times H^2 \times H^2$ fixed point with $SO(2)_R$ symmetry in IR as $r \to -4$ for $g_1 = 16h$, $h = 1$, and $\phi_3 = 0$ along the flow.

As in the case of AdS_3 solutions with $SO(2) \times SO(2)$ symmetry, the above solution can be uplifted to eleven dimensions by setting $g_2 = g_1$. The eleven-dimensional metric can be obtained from (3.105) by setting $\phi = 0$ and $A^6_{(1)} = 0$, or equivalently $A^{12} = A^{34} \equiv A^3$. The result is given by

$$\begin{aligned} d\hat{s}^2_{11} &= \Delta^{\frac{1}{3}} \left[e^{2U} dx^2_{1,1} + dr^2 + e^{2V} ds^2_{\Sigma^2_{k_1}} + e^{2W} ds^2_{\Sigma^2_{k_2}} \right] \\ &\quad + \frac{2}{g^2} \Delta^{-\frac{2}{3}} \left(e^{-2\sigma} \cos^2 \xi + e^{\frac{\sigma}{2}} \sin^2 \xi \right) d\xi^2 + \frac{1}{2g^2} \Delta^{-\frac{2}{3}} e^{\frac{\sigma}{2}} \cos^2 \xi \left[d\psi^2 \right. \\ &\quad \left. + \cos^2 \psi (d\alpha - gA^3)^2 + \sin^2 \psi (d\beta - gA^3)^2 \right] \end{aligned} \tag{3.134}$$

with

$$\Delta = e^{2\sigma} \sin^2 \xi + e^{-\frac{\sigma}{2}} \cos^2 \xi \,. \tag{3.135}$$

Note also that the seven-dimensional solution in this case has recently been discussed in [87] in the context of massive type IIA theory.

3.2.2 Supersymmetric $AdS_3 \times K^4$ Solutions

We repeat the same analysis for M^4 being a Kahler four-cycle and look for solutions of the form $AdS_3 \times K^4_k$. For the constant $k = 1, 0, -1$, the Kahler four-cycle becomes a two-dimensional complex space CP^2, a four-dimensional flat space $\mathbb{R}^4$, or a two-dimensional complex hyperbolic space CH^2, respectively. Apart from an $SO(2)_R$ gauge field used in the previous case, supersymmetric $AdS_3 \times K^4$ solutions can be obtained from the twist using the full $SO(3)_R$ gauge fields.

3.2.2.1 AdS_3 Vacua with $SO(2) \times SO(2)$ Symmetry

As in $\Sigma^2 \times \Sigma^2$ case, we begin with AdS_3 vacua with $SO(2) \times SO(2)$ symmetry and take the following ansatz for the seven-dimensional metric

$$ds^2_7 = e^{2U(r)} dx^2_{1,1} + dr^2 + e^{2V(r)} ds^2_{K^4_k} \,. \tag{3.136}$$

The metric on the Kahler four-cycle K^4_k is given by

$$ds^2_{K^4_k} = \frac{d\varphi^2}{\tilde{f}^2_k(\varphi)} + \frac{\varphi^2}{\tilde{f}_k(\varphi)} (\tau^2_1 + \tau^2_2) + \frac{\varphi^2}{\tilde{f}^2_k(\varphi)} \tau^2_3 \tag{3.137}$$

with $\varphi \in [0, \frac{\pi}{2}]$ and the function $\tilde{f}_k(\varphi)$ defined by

$$\tilde{f}_k(\varphi) = 1 + k\varphi^2\,. \tag{3.138}$$

τ_i, $i = 1, 2, 3$, are $SU(2)$ left-invariant one-forms satisfying $d\tau_i = \frac{1}{2}\varepsilon_{ijl}\tau_j \wedge \tau_l$. Their explicit form is given by

$$\begin{aligned}
\tau_1 &= -\sin\chi d\theta + \cos\chi \sin\theta d\psi,\\
\tau_2 &= \cos\chi d\theta + \sin\chi \sin\theta d\psi,\\
\tau_3 &= d\chi + \cos\theta d\psi\,.
\end{aligned} \tag{3.139}$$

The ranges of the coordinates are $\theta \in [0, \pi]$, $\psi \in [0, 2\pi]$, and $\chi \in [0, 4\pi]$.

By choosing the following choice of vielbein

$$\begin{aligned}
&e^{\hat{a}} = e^U dx^a, \qquad e^{\hat{3}} = e^V \frac{\varphi}{\sqrt{\tilde{f}_k(\varphi)}}\tau_1, \qquad e^{\hat{4}} = e^V \frac{\varphi}{\sqrt{\tilde{f}_k(\varphi)}}\tau_2,\\
&e^{\hat{2}} = dr, \qquad e^{\hat{5}} = e^V \frac{\varphi}{\tilde{f}_k(\varphi)}\tau_3, \qquad e^{\hat{6}} = e^V \frac{1}{\tilde{f}_k(\varphi)}d\varphi,
\end{aligned} \tag{3.140}$$

we find non-vanishing components of the spin connection of the form

$$\begin{aligned}
&\omega^{\hat{a}}{}_{\hat{2}} = U' e^{\hat{a}}, \qquad \omega^{\hat{m}}{}_{\hat{2}} = V' e^{\hat{m}}, \qquad m = 3, 4, 5, 6,\\
&\omega^{\hat{3}}{}_{\hat{6}} = \omega^{\hat{4}}{}_{\hat{5}} = e^{-V}\frac{1}{\sqrt{\tilde{f}_k(\varphi)}}\tau_1, \qquad \omega^{\hat{3}}{}_{\hat{4}} = e^{-V}\frac{(2k\varphi^2 + 1)}{\tilde{f}_k(\varphi)}\tau_3,\\
&\omega^{\hat{4}}{}_{\hat{6}} = \omega^{\hat{5}}{}_{\hat{3}} = e^{-V}\frac{1}{\sqrt{\tilde{f}_k(\varphi)}}\tau_2, \qquad \omega^{\hat{6}}{}_{\hat{5}} = e^{-V}\frac{(k\varphi^2 - 1)}{\tilde{f}_k(\varphi)}\tau_3\,.
\end{aligned} \tag{3.141}$$

We can now perform the twist by turning on $SO(2) \times SO(2)$ gauge fields with the following ansatz

$$A^3_{(1)} = e^{-V} p_1 \frac{3\varphi^2}{\sqrt{\tilde{f}_k(\varphi)}}\tau_3 \qquad \text{and} \qquad A^6_{(1)} = e^{-V} p_2 \frac{3\varphi^2}{\sqrt{\tilde{f}_k(\varphi)}}\tau_3\,. \tag{3.142}$$

The corresponding two-form field strengths are given by

$$F^3_{(2)} = 3e^{-2V} p_1 J_{(2)} \qquad \text{and} \qquad F^6_{(2)} = 3e^{-2V} p_2 J_{(2)} \tag{3.143}$$

where $J_{(2)}$ is the Kahler structure defined by

$$J_{(2)} = e^{\hat{3}} \wedge e^{\hat{4}} - e^{\hat{5}} \wedge e^{\hat{6}}\,. \tag{3.144}$$

To implement the twist, we impose the projectors on the Killing spinors (2.42)

$$\gamma^{\hat{3}\hat{4}}\epsilon = -\gamma^{\hat{5}\hat{6}}\epsilon = i\sigma^3\epsilon \tag{3.145}$$

together with the following twist condition

$$g_1 p_1 = k\,. \tag{3.146}$$

As in the previous cases, we need to turn on the three-form field corresponding to

$$G_{(4)} = \frac{9}{8\sqrt{2}h}e^{-4V}(p_1^2 - p_2^2)e^{\hat{3}}\wedge e^{\hat{4}}\wedge e^{\hat{5}}\wedge e^{\hat{6}}. \tag{3.147}$$

With all these and the γ^r projector (3.91), we find the following BPS equations

$$\begin{aligned}
U' &= \frac{1}{5}e^{\frac{\sigma}{2}}\Big[(g_1 e^{-\sigma}\cosh\phi + 4he^{-\frac{5\sigma}{2}}) - 6e^{-2V}(p_1\cosh\phi + p_2\sinh\phi) \\
&\quad + \frac{27}{8h}e^{-\frac{3\sigma}{2}-4V}(p_1^2 - p_2^2)\Big], && (3.148)\\
V' &= \frac{1}{5}e^{\frac{\sigma}{2}}\Big[(g_1 e^{-\sigma}\cosh\phi + 4he^{-\frac{5\sigma}{2}}) + 9e^{-2V}(p_1\cosh\phi + p_2\sinh\phi) \\
&\quad - \frac{9}{4h}e^{-\frac{3\sigma}{2}-4V}(p_1^2 - p_2^2)\Big], && (3.149)\\
\sigma' &= \frac{2}{5}e^{\frac{\sigma}{2}}\Big[(g_1 e^{-\sigma}\cosh\phi - 16he^{-\frac{5\sigma}{2}}) - 6e^{-2V}(p_1\cosh\phi + p_2\sinh\phi) \\
&\quad - \frac{9}{4h}e^{-\frac{3\sigma}{2}-4V}(p_1^2 - p_2^2)\Big], && (3.150)\\
\phi' &= -g_1 e^{-\frac{\sigma}{2}}\sinh\phi - 6e^{\frac{\sigma}{2}-2V}(p_1\sinh\phi + p_2\cosh\phi) && (3.151)
\end{aligned}$$

with ϕ being the $SO(2)\times SO(2)$ singlet scalar in (3.85).

The BPS equations admit an $AdS_3\times CH^2$ fixed point given by

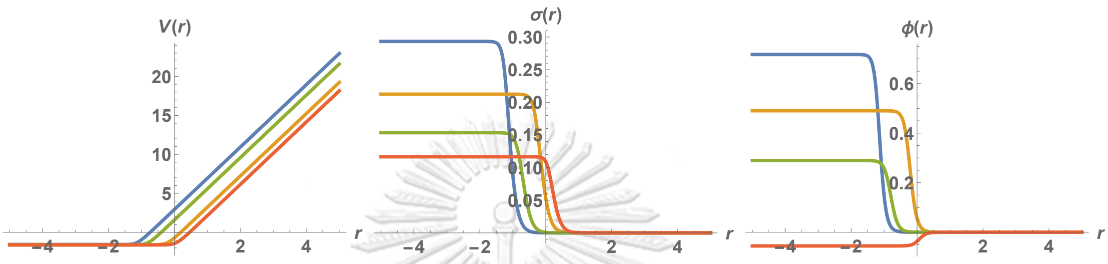
$$\begin{aligned}
\sigma &= \frac{2}{5}\ln\left[\frac{g_1 p_1^2}{12h\sqrt{p_1^4 - 10p_1^2p_2^2 + 9p_2^4}}\right], \quad \phi = \frac{1}{2}\ln\left[\frac{p_1^2 + 2p_1p_2 - 3p_2^2}{p_1^2 - 2p_1p_2 - 3p_2^2}\right], \\
V &= \frac{1}{10}\ln\left[\frac{3^8(p_1^2 - p_2^2)^4}{16h^2 g_1^3(9p_1p_2^2 - p_1^3)}\right], \quad L_{AdS_3} = \left[\frac{8(p_1^5 - 10p_1^3p_2^2 + 9p_1p_2^4)^2}{3hg_1^4(p_1^2 - 3p_2^2)^5}\right]^{\frac{1}{5}}. && (3.152)
\end{aligned}$$

This AdS_3 solution preserves four supercharges and exists for

$$-\frac{1}{48h} < p_2 < \frac{1}{48h} \tag{3.153}$$

with $g_1 = 16h$, $k = -1$, and $h > 0$. The $AdS_3\times CH^2$ fixed point is dual to a two-dimensional $N = (2,0)$ SCFT with $SO(2)\times SO(2)$ symmetry.

Examples of RG flows from the $SO(4)$ AdS_7 critical point to these fixed points for $g_1 = 16h$, $h = 1$, and different values of p_2 are shown in Figure 3.10. As in the $\Sigma^2 \times \Sigma^2$ case, the $AdS_3 \times CH^2$ fixed points and the associated RG flows can be uplifted to eleven dimensions by setting $g_2 = g_1$. The eleven-dimensional metric can be obtained from (3.105) by replacing $e^{2V} ds^2_{\Sigma^2_{k_1}} + e^{2W} ds^2_{\Sigma^2_{k_2}}$ by $e^{2V} ds^2_{K^4_k}$ and using the gauge fields in (3.142). We will not repeat it here.



(a) V solution (b) σ solution (c) ϕ solution

Figure 3.10: Numerical solutions from the $SO(4)$ AdS_7 vacuum in UV as $r \to 4$ to $AdS_3 \times CH^2$ fixed points with $SO(2) \times SO(2)$ symmetry in IR as $r \to -4$ for $g_1 = 16h$ and $h = 1$. The blue, orange, green, and red curves refer to $p_2 = -\frac{1}{64}, -\frac{1}{80}, -\frac{1}{120}, \frac{1}{580}$.

3.2.2.2 AdS_3 Vacua with $SO(2)_{\text{diag}}$ Symmetry

We next consider the solutions with smaller unbroken symmetry $SO(2)_{\text{diag}} \subset SO(2) \times SO(2)$ by imposing $g_2 p_2 = g_1 p_1$ and using the coset representative given by (3.108). As in the previous cases, $\phi_1 = 0$ or $\phi_3 = 0$ is required in order to have compatibility between BPS equations and the field equations. We will consider the case of $\phi_3 = 0$ with the following BPS equations

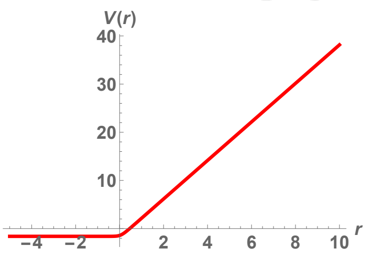
$$\begin{aligned} U' &= \frac{1}{5} e^{\frac{\sigma}{2}} \Bigg[\left(g_1 e^{-\sigma} \cosh^2\phi_1 \cosh\phi_2 + g_2 e^{-\sigma} \sinh^2\phi_1 \sinh\phi_2 + 4h e^{\frac{3\sigma}{2}} \right) \\ &\quad -6e^{-2V} \left(\cosh\phi_2 + \frac{g_1}{g_2} \sinh\phi_2 \right) p_1 - \frac{27}{8hg_2^2} e^{-\frac{3\sigma}{2} - 4V} (g_1^2 - g_2^2) p_1^2 \Bigg], \quad (3.154) \\ V' &= \frac{1}{5} e^{\frac{\sigma}{2}} \Bigg[\left(g_1 e^{-\sigma} \cosh^2\phi_1 \cosh\phi_2 + g_2 e^{-\sigma} \sinh^2\phi_1 \sinh\phi_2 + 4h e^{\frac{3\sigma}{2}} \right) \\ &\quad +9e^{-2V} \left(\cosh\phi_2 + \frac{g_1}{g_2} \sinh\phi_2 \right) p_1 + \frac{9}{4hg_2^2} e^{-\frac{3\sigma}{2} - 4V} (g_1^2 - g_2^2) p_1^2 \Bigg], \quad (3.155) \end{aligned}$$

$$\sigma' = \frac{2}{5}e^{\frac{\sigma}{2}}\left[\left(g_1 e^{-\sigma}\cosh^2\phi_1\cosh\phi_2 + g_2 e^{-\sigma}\sinh^2\phi_1\sinh\phi_2 - 16he^{\frac{3\sigma}{2}}\right)\right.$$
$$\left. -6e^{-2V}\left(\cosh\phi_2 + \frac{g_1}{g_2}\sinh\phi_2\right)p_1 + \frac{9}{4hg_2^2}e^{-\frac{3\sigma}{2}-4V}(g_1^2-g_2^2)p_1^2\right], \quad (3.156)$$
$$\phi_1' = -e^{-\frac{\sigma}{2}}\cosh\phi_1\sinh\phi_1(g_1\cosh\phi_2 + g_2\sinh\phi_2), \quad (3.157)$$
$$\phi_2' = -e^{\frac{\sigma}{2}}\left[\left(g_1 e^{-\sigma}\cosh^2\phi_1\sinh\phi_2 + g_2 e^{-\sigma}\sinh^2\phi_1\cosh\phi_2\right)\right.$$
$$\left. +6e^{-2V}\left(\sinh\phi_2 + \frac{g_1}{g_2}\cosh\phi_2\right)p_1\right]. \quad (3.158)$$

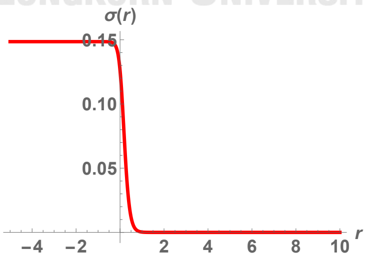
There exist two classes of $AdS_3 \times CH^2$ fixed points preserving four supercharges and corresponding to $N = (2, 0)$ SCFTs with $SO(2)_{\text{diag}}$ symmetry in two dimensions. The first class is given by

$$\phi_1 = 0, \qquad \sigma = \frac{2}{5}\phi_2 + \frac{2}{5}\ln\left[\frac{g_1 g_2^2}{12h(g_2^2 + 2g_1g_2 - 3g_1^2)}\right],$$
$$\phi_2 = \frac{1}{2}\ln\left[\frac{3g_1^2 - 2g_1g_2 - g_2^2}{3g_1^2 + 2g_1g_2 - g_2^2}\right],$$
$$V = \frac{1}{10}\ln\left[\frac{3^8(g_1^2-g_2^2)^4}{16h^2g_1^8g_2^6(g_2^2-9g_1^2)}\right],$$
$$L_{AdS_3} = \left[\frac{8(9g_1^4g_2 - 10g_1^2g_2^3 + g_2^5)^2}{3hg_1^4(g_2^2-3g_1^2)^5}\right]^{\frac{1}{5}} \quad (3.159)$$

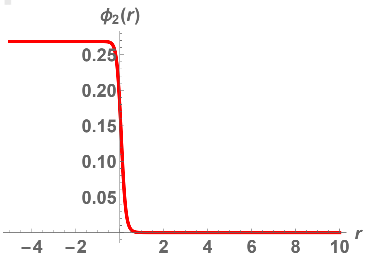
with $g_2 > 3g_1$ or $g_2 < -3g_1$ for good AdS_3 vacua to exist. An RG flow from the $SO(4)$ AdS_7 crtical point to this fixed point is given in Figure 3.11 with $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$.



(a) V solution



(b) σ solution



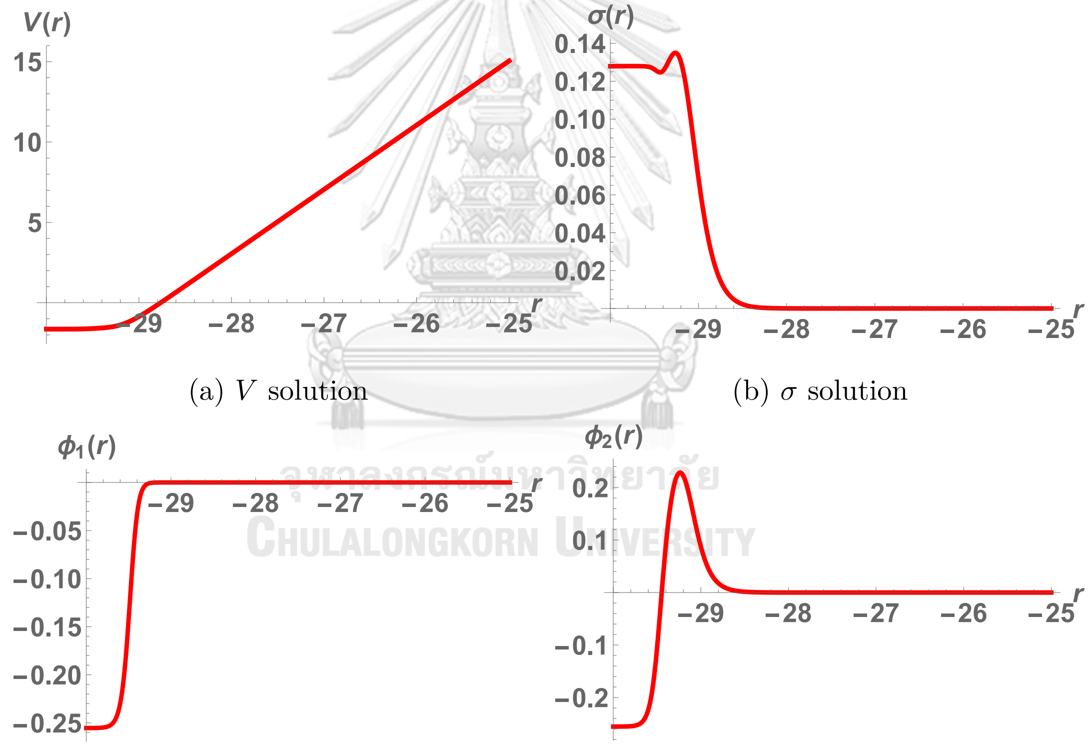
(c) ϕ_2 solution

Figure 3.11: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 10$ to an $AdS_3 \times CH^2$ fixed point in IR as $r \to -4$ with $SO(2)_{\text{diag}}$ symmetry for $g_2 = 4g_1$, $g_1 = 16h$, $h = 1$, and $\phi_1 = 0$ along the flow.

Another class of $AdS_3 \times CH^2$ fixed points is given by

$$\sigma = \frac{2}{5}\ln\left[\frac{g_1 g_2}{12h\sqrt{(g_2+g_1)(g_2-g_1)}}\right], \qquad \phi_1 = \phi_2 = \frac{1}{2}\ln\left[\frac{g_2-g_1}{g_2+g_1}\right],$$
$$V = \frac{1}{5}\ln\left[\frac{3^4(g_1^2-g_2^2)^2}{4hg_1^4g_2^4}\right], \qquad L_{AdS_3} = \left[\frac{8(g_1^2-g_2^2)^2}{3hg_1^4g_2^4}\right]^{\frac{1}{5}}. \tag{3.160}$$

To obtain good AdS_3 vacua, $g_2 > g_1$ is needed. Examples of RG flows from the $SO(4)$ and $SO(3)_{\text{diag}}$ AdS_7 vacua to these fixed points for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$ are shown in Figures 3.12, 3.13, and 3.14. All of these $AdS_3 \times CH^2$ fixed points and RG flows cannot be uplifted to eleven dimensions due to $g_1 \neq g_2$, so we also do not have a clear holographic interpretation in this case.



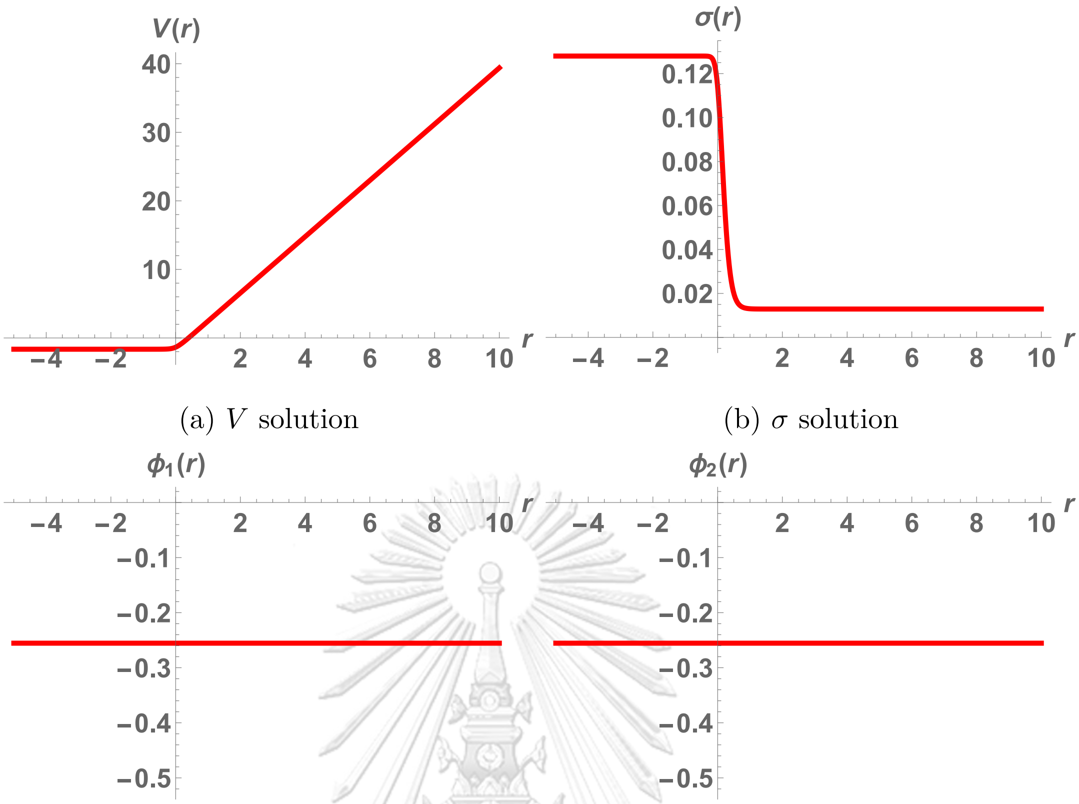
(a) V solution

(b) σ solution

(c) ϕ_1 solution

(d) ϕ_2 solution

Figure 3.12: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to -25$ to an $AdS_3 \times CH^2$ fixed point with $SO(2)_{\text{diag}}$ symmetry in IR as $r \to -30$ for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$.



(a) V solution (b) σ solution

(c) ϕ_1 solution (d) ϕ_2 solution

Figure 3.13: A numerical solution from the $SO(3)_{\text{diag}}$ AdS_7 vacuum in UV as $r \to 10$ to an $AdS_3 \times CH^2$ fixed point with $SO(2)_{\text{diag}}$ symmetry in IR as $r \to -4$ for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$.

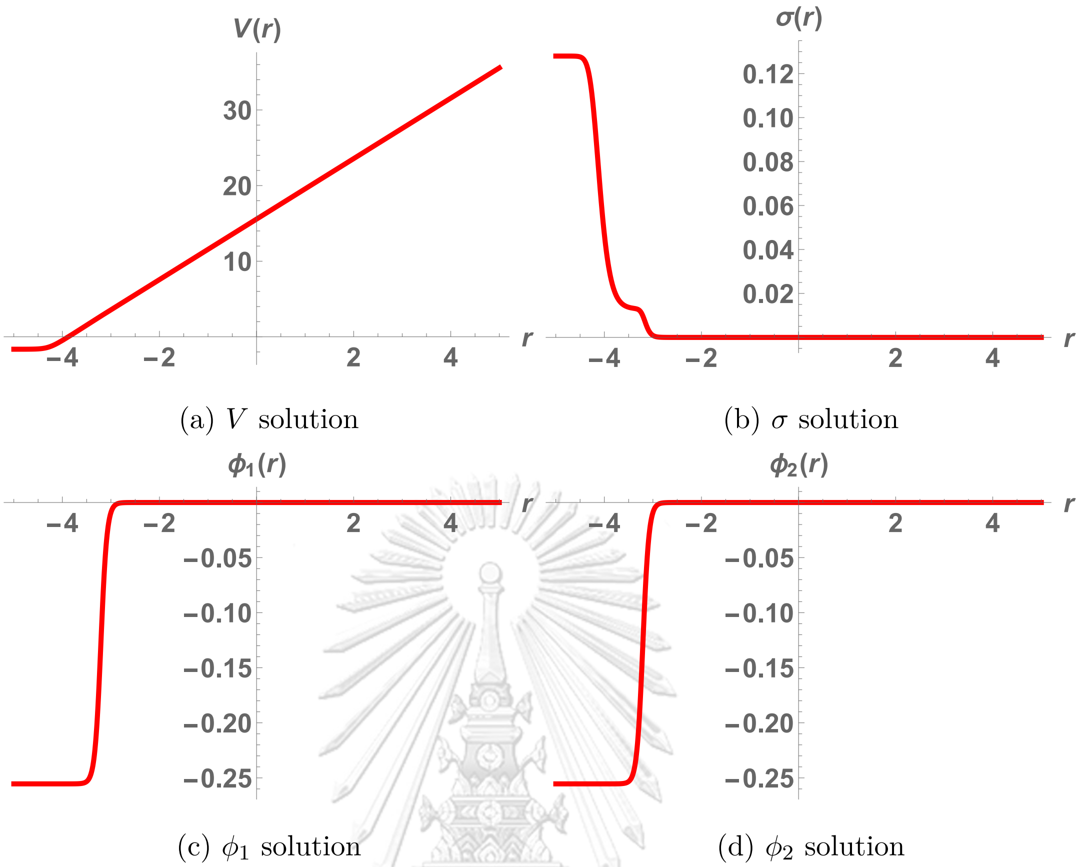
3.2.2.3 AdS_3 Vacua with $SO(2)_R$ Symmetry

By setting $p_2 = 0$ in the $SO(2) \times SO(2)$ case, we obtain solutions with $SO(2)_R \subset SO(3)_R$ symmetry. As in the previous case, the three $SO(2)_R$ singlet scalars need to vanish in order for AdS_3 fixed points to exist. We will accordingly set all vector multiplet scalars to zero for brevity. The resulting BPS equations are given by

$$U' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[g_1 e^{-\sigma} + 4he^{\frac{3\sigma}{2}} - 6e^{-2V}p_1 + \frac{27}{8h}e^{-4V}p_1^2\right], \qquad (3.161)$$

$$V' = \frac{1}{5}e^{\frac{\sigma}{2}}\left[g_1 e^{-\sigma} + 4he^{\frac{3\sigma}{2}} + 9e^{-2V}p_1 - \frac{9}{4h}e^{-4V}p_1^2\right], \qquad (3.162)$$

$$\sigma' = \frac{2}{5}e^{\frac{\sigma}{2}}\left[g_1 e^{-\sigma} - 14he^{\frac{3\sigma}{2}} - 6e^{-2V}p_1 - \frac{9}{4h}e^{-4V}p_1^2\right]. \qquad (3.163)$$



(a) V solution (b) σ solution

(c) ϕ_1 solution (d) ϕ_2 solution

Figure 3.14: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 4$ to the $SO(3)_{\text{diag}}$ AdS_7 critical point and then to an $AdS_3 \times CH^2$ fixed point with $SO(2)_{\text{diag}}$ symmetry in IR as $r \to -4$ for $g_2 = 4g_1$, $g_1 = 16h$, and $h = 1$.

Imposing the twist condition (3.146), we find an AdS_3 solution for $k = -1$,

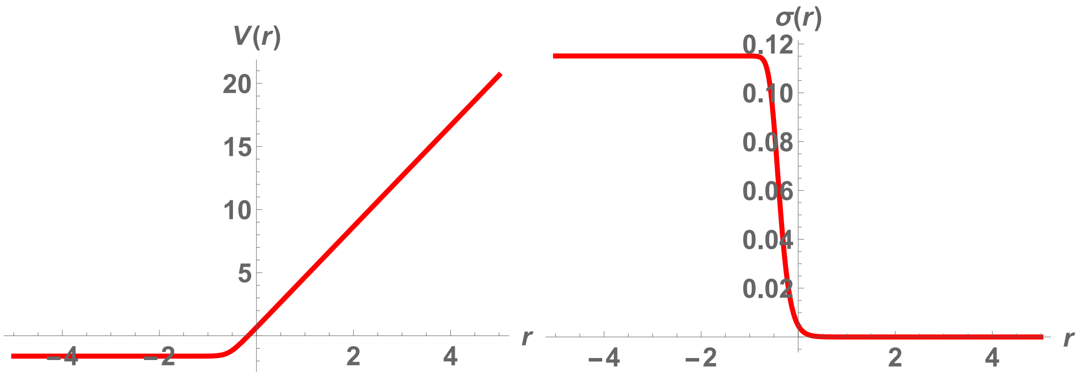
$$\sigma = \frac{2}{5}\ln\left[\frac{g_1}{12h}\right], \qquad V = \frac{1}{10}\ln\left[\frac{3^8}{16h^2 g_1^8}\right], \qquad L_{AdS_3} = \left[\frac{8}{3hg_1^4}\right]^{\frac{1}{5}}. \tag{3.164}$$

An RG flow from the $SO(4)$ AdS_7 critical point to this $AdS_3 \times CH^2$ fixed point for $g_1 = 16h$ and $h = 1$ is given in Figure 3.15.

3.2.2.4 AdS_3 Vacua with $SO(3)_{\text{diag}}$ Symmetry

For Kahler four-cycles with $SU(2)\times U(1)$ spin connection, we can also perform the twist by identifying $SO(3) \sim SU(2) \subset SU(2)\times U(1)$ with the unbroken symmetry $SO(3)_{\text{diag}} \subset SO(3) \times SO(3)$. In this case, we use the metric on K_k^4 in the form

$$ds^2_{K_k^4} = d\varphi^2 + f_k(\varphi)^2(\tau_1^2 + \tau_2^2 + \tau_3^2) \tag{3.165}$$



(a) V solution (b) σ solution

Figure 3.15: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 4$ to an $AdS_3 \times CH^2$ fixed point with $SO(2)_R$ symmetry in IR as $r \to -4$ for $g_1 = 16h$ and $h = 1$.

with τ_i, $i = 1, 2, 3$, being the $SU(2)$ left-invariant one-forms given in (3.139) and $f_k(\varphi)$ defined in (3.80).

With the seven-dimensional vielbein

$$\begin{aligned} e^{\hat{a}} &= e^U dx^a, \qquad e^{\hat{2}} = dr, \qquad e^{\hat{3}} = e^V f_k(\varphi)\tau_1, \\ e^{\hat{4}} &= e^V f_k(\varphi)\tau_2, \qquad e^{\hat{5}} = e^V f_k(\varphi)\tau_3, \qquad e^{\hat{6}} = e^V d\varphi, \end{aligned} \tag{3.166}$$

we can compute the following non-vanishing components of the spin connection

$$\begin{aligned} \omega^{\hat{a}}{}_{\hat{2}} &= U' e^{\hat{a}}, \qquad \omega^{\hat{m}}{}_{\hat{2}} = V' e^{\hat{m}}, \qquad m = 3, 4, 5, 6, \\ \omega^{\hat{3}}{}_{\hat{6}} &= e^{-V} f_k'(\varphi)\tau_1, \quad \omega^{\hat{4}}{}_{\hat{6}} = e^{-V} f_k'(\varphi)\tau_2, \quad \omega^{\hat{5}}{}_{\hat{6}} = e^{-V} f_k'(\varphi)\tau_3, \\ \omega^{\hat{4}}{}_{\hat{5}} &= e^{-V}\tau_1 \qquad \omega^{\hat{5}}{}_{\hat{3}} = e^{-V}\tau_2 \qquad \omega^{\hat{3}}{}_{\hat{4}} = e^{-V}\tau_3. \end{aligned} \tag{3.167}$$

We then turn on the $SO(3)_{\text{diag}}$ gauge fields as follow

$$A^i_{(1)} = \frac{g_2}{g_1} A^{i+3}_{(1)} = \frac{p}{k} e^{-V} (f_k'(\varphi) + 1)\tau_i \tag{3.168}$$

with the two-form field strengths given by

$$F^1_{(2)} = \frac{g_2}{g_1} F^4_{(2)} = e^{-2V} p \, (e^{\hat{3}} \wedge e^{\hat{6}} + e^{\hat{4}} \wedge e^{\hat{5}}), \tag{3.169}$$

$$F^2_{(2)} = \frac{g_2}{g_1} F^5_{(2)} = e^{-2V} p \, (e^{\hat{3}} \wedge e^{\hat{5}} + e^{\hat{4}} \wedge e^{\hat{6}}), \tag{3.170}$$

$$F^3_{(2)} = \frac{g_2}{g_1} F^6_{(2)} = e^{-2V} p \, (e^{\hat{3}} \wedge e^{\hat{4}} + e^{\hat{5}} \wedge e^{\hat{6}}). \tag{3.171}$$

As in the previous cases, we also need a non-vanishing four-form field strength

$$G_{(4)}=\frac{3}{8\sqrt{2}hg_2^2}e^{-4V}(g_1^2-g_2^2)p^2e^{\hat{3}}\wedge e^{\hat{4}}\wedge e^{\hat{5}}\wedge e^{\hat{6}} \qquad (3.172)$$

together with the twist condition

$$g_1p=k, \qquad (3.173)$$

and the following projectors

$$\gamma^r\epsilon=-\gamma^{\hat{3}\hat{4}\hat{5}\hat{6}}\epsilon=\epsilon,\qquad \gamma^{\hat{3}\hat{4}}\epsilon=i\sigma^3\epsilon,\quad \text{and}\quad \gamma^{\hat{4}\hat{5}}\epsilon=i\sigma^1\epsilon. \qquad (3.174)$$

Hence, the resulting AdS_3 fixed points preserve two supercharges corresponding to $N=(1,0)$ superconformal symmetry in two dimensions.

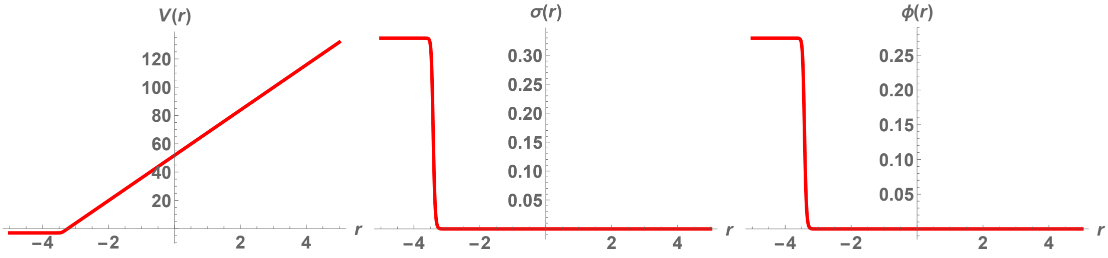
With all these and the $SO(3)_{\text{diag}}$ coset representative (2.33), we find the following BPS equations

$$\begin{aligned}
U' &= \frac{1}{5}e^{\frac{\sigma}{2}}\Bigg[(g_1e^{-\sigma}\cosh^3\phi+g_2e^{-\sigma}\sinh^3\phi+4he^{\frac{3\sigma}{2}})-\frac{9p^2}{8hg_2^2}e^{-\frac{3\sigma}{2}-4V}(g_1^2-g_2^2)\\
&\quad -6pe^{-2V}\left(\cosh\phi+\frac{g_1}{g_2}\sinh\phi\right)\Bigg], \qquad (3.175)\\
V' &= \frac{1}{5}e^{\frac{\sigma}{2}}\Bigg[(g_1e^{-\sigma}\cosh^3\phi+g_2e^{-\sigma}\sinh^3\phi+4he^{\frac{3\sigma}{2}})+\frac{3p^2}{4hg_2^2}e^{-\frac{3\sigma}{2}-4V}(g_1^2-g_2^2)\\
&\quad +9pe^{-2V}\left(\cosh\phi+\frac{g_1}{g_2}\sinh\phi\right)\Bigg], \qquad (3.176)\\
\sigma' &= \frac{2}{5}e^{\frac{\sigma}{2}}\Bigg[(g_1e^{-\sigma}\cosh^3\phi+g_2e^{-\sigma}\sinh^3\phi-16he^{\frac{3\sigma}{2}})+\frac{3p^2}{4hg_2^2}e^{-\frac{3\sigma}{2}-4V}(g_1^2-g_2^2)\\
&\quad -6pe^{-2V}\left(\cosh\phi+\frac{g_1}{g_2}\sinh\phi\right)\Bigg], \qquad (3.177)\\
\phi' &= -\frac{1}{2g_2}e^{-\frac{\sigma}{2}}(g_1\cosh\phi+g_2\sinh\phi)(g_2\sinh 2\phi+4pe^{\sigma-2V}). \qquad (3.178)
\end{aligned}$$

We now look for AdS_3 fixed points for the case of $g_2=g_1$ that can be embedded in eleven dimensions. Setting $g_2=g_1$ in the above equations, we find the following $AdS_3\times CH^2$ fixed point

$$\sigma=\frac{2}{5}\ln\left[\frac{3^{\frac{3}{4}}g_1}{16h}\right],\quad \phi=\frac{1}{4}\ln 3,\quad V=\frac{1}{5}\ln\left[\frac{18}{hg_1^4}\right],\quad L_{AdS_3}=\left[\frac{64}{27hg_1^4}\right]^{\frac{1}{5}}. \qquad (3.179)$$

An RG flow interpolating between the $SO(4)$ AdS_7 vacuum and this $AdS_3\times CH^2$ fixed point is shown in Figure 3.16 for $g_1=16h$ and $h=1$.



(a) V solution (b) σ solution (c) ϕ solution

Figure 3.16: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 4$ to an $AdS_3 \times CH^2$ fixed point with $SO(3)_{\text{diag}}$ symmetry in IR as $r \to -4$ for $g_1 = g_2 = 16h$ and $h = 1$.

We can uplift this solution to eleven dimensions by first choosing the S^3 coordinates

$$\mu^\alpha = (\cos\psi \hat{\mu}^p, \sin\psi), \qquad p, q, \ldots = 1, 2, 3 \tag{3.180}$$

with $\hat{\mu}^p$ being coordinates on S^2 satisfying $\hat{\mu}^p \hat{\mu}^p = 1$. Using the $SL(4,\mathbb{R})/SO(4)$ matrix

$$\tilde{T}^{-1}_{\alpha\beta} = \text{diag}(e^\phi, e^\phi, e^\phi, e^{-3\phi}) = (\delta_{pq} e^\phi, e^{-3\phi}), \tag{3.181}$$

we find the eleven-dimensional metric

$$\begin{aligned} d\hat{s}^2_{11} \;=\; & \Delta^{\frac{1}{3}} \left[e^{2U} dx^2_{1,1} + dr^2 + e^{2V} [d\varphi^2 + f_k(\varphi)^2 (\tau_1^2 + \tau_2^2 + \tau_3^2)] \right] \\ & + \frac{2}{g^2} \Delta^{-\frac{2}{3}} e^{-2\sigma} \left[\cos^2\xi + e^{\frac{5}{2}\sigma} \sin^2\xi (e^\phi \cos^2\psi + e^{-3\phi} \sin^2\psi) \right] d\xi^2 \\ & + \frac{1}{2g^2} \Delta^{-\frac{2}{3}} e^{\frac{\sigma}{2}} \cos^2\xi \left[(e^{-3\phi} \cos^2\psi + e^\phi \sin^2\psi) d\psi^2 + e^\phi \cos^2\psi D\hat{\mu}^a D\hat{\mu}^a \right] \\ & + \frac{1}{g^2} \Delta^{-\frac{2}{3}} e^{\frac{\sigma}{2}} \sin\xi \sin\psi \cos\psi (e^\phi - e^{-3\phi}) d\xi d\psi \end{aligned} \tag{3.182}$$

with Δ given by

$$\Delta = e^{-\frac{\sigma}{2}} \cos^2\xi (e^{-\phi} \cos^2\psi + e^{3\phi} \sin^2\psi) + e^{2\sigma} \sin^2\xi \tag{3.183}$$

and $D\hat{\mu}^p = d\hat{\mu}^p + g A^{pq} \hat{\mu}^q$. The gauge fields A^{pq} are given by

$$A^{12} = 2A^3_{(1)}, \qquad A^{13} = -2A^2_{(1)}, \qquad A^{23} = -2A^1_{(1)}. \tag{3.184}$$

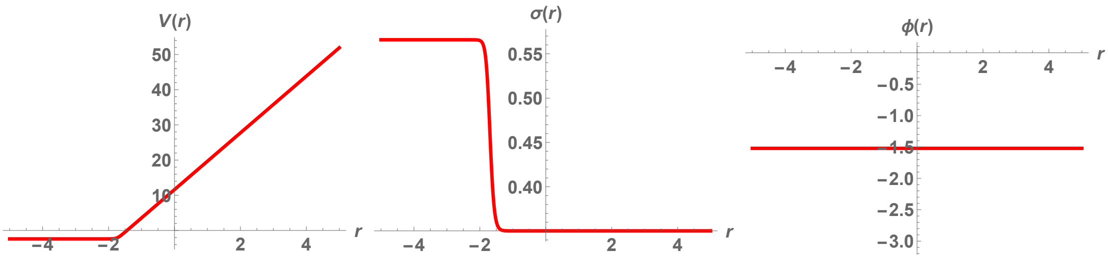
For $g_2 \neq g_1$, we find the following AdS_3 fixed points

$$\sigma = \frac{2}{5}\ln\left[\frac{3g_1g_2}{28h\sqrt{(g_2+g_1)(g_2-g_1)}}\right], \qquad \phi = \frac{1}{2}\ln\left[\frac{g_2-g_1}{g_2+g_1}\right],$$
$$V = \frac{1}{10}\ln\left[\frac{3087(g_1^2-g_2^2)^4}{16h^2g_1^8g_2^8}\right], \qquad L_{AdS_3} = \left[\frac{24(g_1^2-g_2^2)^2}{7g_1^4g_2^4h}\right]^{\frac{1}{5}}. \qquad (3.185)$$

These are $AdS_3 \times CH^2$ solutions with the condition $g_2 > g_1$. Finally, we can numerically find RG flow solutions connecting these fixed points to the AdS_7 vacua with $SO(4)$ and $SO(3)_{\text{diag}}$ symmetries. Examples of these solutions for $g_2 = 1.1g_1$, $g_1 = 16h$, and $h = 1$ are given in Figures 3.17, 3.18, and 3.19.

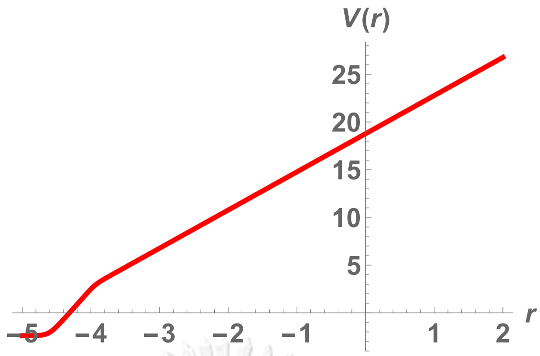
(a) V solution (b) σ solution (c) ϕ solution

Figure 3.17: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to -20$ to an $AdS_3 \times CH^2$ fixed point with $SO(3)_{\text{diag}}$ symmetry in IR as $r \to -30$ for $g_2 = 1.1g_1$, $g_1 = 16h$, and $h = 1$.

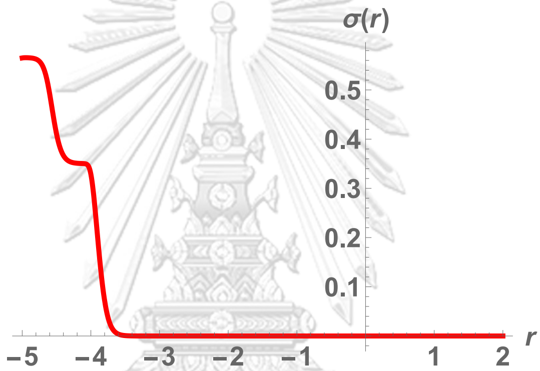


(a) V solution (b) σ solution (c) ϕ solution

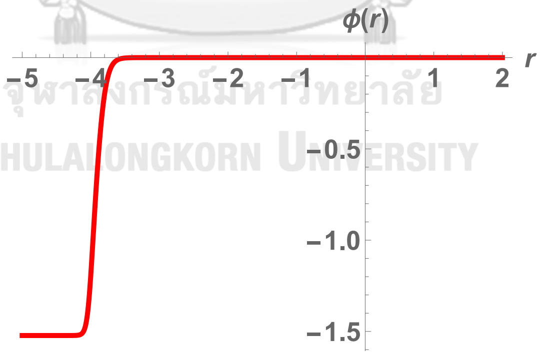
Figure 3.18: A numerical solution from the $SO(3)_{\text{diag}}$ AdS_7 vacuum in UV as $r \to 4$ to an $AdS_3 \times CH^2$ fixed point with $SO(3)_{\text{diag}}$ symmetry in IR as $r \to -4$ for $g_2 = 1.1g_1$, $g_1 = 16h$, and $h = 1$.



(a) V solution



(b) σ solution



(c) ϕ solution

Figure 3.19: A numerical solution from the $SO(4)$ AdS_7 vacuum in UV as $r \to 2$ to the $SO(3)_{\text{diag}}$ AdS_7 critical point and then to an $AdS_3 \times CH^2$ fixed point with $SO(3)_{\text{diag}}$ symmetry in IR as $r \to -5$ for $g_2 = 1.1g_1$, $g_1 = 16h$, and $h = 1$.

CHAPTER IV

SUPERSYMMETRIC SOLUTIONS OF MAXIMAL GAUGED SUPERGRAVITY

In the maximal case with thirty-two supercharges, the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence has been explored in great detail both from the M-theory point of view and the effective $N = 4$, $SO(5)$ gauged supergravity in seven dimensions. In the context of M-theory, a six-dimensional $N = (2, 0)$ SCFT emerges as a world-volume theory of M5-branes in the near horizon limit. On the other hand, in the low-energy limit, this $N = (2, 0)$ SCFT is dual to the maximally supersymmetric AdS_7 vacuum of the $SO(5)$ gauged supergravity, see [88] for example. General discussions about supersymmetric DWs in the $SO(5)$ gauged theory have already been given in [48–52]. Moreover, supersymmetric solutions of the $SO(5)$ gauged supergravity corresponding to holographic RG flows across dimensions from the six-dimensional $N = (2, 0)$ SCFT to SCFTs in lower dimensions have been extensively studied in [53–59, 86] in the context of wrapped M5-branes.

Apart from $SO(5)$, seven-dimensional maximal $N = 4$ supergravity can be gauged by various possible gauge groups in the embedding tensor formalism, as reviewed in Section 2.2. Among many viable gauge groups, only the $SO(5)$ gauged theory admits a maximally supersymmetric AdS_7 vacuum. For other gauge groups, their vacua are given by supersymmetric (flat) DWs dual to $N = (2, 0)$ SQFTs in six dimensions according to the DW/QFT correspondence. However, as pointed out in [52], a systematic study of these DWs in other gauge groups has not appeared so far.

We will look for a complete classification of supersymmetric solutions of seven-dimensional maximal gauged supergravity with several gauge groups within this chapter. Starting from finding a large class of supersymmetric (flat) DW solutions, charged DWs and twisted solutions can be obtained by extending from these vacua. Supersymmetric solutions found in this chapter may give a huge comprehension of the AdS_7/CFT_6 correspondence and also the general DW_7/QFT_6 duality. Moreover, the solutions with $CSO(p, q, 5-p-q)$ and $CSO(p, q, 4-p-q)$ gauge groups are of particular interest since they can be embedded in eleven-dimensional supergravity and type IIB theory by consistent truncations on $H^{p,q} \circ T^{5-p-q}$ [64] and $H^{p,q} \circ T^{4-p-q}$ [65], respectively. These solutions have higher dimensional origins and could be interpreted as different brane configurations in string/M-theory.

4.1 Flat Domain Wall Solutions

As pointed out in [52], each of the two components of the embedding tensor transforming in $\mathbf{15}$ and $\overline{\mathbf{40}}$ representations leads to half-supersymmetric DWs of seven-dimensional maximal gauged supergravity. These $\mathbf{15}$ and $\overline{\mathbf{40}}$ parts give rise to DWs respectively supporting tensor and vector multiplets on their world-volumes. Moreover, when both representations of the embedding tensor are present simultaneously, the DWs are only $\frac{1}{4}$-supersymmetric. We provide a systematic study of these supersymmetric DWs from several gauge groups in this section.

Although solutions with $CSO(p, q, 5-p-q)$ and $CSO(p, q, 4-p-q)$ gauge groups can be embedded respectively in eleven and ten dimensions, their complete truncation ansatze have not been constructed. In the following analyses, we will give uplifted solutions of the DWs only for $SO(5)$ and $CSO(4, 0, 1)$ gauge groups in which the truncation ansatze have been completely constructed long ago in [26,27] and [77]. We leave uplifting the solutions from other gauge groups for future work.

4.1.1 Gaugings in 15 Representation

In this section, we consider supersymmetric DWs from $CSO(p, q, 5-p-q)$ gauge groups resulting from the embedding tensor in **15** representation. As in other standard DWs, all tensor fields vanish while the metric takes the standard form given in (2.37) with the vielbein

$$e^{\hat{m}} = e^{U(r)}dx^m, \qquad e^{\hat{6}} = dr \tag{4.1}$$

in which x^m with $m = 0, 1, ..., 5$ are the coordinates on six-dimensional Minkowski space. For scalar fields, we follow the approach introduced in [76] by restricting ourselves to a subset of scalars invariant under a certain residual symmetry $H_0 \subset G_0 = CSO(p, q, 5-p-q)$. To obtain an explicit parametrization of the $SL(5)/SO(5)$ coset representative $\mathcal{V}_M{}^A$, we introduce $GL(5)$ matrices of the form

$$(e_{MN})_K{}^L = \delta_{MK}\delta^L_N\,. \tag{4.2}$$

Non-compact generators of $SL(5)$ are symmetric traceless matrices defined in terms of these $GL(5)$ matrices.

We use the following convenient choice of $SO(5)$ gamma matrices

$$\begin{aligned} \Gamma_1 &= -\sigma_2 \otimes \sigma_2, \qquad \Gamma_2 = \mathbf{1}_2 \otimes \sigma_1, \qquad \Gamma_3 = \mathbf{1}_2 \otimes \sigma_3, \\ \Gamma_4 &= \sigma_1 \otimes \sigma_2, \qquad \Gamma_5 = \sigma_3 \otimes \sigma_2 \end{aligned} \tag{4.3}$$

together with the $USp(4)$ symplectic form given by

$$\Omega_{ab} = \Omega^{ab} = \mathbf{1}_2 \otimes i\sigma_2 \tag{4.4}$$

where $\mathbf{1}_2$ is a (2×2) identity matrix and $\{\sigma_1, \sigma_2, \sigma_3\}$ are the usual Pauli matrices (B.3). We are now in a position to set up BPS equations and look for DW solutions with different unbroken symmetries.

4.1.1.1 $SO(4)$ Symmetric Domain Walls

We start with a simple solution with $SO(4)$ unbroken symmetry. The gauge groups that contain $SO(4)$ as a subgroup are $SO(5)$, $SO(4,1)$, and $CSO(4,0,1)$.

To incorporate all of these gauge groups in a single framework, we write the embedding tensor in the form

$$Y_{MN} = \text{diag}(1, 1, 1, 1, \rho) \tag{4.5}$$

with $\rho = 1, -1, 0$ corresponding to $SO(5)$, $SO(4,1)$, and $CSO(4,0,1)$ gauge groups, respectively.

There exists one $SO(4)$ singlet scalar ϕ corresponding to the non-compact generator

$$\hat{Y} = e_{11} + e_{22} + e_{33} + e_{44} - 4e_{55}\,. \tag{4.6}$$

The $SL(5)/SO(5)$ coset representative can be written as

$$\mathcal{V} = e^{\phi\hat{Y}}. \tag{4.7}$$

For this $SO(4)$ singlet scalar, the scalar potential is given by

$$\mathbf{V} = -\frac{g^2}{64}e^{-4\phi}(8 + 8\rho e^{10\phi} - \rho^2 e^{20\phi}). \tag{4.8}$$

This potential admits two AdS_7 critical points with $SO(5)$ and $SO(4)$ unbroken symmetries only for $\rho = 1$ corresponding to $SO(5)$ gauge group. These vacua and their cosmological constants are given respectively by

$$\phi = 0 \qquad \text{and} \qquad \mathbf{V}_0 = -\frac{15}{64}g^2 \tag{4.9}$$

and

$$\phi = \frac{1}{10}\ln 2 \qquad \text{and} \qquad \mathbf{V}_0 = -\frac{5g^2}{16 \times 2^{2/5}}\,. \tag{4.10}$$

According to the previous studied [33], the former preserves all SUSY while the latter is non-supersymmetric and unstable.

To setup BPS equations, we impose

$$\gamma^{\hat{6}}\epsilon^a = \epsilon^a, \tag{4.11}$$

and obtain the following BPS equations from $\delta\psi^a_m = 0$ and $\delta\chi^{abc} = 0$ conditions

$$U' = \frac{g}{40}e^{-2\phi}(4 + \rho e^{10\phi}), \tag{4.12}$$

$$\phi' = \frac{g}{20}e^{-2\phi}(1 - \rho e^{10\phi}). \tag{4.13}$$

The condition $\delta\psi^a_r = 0$ gives the usual solution for the Killing spinors

$$\epsilon^a = e^{\frac{U}{2}}\epsilon^a_0 \tag{4.14}$$

with ϵ^a_0 being constant SM spinors satisfying $\gamma^{\hat{6}}\epsilon^a_0 = \epsilon^a_0$. The solution is then half-supersymmetric.

With the new radial coordinate $\tilde{r}$ defined by $\frac{d\tilde{r}}{dr} = e^{3\phi}$, the above BPS equations can be readily solved with the solution

$$U = 2\phi - \frac{1}{4}\ln(1-\rho e^{10\phi}), \tag{4.15}$$

$$e^{5\phi} = \frac{1}{\sqrt{\rho}}\tanh\left[\frac{\sqrt{\rho}}{4}(g\tilde{r}+C)\right]. \tag{4.16}$$

The integration constant C can be eliminated by shifting the radial coordinate $\tilde{r}$. We have also neglected an additive integration constant for U since it can be absorbed by rescaling the flat coordinates x^m.

Note that for $\rho = -1$, the solution for ϕ can be written as

$$e^{5\phi} = \tan\left[\frac{1}{4}(g\tilde{r}+C)\right]. \tag{4.17}$$

For $\rho = 0$, we find

$$e^{5\phi} = \frac{1}{4}(g\tilde{r}+C). \tag{4.18}$$

4.1.1.2 $SO(3)\times SO(2)$ Symmetric Domain Walls

We now consider $SO(3)\times SO(2)$ symmetric case, which is possible only for $SO(5)$ and $SO(3,2)$ gauge groups. The embedding tensor, in this case, is written as

$$Y_{MN} = \text{diag}(1,1,1,\sigma,\sigma) \tag{4.19}$$

with $\sigma = 1$ and $\sigma = -1$ corresponding to $SO(5)$ and $SO(3,2)$, respectively.

There again exists only one $SO(3)\times SO(2)$ singlet scalar corresponding to the non-compact generator

$$\tilde{Y} = 2e_{11} + 2e_{22} + 2e_{33} - 3e_{44} - 3e_{55}\,. \tag{4.20}$$

With the coset representative

$$\mathcal{V} = e^{\phi\tilde{Y}}, \tag{4.21}$$

we find the scalar potential admitting an AdS_7 critical point at $\phi = 0$ for $\sigma = 1$

$$\mathbf{V} = -\frac{3}{64}g^2 e^{-8\phi}(1 + 4\sigma e^{10\phi}). \tag{4.22}$$

The BPS equations are given by

$$\phi' = -\frac{1}{20} g e^{-4\phi}(\sigma e^{10\phi} - 1), \tag{4.23}$$

$$U' = \frac{1}{40} g e^{-4\phi}(3 + 2\sigma e^{10\phi}). \tag{4.24}$$

Defining a new radial coordinate $\tilde{r}$ by the relation $\frac{d\tilde{r}}{dr} = e^{\phi}$, we obtain the solution very similar to the previous $SO(4)$ case

$$U = \frac{3}{2}\phi - \frac{1}{4}\ln(1 - \sigma e^{10\phi}), \tag{4.25}$$

$$e^{5\phi} = \frac{1}{\sqrt{\sigma}} \tanh\left[\frac{\sqrt{\sigma}}{4}(g\tilde{r} + C)\right]. \tag{4.26}$$

4.1.1.3 $SO(3)$ Symmetric Domain Walls

We then find more interesting solutions when the residual symmetry of the solutions is smaller. Supersymmetric DW solutions with $SO(3)$ symmetry are considered in this case. There are many gauge groups containing an $SO(3)$ subgroup corresponding to the embedding tensor given by

$$Y_{MN} = \text{diag}(1, 1, 1, \sigma, \rho). \tag{4.27}$$

There are three scalar fields invariant under $SO(3)$ symmetry generated by gauge generators X_{MN}, $M, N = 1, 2, 3$. These singlets correspond to the following non-compact generators of $SL(5)$

$$\hat{Y}_1 = 2e_{1,1} + 2e_{2,2} + 2e_{3,3} - 3e_{4,4} - 3e_{5,5},$$

$$\hat{Y}_2 = e_{4,5} + e_{5,4},$$

$$\hat{Y}_3 = e_{4,4} - e_{5,5}\,. \tag{4.28}$$

Using the parametrization of the coset representative

$$\mathcal{V} = e^{\phi_1 \hat{Y}_1 + \phi_2 \hat{Y}_2 + \phi_3 \hat{Y}_3}, \tag{4.29}$$

we obtain the scalar potential

$$\begin{aligned}\mathbf{V} = & -\frac{g^2}{64}\Big[3e^{-8\phi_1}+6e^{2\phi_1}\left[(\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3+(\rho-\sigma)\sinh 2\phi_3\right] \\ & +\frac{1}{4}e^{12\phi_1}\Big[\rho^2+10\rho\sigma+\sigma^2-(3\rho^2-2\rho\sigma+3\sigma^2)\cosh 4\phi_3 \\ & -(\rho+\sigma)^2\cosh 4\phi_2(1+\cosh 4\phi_3)-4(\rho^2-\sigma^2)\cosh 2\phi_2\sinh 4\phi_3\Big]\Big]\,. \quad (4.30)\end{aligned}$$

This potential admits two AdS_7 critical point for $\rho = \sigma = 1$. The first one is at $\phi_1 = \phi_2 = \phi_3 = 0$ corresponding to the $N = 4$ supersymmetric AdS_7 with $SO(5)$ symmetry given in (4.9). Another critical point is given by

$$\phi_1 = \frac{1}{20}\ln 2, \qquad \phi_2 = \frac{1}{4}\ln 2, \qquad \phi_3 = 0, \qquad \mathbf{V}_0 = -\frac{5g^2}{16\times 2^{2/5}}\,. \qquad (4.31)$$

This is the same non-supersymmetric and unstable critical point given in (4.10) for which the residual symmetry is enlarged to $SO(4)$ due to $\phi_2 = 5\phi_1$.

Using the same procedure as in the previous cases, we find the following BPS equations

$$\begin{aligned}U' &= \frac{g}{40}e^{-4\phi_1}\left[3+e^{10\phi_1}\left[(\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3+(\rho-\sigma)\sinh 2\phi_3\right]\right], \quad (4.32)\\ \phi_1' &= \frac{g}{40}e^{-4\phi_1}\left[2-e^{10\phi_1}\left[(\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3-(\rho-\sigma)\sinh 2\phi_3\right]\right], \quad (4.33)\\ \phi_2' &= -\frac{g}{8}e^{6\phi_1}(\rho+\sigma)\sinh 2\phi_2\ \mathrm{sech}2\phi_3, \quad (4.34)\\ \phi_3' &= -\frac{g}{8}e^{6\phi_1}\left[(\rho-\sigma)\cosh 2\phi_3+(\rho+\sigma)\cosh 2\phi_2\sinh 2\phi_3\right]. \quad (4.35)\end{aligned}$$

Explicit solutions to these equations can be obtained when we examine various specific values of ρ and σ separately.

(1) Domain walls in $CSO(3, 0, 2)$ gauge group

We begin with the simplest case for $\rho = \sigma = 0$ corresponding to non-semisimple $CSO(3, 0, 2)$ gauge group. In this case, we find $\phi_2' = \phi_3' = 0$. Furthermore, it can be checked that $\frac{\partial \mathbf{V}}{\partial \phi_2} = \frac{\partial \mathbf{V}}{\partial \phi_3} = 0$ at $\phi_2 = \phi_3 = 0$. Therefore, the scalars ϕ_2 and ϕ_3 can be consistently truncated out. After setting $\phi_2 = \phi_3 = 0$, we find a DW solution

$$\phi_1 = \frac{1}{4}\ln\left[\frac{gr}{5}+C\right] \qquad \text{and} \qquad U = \frac{3}{8}\ln\left[\frac{gr}{5}+C\right]. \qquad (4.36)$$

(2) Domain walls in $CSO(4,0,1)$ and $CSO(3,1,1)$ gauge groups

For $\rho = 0$ and $\sigma \neq 0$, the gauge group is either $CSO(4,0,1)$ or $CSO(3,1,1)$ depending on the value of $\sigma = 1$ or $\sigma = -1$. With a new radial coordinate $\tilde{r}$ defined by $\frac{d\tilde{r}}{dr} = e^{6\phi_1}$, a DW solution to the BPS equations can be found

$$\phi_2 = \frac{1}{4}\ln\left[\frac{g^2\tilde{r}^2 + (C_2 - 8)^2}{g^2\tilde{r}^2 + C_2^2}\right], \tag{4.37}$$

$$\phi_3 = \frac{1}{4}\ln\left[\frac{e^{2\phi_2} - e^{4\phi_2 + C_3} + e^{C_3} + 1}{e^{2\phi_2} + e^{4\phi_2 + C_3} - e^{C_3} - 1}\right], \tag{4.38}$$

$$\phi_1 = \frac{1}{10}\ln\left[\frac{2(e^{C_1} - e^{4\phi_2 + C_1} - 1)}{\sigma\sqrt{(e^{4\phi_2} - 1)(1 + 2e^{C_3} + e^{2C_3} - e^{4\phi_2 + 2C_3})}}\right], \tag{4.39}$$

$$U = -\phi_1 - \ln(e^{4\phi_2} - 1) + \ln(e^{C_1} - e^{4\phi_2 + C_1} - 1). \tag{4.40}$$

(3) Domain walls in $SO(4,1)$ gauge group

In this case with $\sigma = -\rho = 1$, we find that ϕ_2 can be consistently truncated out in the same way as in the $CSO(3,0,2)$ case. With $\phi_2 = 0$ and the new radial coordinate $\tilde{r}$ defined by $\frac{d\tilde{r}}{dr} = e^{6\phi_1}$, we find a DW solution

$$e^{2\phi_3} = \tan\left[\frac{g\tilde{r}}{4} + C_3\right], \tag{4.41}$$

$$\phi_1 = -\frac{1}{5}\phi_3 + \frac{1}{10}\ln\left[C_1(1 + e^{4\phi_3}) - 1\right], \tag{4.42}$$

$$U = \frac{1}{5}\phi_3 - \frac{1}{4}\ln(1 + e^{4\phi_3}) + \frac{3}{20}\ln\left[C_1(1 + e^{4\phi_3}) - 1\right]. \tag{4.43}$$

(4) Domain walls in $SO(5)$ and $CSO(3,2)$ gauge groups

We now look at the last possibility $\rho = \sigma = \pm 1$ corresponding to $SO(5)$ and $SO(3,2)$ gauge groups. In term of the new radial coordinate $\tilde{r}$ as defined in the previous cases, we obtain a DW solution

$$\phi_2 = \frac{1}{4}\ln\left[\frac{1 + e^{g\sigma\tilde{r}} + 4e^{g\sigma\tilde{r} + 2C_3} - 2e^{\frac{1}{2}g\sigma\tilde{r}}}{1 + e^{g\sigma\tilde{r}} + 4e^{g\sigma\tilde{r} + 2C_3} + 2e^{\frac{1}{2}g\sigma\tilde{r}}}\right], \tag{4.44}$$

$$\phi_3 = \frac{1}{4}\ln\left[\frac{e^{2\phi_2} + e^{4\phi_2 + C_3} - e^{C_3}}{e^{2\phi_2} - e^{4\phi_2 + C_3} + e^{C_3}}\right], \tag{4.45}$$

$$\phi_1 = \frac{1}{10}\ln\left[\sigma\left[1 + C_1(e^{4\phi_2} - 1)\right]\sqrt{e^{8\phi_2 + 2C_3} + e^{2C_3} - e^{4\phi_2} - 2e^{4\phi_2 + 2C_3}}\right], \tag{4.46}$$

$$U = -\phi_1 + \frac{1}{4}\ln(e^{4\phi_2} - 1) - \frac{1}{4}\ln[1 + C_1(e^{4\phi_2} - 1)]. \tag{4.47}$$

4.1.1.4 $SO(2) \times SO(2)$ Symmetric Domain Walls

We finally consider supersymmetric DWs with $SO(2)\times SO(2)$ symmetry generated by gauge generators X_{12} and X_{34}. There are two $SO(2) \times SO(2)$ invariant scalars corresponding to the non-compact generators

$$\tilde{Y}_1 = e_{11} + e_{22} - 2e_{55} \quad \text{and} \quad \tilde{Y}_2 = e_{33} + e_{44} - 2e_{55}\,. \tag{4.48}$$

In this case, the embedding tensor takes the form of

$$Y_{MN} = \text{diag}(1, 1, \sigma, \sigma, \rho) \tag{4.49}$$

which give rise to various gauge groups with an $SO(2)\times SO(2)$ subgroup i.e. $SO(5)$ ($\sigma = \rho = 1$), $SO(4, 1)$ ($\sigma = -\rho = 1$), $SO(3, 2)$ ($\sigma = -\rho = -1$), $CSO(4, 0, 1)$ ($\sigma = 1, \rho = 0$), and $CSO(2, 2, 1)$ ($\sigma = -1, \rho = 0$).

With the parametrization of the coset representative

$$\mathcal{V} = e^{\phi_1\tilde{Y}_1+\phi_2\tilde{Y}_2}, \tag{4.50}$$

we find the scalar potential

$$\mathbf{V} = -\frac{1}{64}g^2e^{-2(\phi_1+\phi_2)}\left[8\sigma - \rho^2e^{10(\phi_1+\phi_2)} + 4\rho(e^{4\phi_1+6\phi_2} + \sigma e^{6\phi_1+4\phi_2})\right]. \tag{4.51}$$

Only for $SO(5)$ gauge group, there are $N = 4$ supersymmetric and non-supersymmetric AdS_7 critical points given in (4.9) and (4.10) at $\phi_1 = \phi_2 = 0$ and $\phi_1 = \phi_2 = \frac{1}{10}\ln 2$, respectively.

The BPS equations, in this case, read

$$U' = \frac{g}{40}\left[2e^{-2\phi_1} + 2\sigma e^{-2\phi_2} + \rho e^{4(\phi_1+\phi_2)}\right], \tag{4.52}$$

$$\phi_1' = \frac{g}{20}\left[3e^{-2\phi_1} - \rho e^{4(\phi_1+\phi_2)} - 2\sigma e^{-2\phi_2}\right], \tag{4.53}$$

$$\phi_2' = \frac{g}{20}\left[3\sigma e^{-2\phi_2} - 2e^{-2\phi_1} - \rho e^{4(\phi_1+\phi_2)}\right]. \tag{4.54}$$

Defining a new radial coordinate $\tilde{r}$ by $\frac{d\tilde{r}}{dr} = e^{-2\phi_1}$, we find a DW solution

$$\phi_2 = -\frac{3}{2}\phi_1 - \frac{1}{4}\ln\left[\rho - \rho e^{C_2-\frac{g\tilde{r}}{2}}\right], \tag{4.55}$$

$$\phi_1 = -\frac{1}{10}\ln\left[\rho - \rho e^{C_1-\frac{g\tilde{r}}{2}}\right] - \frac{1}{5}\ln\left[\sigma - \sigma e^{C_2-\frac{g\tilde{r}}{2}}\right], \tag{4.56}$$

$$U = \frac{g\tilde{r}}{8} + \frac{1}{10}\ln\left[1 - e^{C_1-\frac{g\tilde{r}}{2}}\right] + \frac{1}{20}\ln\left[1 - e^{C_2-\frac{g\tilde{r}}{2}}\right]. \tag{4.57}$$

4.1.1.5 Uplift to Eleven Dimensions and Holographic RG Flows

For the $SO(5)$ gauge group, supersymmetric DW solutions obtained previously are asymptotic to the $N = 4$ supersymmetric AdS_7 vacuum dual to a six-dimensional $N = (2,0)$ SCFT with $SO(5)$ symmetry. According to the AdS/CFT duality, the solutions can be interpreted as holographic RG flows from the $N = (2,0)$ SCFT to six-dimensional SQFTs in the IR, see [42, 43] for examples. Furthermore, these DWs can be uplifted to be solutions of eleven-dimensional supergravity. We will consider these holographic RG flows together with their uplift to eleven dimensions in this section.

4.1.1.5.1 RG Flow Preserving $SO(4)$ Symmetry

We first consider a simple $SO(4)$ symmetric solution. With $\frac{d\tilde{r}}{dr} = e^{3\phi}$, the DW solution for $SO(5)$ gauge group reads

$$\phi = \frac{1}{5}\ln\left[\frac{1-e^{\frac{1}{2}(C-g\tilde{r})}}{1+e^{\frac{1}{2}(C-g\tilde{r})}}\right], \tag{4.58}$$

$$U = 2\phi - \frac{1}{4}\ln\left(1-e^{10\phi}\right). \tag{4.59}$$

As $\tilde{r} \to \infty$, we find $\phi \to 0$ and $\tilde{r} \sim r$ with an asymptotic behavior

$$\phi \sim e^{-\frac{1}{2}gr} \sim e^{-\frac{4r}{L}} \quad \text{and} \quad U \sim \frac{1}{8}gr \sim \frac{r}{L_{AdS_7}}, \quad L_{AdS_7} = \frac{8}{g} \tag{4.60}$$

which indicates that the solution is asymptotic to the $N = 4$ supersymmetric AdS_7 critical point. As $g\tilde{r} \to C$, the solution is singular with the following behavior

$$\phi \sim \frac{1}{5}\ln(g\tilde{r} - C) \quad \text{and} \quad U \sim 2\phi \sim \frac{2}{5}\ln(g\tilde{r} - C). \tag{4.61}$$

We can verify that the scalar potential is bounded above with $\mathbf{V} \to -\infty$ as $\phi \to -\infty$. According to the criterion of [85], this singularity is physically acceptable.

Moreover, we can use the truncation ansatz, reviewed in Appendix C.1, to uplift this solution to eleven dimensions. Using the $SL(5)/SO(5)$ coset

$$\mathcal{M}_{MN} = \text{diag}(e^{-8\phi}, e^{2\phi}, e^{2\phi}, e^{2\phi}, e^{2\phi}), \tag{4.62}$$

and the coordinates on S^4

$$\mu^M = (\mu^0, \mu^i) = (\cos\xi, \sin\xi\hat{\mu}^i), \qquad i = 1, 2, 3, 4 \tag{4.63}$$

with $\hat{\mu}^i$ being coordinates on S^3 satisfying $\hat{\mu}^i\hat{\mu}^i = 1$, we find the eleven-dimensional metric and four-form field strength tensor

$$\begin{aligned} d\hat{s}^2_{11} &= \Delta^{\frac{1}{3}}\left(e^{2U}dx^2_{1,5} + dr^2\right) + \frac{16}{g^2}\Delta^{-\frac{2}{3}}\left[e^{-8\phi}\sin^2\xi d\xi^2 \right. \\ &\quad \left. + e^{2\phi}(\cos^2\xi d\xi^2 + \sin^2\xi d\Omega^2_{(3)})\right], \qquad (4.64) \\ \hat{F}_{(4)} &= \frac{64}{g^3}\Delta^{-2}\sin^4\xi\left(\mathcal{U}\sin\xi d\xi - 10e^{6\phi}\phi'\cos\xi dr\right)\wedge\epsilon_{(3)} \qquad (4.65) \end{aligned}$$

with $d\Omega^2_{(3)}$ being the metric on a unit S^3 and

$$\begin{aligned} \Delta &= e^{8\phi}\cos^2\xi + e^{-2\phi}\sin^2\xi, \qquad \epsilon_{(3)} = \frac{1}{3!}\varepsilon_{ijkl}\hat{\mu}^i d\hat{\mu}^j\wedge d\hat{\mu}^k\wedge d\hat{\mu}^l, \\ \mathcal{U} &= (e^{16\phi} - 4e^{6\phi})\cos^2\xi - (e^{6\phi} + 2e^{-4\phi})\sin^2\xi\,. \end{aligned} \tag{4.66}$$

We see that the internal S^4 is deformed by leaving an S^3 inside the S^4 unchanged. The isometry of this S^3 is the $SO(4)$ residual symmetry of the seven-dimensional solution.

With this uplifted solution, we can examine the behavior of the metric component $\hat{g}_{00} = e^{2U}\Delta^{\frac{1}{3}}$ near the IR singularity. A straightforward computation gives

$$\hat{g}_{00} \sim e^{\frac{10}{3}\phi} \to 0 \tag{4.67}$$

which implies the singularity is physical according to the criterion given in [86]. This solution accordingly describes a holographic RG flow from the $N = (2, 0)$ SCFT with $SO(5)$ symmetry to a six-dimensional SQFT in the IR. With the presence of the normalizable mode in (4.60), this RG flow is induced by a vacuum expectation value of an operator of dimension $\Delta = 4$ breaking conformal symmetry and preserving only $SO(4) \subset SO(5)$ R-symmetry. Note that this RG flow has also been studied in [89] in the context of a consistent truncation to half-maximal $N = 2$ gauged supergravity.

4.1.1.5.2 RG Flow Preserving $SO(3) \times SO(2)$ Symmetry

With $\frac{d\tilde{r}}{dr} = e^{\phi}$, the flow solution with $SO(3) \times SO(2)$ symmetry reads

$$\phi = \frac{1}{5}\ln\left[\frac{1-e^{-\frac{1}{2}(g\tilde{r}-C)}}{1+e^{-\frac{1}{2}(g\tilde{r}-C)}}\right], \quad (4.68)$$

$$U = \frac{3}{2}\phi - \frac{1}{4}\ln(1-e^{10\phi}). \quad (4.69)$$

As $\tilde{r} \to \infty$, we find $\phi \to 0$ and $\tilde{r} \sim r$ with the same asymptotic behavior given in (4.60). As $g\tilde{r} \to C$, the solution becomes singular

$$\phi \sim \frac{1}{5}\ln(g\tilde{r}-C) \qquad \text{and} \qquad U \sim \frac{3}{2}\phi \sim \frac{3}{10}\ln(g\tilde{r}-C). \quad (4.70)$$

Near this singularity, we find that the scalar potential is bounded above, $\mathbf{V} \to -\infty$.

The uplifted solution can be obtained by using the S^4 coordinates

$$\mu^M = (\sin\xi\hat{\mu}^a, \cos\xi\cos\alpha, \cos\xi\sin\alpha), \qquad a = 1,2,3 \quad (4.71)$$

with $\hat{\mu}^a\hat{\mu}^a = 1$ and the scalar matrix

$$\mathcal{M}_{MN} = \text{diag}(e^{4\phi}, e^{4\phi}, e^{4\phi}, e^{-6\phi}, e^{-6\phi}). \quad (4.72)$$

We find the eleven-dimensional solution

$$d\hat{s}^2_{11} = \Delta^{\frac{1}{3}}\left(e^{2U}dx^2_{1,5} + dr^2\right) + \frac{16}{g^2}\left[e^{-6\phi}\cos^2\xi d\alpha^2 + (e^{4\phi}\cos^2\xi + e^{-6\phi}\sin^2\xi)d\xi^2 + e^{4\phi}\sin^2\xi d\hat{\mu}^a d\hat{\mu}^a\right], \quad (4.73)$$

$$\hat{F}_{(4)} = \frac{64}{3g^3}\mathcal{U}\sin^3\xi\cos\xi\Delta^{-2}\left(\sin\xi d\xi + 2e^{2\phi}\cos\xi\phi' dr\right) \wedge d\alpha \wedge \epsilon_{(2)} \quad (4.74)$$

where

$$\epsilon_{(2)} = \frac{1}{2}\varepsilon_{abc}\hat{\mu}^a d\hat{\mu}^b \wedge d\hat{\mu}^c. \quad (4.75)$$

We can see that the $SO(3) \times SO(2)$ unbroken symmetry corresponds to the isometry of the S^2 inside the S^4 and the isometry of the S^1 parametrized by the coordinate α.

From the eleven-dimensional metric, we find

$$\hat{g}_{00} \sim e^{\frac{5}{3}\phi} \to 0. \quad (4.76)$$

The singularity is accordingly physical [86], and the entire solution describes an RG flow from the $N = (2,0)$ SCFT to an SQFT with $SO(3) \times SO(2)$ symmetry in six dimensions.

4.1.1.5.3 RG Flow Preserving $SO(2)\times SO(2)$ Symmetry

With $\frac{d\tilde{r}}{dr}=e^{-2\phi_1}$, a DW solution preserving $SO(2)\times SO(2)$ symmetry in $SO(5)$ gauge group is given by

$$\begin{aligned}\phi_1 &= -\frac{1}{10}\ln(1-e^{C_1-\frac{g\tilde{r}}{2}})-\frac{1}{5}\ln(1-e^{C_2-\frac{g\tilde{r}}{2}}), &(4.77)\\ \phi_2 &= -\frac{3}{2}\phi_1-\frac{1}{4}\ln(1-e^{C_1-\frac{g\tilde{r}}{2}}), &(4.78)\\ U &= \frac{1}{8}g\tilde{r}+\frac{1}{20}\ln(1-e^{C_1-\frac{g\tilde{r}}{2}})+\frac{1}{10}\ln(1-e^{C_2-\frac{g\tilde{r}}{2}}). &(4.79)\end{aligned}$$

We can perform an uplift by using

$$\begin{aligned}\mu^M &= (\cos\xi,\sin\xi\cos\psi\cos\alpha,\sin\xi\cos\psi\sin\alpha,\sin\xi\sin\psi\cos\beta,\sin\xi\sin\psi\sin\beta),\\ \mathcal{M}_{MN} &= \text{diag}(e^{-4(\phi_1+\phi_2)},e^{2\phi_1},e^{2\phi_1},e^{2\phi_2},e^{2\phi_2}). \qquad (4.80)\end{aligned}$$

The corresponding eleven-dimensional metric is

$$\begin{aligned}d\hat{s}^2_{11} &= \Delta^{\frac{1}{3}}\left(e^{2U}dx^2_{1,5}+dr^2\right)+\frac{16}{g^2}\Delta^{-\frac{2}{3}}\Big[e^{2\phi_2}\left(\cos^2\xi\sin^2\psi d\xi^2\right.\\ &\quad\left.+\cos^2\psi\sin^2\xi d\psi^2+\sin^2\xi\sin^2\psi d\beta^2+2\cos\xi\cos\psi\sin\xi\sin\psi d\xi d\psi\right)\\ &\quad+e^{2\phi_1}\left(\cos^2\xi\cos^2\psi d\xi^2+\sin^2\xi\sin^2\psi d\psi^2+\sin^2\xi\cos^2\psi d\alpha^2\right.\\ &\quad\left.-2\cos\xi\cos\psi\sin\xi\sin\psi d\xi d\psi\right)+e^{-4(\phi_1+\phi_2)}\sin^2\xi d\xi^2\Big] \qquad (4.81)\end{aligned}$$

where

$$\Delta=e^{4(\phi_1+\phi_2)}\cos^2\xi+e^{-2\phi_1}\sin^2\xi\cos^2\psi+e^{-2\phi_2}\sin^2\xi\sin^2\psi\,. \qquad (4.82)$$

Here, we neglect an explicit form of the four-form field strength, which is much more complicated than the previous cases. The unbroken symmetry $SO(2)\times SO(2)$ corresponds to the isometry of $S^1\times S^1$ parametrized by coordinates α and β.

As $\tilde{r}\to\infty$, the solution becomes

$$\phi_1\sim\phi_2\sim e^{-\frac{4\tilde{r}}{L}}\quad\text{with}\quad\tilde{r}\sim r \qquad (4.83)$$

which again implies that ϕ_1 and ϕ_2 are dual to operators of dimension $\Delta=4$ in the dual $N=(2,0)$ SCFT.

For the IR behaviors, there are two possibilities. As $g\tilde{r}\to 2C_1$, we have

$$\phi_1\sim\phi_2\sim-\frac{1}{10}\ln(g\tilde{r}-2C_1)\quad\text{and}\quad U\sim-\frac{1}{2}\phi_1\sim\frac{1}{20}\ln(g\tilde{r}-2C_1). \qquad (4.84)$$

Near this singularity, the scalar potential is unbounded above, $\mathbf{V} \to \infty$, and the eleven-dimensional metric gives

$$\hat{g}_{00} \sim e^{\frac{5}{3}\phi_1} \to \infty\,. \tag{4.85}$$

This singularity is then unphysical.

As $g\tilde{r} \to 2C_2$, we have

$$\begin{aligned} \phi_1 &\sim -\frac{1}{5}\ln(g\tilde{r} - 2C_2), \quad \phi_2 \sim -\frac{3}{2}\phi_1 \sim \frac{3}{10}\ln(g\tilde{r} - 2C_2), \\ U &\sim -\frac{1}{2}\phi_1 \sim \frac{1}{10}\ln(g\tilde{r} - 2C_2). \end{aligned} \tag{4.86}$$

Near the singularity, we find $\mathbf{V} \to -\infty$ and

$$\hat{g}_{00} \sim \text{constant}\,. \tag{4.87}$$

In this case, the singularity is physical, and the DW solution describes an RG flow from the $N = (2, 0)$ SCFT to a six-dimensional SQFT in the IR with $SO(2) \times SO(2)$ symmetry.

4.1.1.5.4 RG Flow Preserving $SO(3)$ Symmetry

The solution is rather complicated in this case. Therefore, we will examine only a truncation of the full solution. By making a consistent truncation $\phi_3 = 0$, we find a simple solution to the truncated BPS equations

$$U = \frac{1}{5}\phi_2 - \frac{1}{4}\ln(1 - e^{4\phi_2}) + \frac{3}{20}\ln\left[1 + C_1(e^{4\phi_2} - 1)\right], \tag{4.88}$$

$$\phi_1 = -\frac{1}{5}\phi_2 + \frac{1}{10}\ln\left[1 + C_1(e^{4\phi_2} - 1)\right], \tag{4.89}$$

$$\phi_2 = \frac{1}{2}\ln\left[\frac{1 - e^{\frac{1}{2}(C - g\tilde{r})}}{1 + e^{\frac{1}{2}(C - g\tilde{r})}}\right] \tag{4.90}$$

in which the new radial coordinate $\tilde{r}$ is defined by $\frac{d\tilde{r}}{dr} = e^{6\phi_1}$.

Near the $N = 4$ supersymmetric AdS_7 critical point in the UV as $\tilde{r} \to \infty$, we find, as in the previous cases,

$$\phi_1 \sim \phi_2 \sim e^{-\frac{4\tilde{r}}{L}} \quad \text{with} \quad \tilde{r} \sim r, \tag{4.91}$$

and, near the IR singularity as $g\tilde{r} \to C$, the solution becomes singular

$$\begin{aligned} &\phi_2 \sim \frac{1}{2}\ln(g\tilde{r}-C), \qquad \phi_1 \sim -\frac{1}{5}\phi_2 \sim -\frac{1}{10}\ln(g\tilde{r}-C), \\ &U \sim \frac{1}{5}\phi_2 \sim \frac{1}{10}\ln(g\tilde{r}-C). \end{aligned} \tag{4.92}$$

In this case, $\mathbf{V} \to \infty$ near the singularity, and the (00)-component of the eleven-dimensional metric is

$$\hat{g}_{00} \sim e^{-\frac{2}{3}\phi_2} \to \infty\,. \tag{4.93}$$

The singularity is then unphysical. We will not give the corresponding eleven-dimensional solution in this case. Moreover, it can be checked that another truncation with $\phi_2 = 0$ also gives a similar result.

4.1.1.6 Uplifted Solutions to Type IIA Supergravity

We now consider the uplift to type IIA supergravity of the DW solutions in $CSO(4,0,1)$ gauge group. Relevant formulae, including the truncation ansatze and useful relations, are summarized in Appendix C.2. As gaugings in $\overline{\mathbf{40}}$ representation, we also decompose the $SL(5)/SO(5)$ coset $\mathcal{M}_{MN}$ into the $SL(4)/SO(4)$ submanifold $\widetilde{\mathcal{M}}_{ij}$ given in (2.94) in this case. Moreover, all axion scalars disappear, $b_i = \chi_i = 0$, in these solutions so that only the metric, dilaton, and three-form field strength in type IIA supergravity are non-vanishing. As expected for DWs in seven dimensions, the resulting solutions should describe Neveu Schwarz five-branes (NS5-branes) in the transverse space with different symmetries.

4.1.1.6.1 Solution with $SO(4)$ Symmetry

In this case, we simply have $\widetilde{\mathcal{M}}_{ij} = \delta_{ij}$ and

$$\begin{aligned} d\hat{s}^2_{10} &= e^{\frac{3}{2}\phi_0}\left(e^{2U}dx^2_{1,5} + dr^2\right) + \frac{16}{g^2}e^{-\frac{5}{2}\phi_0}d\Omega^2_{(3)}, \\ \hat{F}_{(3)} &= \frac{128}{g^3}\epsilon_{(3)}, \qquad \hat{\varphi} = 5\phi_0\,. \end{aligned} \tag{4.94}$$

Solving the BPS equations in (4.12) and (4.13) by renaming ϕ to ϕ_0 and setting $\rho = 0$, we find the following solution

$$\phi_0 = \frac{1}{2}\ln\left[\frac{gr}{10} + C\right] \qquad \text{and} \qquad U = \ln\left[\frac{gr}{10} + C\right]. \tag{4.95}$$

We identify the resulting ten-dimensional solution with the "near horizon" geometry of NS5-branes in the transverse space $\mathbb{R}^4$.

4.1.1.6.2 Solution with $SO(3)$ Symmetry

With $\rho = 0$ and $\sigma = 1$, the following $SO(3)$ symmetric solution can be obtained from the BPS equations (4.32) to (4.35) by setting $\phi_2 = 0$

$$U = \frac{1}{5}\phi_3 + \frac{3}{20}\ln(C_1 + e^{4\phi_3}), \tag{4.96}$$

$$\phi_1 = -\frac{1}{5}\phi_3 + \frac{1}{10}\ln(C_1 + e^{4\phi_3}), \tag{4.97}$$

$$2gC^{\frac{3}{5}}r = 5e^{\frac{16}{5}\phi_3}\ {}_2F_1\left(\frac{3}{5}, \frac{4}{5}, \frac{9}{5}, -\frac{e^{4\phi_3}}{C_1}\right). \tag{4.98}$$

In this solution, ${}_2F_1$ is the hypergeometric function.

To uplift the solution, we parametrize the $SL(4)/SO(4)$ coset by

$$\widetilde{\mathcal{M}}_{ij} = \text{diag}(e^{2\phi}, e^{2\phi}, e^{2\phi}, e^{-6\phi}). \tag{4.99}$$

In this case, the dilaton ϕ_0 and the $SO(3)$ singlet ϕ are related to ϕ_1 and ϕ_3 by

$$\phi_0 = -\frac{1}{4}(3\phi_1 + \phi_3) \qquad \text{and} \qquad \phi = \frac{1}{4}(5\phi_1 - \phi_3). \tag{4.100}$$

Choosing a specific form of the S^3 coordinates to be

$$\mu^i = (\sin\xi\hat{\mu}^a, \cos\xi), \qquad a = 1, 2, 3 \tag{4.101}$$

with $\hat{\mu}^a$ being the coordinates on S^2 subject to the condition $\hat{\mu}^a\hat{\mu}^a = 1$, we find the following ten-dimensional fields

$$\begin{aligned} d\hat{s}^2_{10} &= \frac{16}{g^2}e^{-\frac{5}{2}\phi_0}\Delta^{-\frac{3}{4}}\left[\left(e^{-6\phi}\sin^2\xi + e^{2\phi}\cos^2\xi\right)d\xi^2 \sin^2\xi e^{2\phi}d\hat{\mu}^a d\hat{\mu}^a\right] \\ &\quad + e^{\frac{3}{2}\phi_0}\Delta^{\frac{1}{4}}\left(e^{2U}dx^2_{1,5} + dr^2\right), \qquad e^{2\hat{\varphi}} = \Delta^{-1}e^{10\phi_0}, \\ \hat{F}_{(3)} &= \frac{64}{g^3}\Delta^{-2}\sin^3\xi\left(\mathcal{U}\sin\xi d\xi + 8e^{4\phi}\cos\xi\phi' dr\right)\wedge\epsilon_{(2)} \end{aligned} \tag{4.102}$$

in which

$$\Delta = e^{6\phi}\cos^2\xi + e^{-2\phi}\sin^2\xi, \qquad \epsilon_{(2)} = \frac{1}{2}\varepsilon_{abc}\hat{\mu}^a d\hat{\mu}^b \wedge d\hat{\mu}^c,$$
$$\mathcal{U} = e^{12\phi}\cos^2\xi - e^{-4\phi}\sin^2\xi - e^{4\phi}(\sin^2\xi + 3\cos^2\xi). \tag{4.103}$$

The residual symmetry $SO(3)$ corresponds to the isometry of $S^2 \subset S^3$.

4.1.1.6.3 Solution with $SO(2) \times SO(2)$ Symmetry

Setting $\sigma = 1$ and $\rho = 0$ and using $\frac{d\tilde{r}}{dr} = e^{-2\phi_2}$, we find the following $SO(2)\times SO(2)$ symmetric DW from the BPS equations given in Section 4.1.1.4

$$U = \frac{1}{20}g\tilde{r} + \frac{1}{10}\ln(C_1 + e^{\frac{1}{2}g\tilde{r}}), \tag{4.104}$$
$$\phi_1 = C_2 - \frac{1}{10}g\tilde{r} + \frac{3}{10}\ln(C_1 + e^{\frac{1}{2}g\tilde{r}}), \tag{4.105}$$
$$\phi_2 = C_2 + \frac{3}{20}g\tilde{r} - \frac{1}{5}\ln(C_1 + e^{\frac{1}{2}g\tilde{r}}). \tag{4.106}$$

To uplift the solution, we use the following parametrization of $SL(4)/SO(4)$ coset

$$\widetilde{\mathcal{M}}_{ij} = \text{diag}(e^{2\phi}, e^{2\phi}, e^{-2\phi}, e^{-2\phi}) \tag{4.107}$$

where ϕ_0 and ϕ are associated to ϕ_1 and ϕ_2 through the relations

$$\phi_0 = -\frac{1}{2}(\phi_1 + \phi_2) \qquad \text{and} \qquad \phi = \frac{1}{2}(\phi_1 - \phi_2). \tag{4.108}$$

Choosing the coordinates on S^3 to be

$$\mu^i = (\cos\xi\cos\alpha, \cos\xi\sin\alpha, \sin\xi\cos\beta, \sin\xi\sin\beta), \tag{4.109}$$

we find

$$d\hat{s}^2_{10} = \Delta^{\frac{1}{4}}e^{\frac{3}{2}\phi_0}\left(e^{2U}dx^2_{1,5} + dr^2\right) + \frac{16}{g^2}\Delta^{-\frac{3}{4}}e^{-\frac{5}{2}\phi_0}\big[(e^{2\phi}\sin^2\xi + e^{-2\phi}\cos^2\xi)d\xi^2$$
$$+e^{2\phi}\cos^2\xi d\alpha^2 + e^{-2\phi}\sin^2\xi d\beta^2\big],$$
$$e^{2\hat{\varphi}} = \Delta^{-1}e^{10\phi_0}, \qquad \hat{F}_{(3)} = \frac{128}{g^3}\Delta^{-2}\cos\xi\sin\xi d\alpha\wedge d\xi\wedge d\beta \tag{4.110}$$

with

$$\Delta = e^{-2\phi}\cos^2\xi + e^{2\phi}\sin^2\xi\,. \tag{4.111}$$

In this case, the unbroken $SO(2)\times SO(2)$ symmetry corresponds to the isometry of $S^1 \times S^1$ parametrized by coordinates α and β.

4.1.2 Gaugings in $\overline{\mathbf{40}}$ Representation

In this section, we repeat the same analysis for $CSO(p,q,4-p-q)$ gauge groups obtained from gaugings in $\overline{\mathbf{40}}$ representation. As in Section 2.2.2.2, we decompose the $SL(5)/SO(5)$ coset in term of the $SL(4)/SO(4)$ submanifold given in (2.94). After setting $Y_{MN}=0$ and using the inverse matrix $\mathcal{M}^{MN}$ of the form

$$\mathcal{M}^{MN} = \begin{pmatrix} e^{2\phi_0}\widetilde{\mathcal{M}}^{ij} & -e^{2\phi_0}b^i \\ -e^{2\phi_0}b^j & e^{-8\phi_0}+e^{2\phi_0}b_k b^k \end{pmatrix} \tag{4.112}$$

with $\widetilde{\mathcal{M}}^{ij}$ being the inverse of $\widetilde{\mathcal{M}}_{ij}$ and $b^i = \widetilde{\mathcal{M}}^{ij}b_j$, we can rewrite the scalar Lagrangian as

$$e^{-1}\mathcal{L}_{\text{scalar}} = -8\partial_\mu\phi_0\partial^\mu\phi_0 + \frac{1}{8}\partial_\mu\widetilde{\mathcal{M}}_{ij}\partial^\mu\widetilde{\mathcal{M}}^{ij} - \frac{1}{4}e^{10\phi_0}\widetilde{\mathcal{M}}^{ij}\partial_\mu b_i\partial^\mu b_j - \mathbf{V} \tag{4.113}$$

in which the scalar potential is given in (2.95). Note here that the nilpotent scalars b_i appear quadratically in this Lagrangian so we can consistently truncate them out throughout this section.

4.1.2.1 $SO(4)$ Symmetric Domain Walls

We first consider DW solutions with the largest residual symmetry, $SO(4)\subset CSO(p,q,4-p-q)$. Only $SO(4)$ gauge group contains an $SO(4)$ subgroup. In this case, there are no $SO(4)$ invariant scalars from the $SL(4)/SO(4)$ submanifold, while the embedding tensor is simply $w^{ij}=\delta^{ij}$. Taking the $SL(4)/SO(4)$ coset representative to be $\widetilde{\mathcal{V}}=\mathbf{1}_4$, we find the scalar potential in a particularly simple form

$$\mathbf{V} = -2g^2 e^{4\phi_0}\,. \tag{4.114}$$

To setup BPS equations, we use the same Killing spinors (4.14). However, the appropriate projector for this type of gaugings is different from that in the previous cases in **15** representation and given by

$$(\Gamma_5)^a{}_b\epsilon_0^b = -\gamma^{\hat{6}}\epsilon_0^a\,. \tag{4.115}$$

Note that the appearance of Γ^5 rather than other Γ^A with $A = 1, 2, 3, 4$ in this projection is due to the specific choice of $v^M = \delta^M_5$ in the embedding tensor $Z^{MN,P}$.

It is now straightforward to derive the corresponding BPS equations

$$U' = \frac{2g}{5} e^{-2\phi_0}, \qquad \phi_0' = \frac{g}{5} e^{-2\phi_0}. \tag{4.116}$$

We can readily find the DW solution

$$\phi_0 = \frac{1}{2} \ln \left[\frac{2gr}{5} + C \right], \qquad U = \ln \left[\frac{2gr}{5} + C \right]. \tag{4.117}$$

4.1.2.2 $SO(3)$ Symmetric Domain Walls

In this section, we look for more complicated solutions preserving $SO(3)$ unbroken symmetry generated by X_{ij} with $i, j = 1, 2, 3$. Gauge groups containing an $SO(3)$ subgroup are $SO(4)$, $SO(3, 1)$, and $CSO(3, 0, 1)$ corresponding to

$$w^{ij} = \text{diag}(1, 1, 1, \rho) \tag{4.118}$$

with $\rho = 1, -1, 0$, respectively.

For simplicity, we truncate axions b_i out and consider only ϕ_0 and scalars parameterizing the $SL(4)/SO(4)$ coset. With an explicit form of the $SL(4)/SO(4)$ coset representative

$$\widetilde{\mathcal{V}} = \text{diag}(e^{\phi}, e^{\phi}, e^{\phi}, e^{-3\phi}), \tag{4.119}$$

we obtain the scalar potential

$$\mathbf{V} = -\frac{g^2}{4} e^{-4(\phi_0 + 3\phi)} (3e^{16\phi} + 6\rho e^{8\phi} + \rho^2). \tag{4.120}$$

Using the projector in (4.115), we can derive the following set of BPS equations

$$\begin{aligned} U' &= \frac{g}{10} e^{-2(\phi_0+3\phi)} (3e^{8\phi} + \rho), & (4.121)\\ \phi_0' &= \frac{g}{20} e^{-2(\phi_0+3\phi)} (3e^{8\phi} + \rho), & (4.122)\\ \phi' &= -\frac{g}{4} e^{-2(\phi_0+3\phi)} (e^{8\phi} - \rho). & (4.123) \end{aligned}$$

The solutions for U and ϕ_0 are given by

$$\begin{aligned} U &= \frac{2}{5}\phi - \frac{1}{5} \ln(e^{8\phi} - \rho), & (4.124)\\ \phi_0 &= \frac{1}{5}\phi - \frac{1}{10} \ln(e^{8\phi} - \rho) + C_0\,. & (4.125) \end{aligned}$$

The solution for $\phi(r)$ is given by

$$\phi = -\frac{5}{16}\ln\left[\frac{2}{5}(e^{-2C_0}gr - C)\right] \tag{4.126}$$

for $\rho = 0$ and

$$4gr\rho(e^{8\phi}-\rho)^{\frac{1}{5}} = 5e^{2C+\frac{32}{5}\phi}\left[4 - 3\left(1-\rho e^{8\phi}\right)^{\frac{1}{5}}\ {}_2F_1\left(\frac{1}{5},\frac{4}{5},\frac{9}{5},\rho e^{8\phi}\right)\right] \tag{4.127}$$

for $\rho = \pm 1$.

4.1.2.3 $SO(2)\times SO(2)$ Symmetric Domain Walls

We now consider DW solutions with $SO(2)\times SO(2)$ symmetry in $SO(4)$ and $SO(2,2)$ gauge groups. These gauge groups are altogether characterized by the component of the embedding tensor in the form of

$$w^{ij} = \text{diag}(1,1,\sigma,\sigma), \qquad \sigma = \pm 1\,. \tag{4.128}$$

With the parametrization for the $SL(4)/SO(4)$ coset representative

$$\widetilde{\mathcal{V}} = \text{diag}(e^{\phi}, e^{\phi}, e^{-\phi}, e^{-\phi}), \tag{4.129}$$

the scalar potential and the BPS equations are given by

$$\mathbf{V} = -2g^2\sigma e^{-4\phi_0} \tag{4.130}$$

and

$$\begin{aligned} U' &= \frac{1}{5}ge^{-2\phi_0-2\phi}(e^{4\phi}+\sigma), &(4.131)\\ \phi_0' &= \frac{1}{10}ge^{-2\phi_0-2\phi}(e^{4\phi}+\sigma), &(4.132)\\ \phi' &= \frac{1}{2}ge^{-2\phi_0-2\phi}(e^{4\phi}-\sigma). &(4.133)\end{aligned}$$

The DW solution can be straightforwardly obtained

$$\begin{aligned} U &= 2\phi_0, &(4.134)\\ \phi_0 &= \frac{1}{5}\phi - \frac{1}{10}\ln(e^{4\phi}-\sigma) + C_0, &(4.135)\\ 6gr\sigma(e^{4\phi}-\sigma)^{\frac{1}{5}} &= 5e^{2C_0+\frac{12}{5}\phi}\left[3-2\left(1-\sigma e^{4\phi}\right)^{\frac{1}{5}}\ {}_2F_1\left(\frac{1}{5},\frac{3}{5},\frac{8}{5},\sigma e^{4\phi}\right)\right]. &(4.136)\end{aligned}$$

4.1.2.4 $SO(2)$ Symmetric Domain Walls

We examine $SO(2)$ symmetric solutions as a final example for DW solutions from gaugings in $\overline{\mathbf{40}}$ representation. Again, we truncate axions b_i out and parametrize the $SL(4)/SO(4)$ coset representative as

$$\widetilde{\mathcal{V}} = e^{\phi_1 Y_1 + \phi_2 Y_2 + \phi_3 Y_3} \tag{4.137}$$

in which Y_i, $i = 1, 2, 3$, are non-compact generators commuting with the $SO(2)$ symmetry generated by X_{12}. The explicit form of these generators is given by

$$Y_1 = e_{11} + e_{22} - e_{33} - e_{44}, \qquad Y_2 = e_{34} + e_{43}, \qquad Y_3 = e_{33} - e_{44}\,. \tag{4.138}$$

Several gauge groups containing an $SO(2)$ subgroup are uniformly characterized by the following component of the embedding tensor

$$w^{ij} = \text{diag}(1, 1, \sigma, \rho). \tag{4.139}$$

The scalar potential is computed to be

$$\begin{aligned} \mathbf{V} \;=\; & -\frac{g^2}{16} e^{-4(\phi_0+\phi_1+\phi_3)} \left[8e^{4\phi_1+2\phi_3} \left[\rho - \sigma + (\rho+\sigma)\cosh 2\phi_2\right] \right. \\ & - \left[\rho - \sigma + (\rho+\sigma)\cosh 2\phi_2\right]^2 - 8e^{4\phi_1+6\phi_3}\left[\rho - \sigma - (\rho+\sigma)\cosh 2\phi_2\right] \\ & + e^{4\phi_3}\left[\rho^2 + 10\rho\sigma + \sigma^2 - (\rho+\sigma)^2\cosh 4\phi_2\right] \\ & \left. + e^{8\phi_3}\left[\rho - \sigma - (\rho+\sigma)\cosh 2\phi_2\right]^2 \right]. \end{aligned} \tag{4.140}$$

It should be noted that the scalar potential with $\sigma = \rho = 0$ vanish identically. This leads to a Minkowski vacuum for $CSO(2, 0, 2)$ gauge group.

In this case, the corresponding BPS equations are much more complicated than those obtained in the previous cases

$$U' = \frac{1}{10} g e^{-2(\phi_0+\phi_1)} \left[2e^{4\phi_1} - (\rho-\sigma)\sinh 2\phi_3 + (\rho+\sigma)\cosh 2\phi_3 \cosh 2\phi_2\right], \tag{4.141}$$

$$\phi_0' = \frac{1}{20} g e^{-2(\phi_0+\phi_1)} \left[2e^{4\phi_1} - (\rho-\sigma)\sinh 2\phi_3 + (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3\right], \tag{4.142}$$

$$\phi_1' = -\frac{1}{4} g e^{-2(\phi_0+\phi_1)} \left[2e^{4\phi_1} + (\rho-\sigma)\sinh 2\phi_3 - (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3\right], \tag{4.143}$$

$$\phi_2' = -\frac{1}{2} g e^{-2(\phi_0+\phi_1)} (\rho+\sigma)\sinh 2\phi_2 \ \text{sech}\ 2\phi_3, \tag{4.144}$$

$$\phi_3' = \frac{1}{2} g e^{-2(\phi_0+\phi_1)} \left[(\rho-\sigma)\cosh 2\phi_3 - (\rho+\sigma)\cosh 2\phi_2 \sinh 2\phi_3\right]. \tag{4.145}$$

We are unable to solve these equations completely for arbitrary values of the parameters ρ and σ. However, the solutions can be separately found for specific values of ρ and σ.

(1) Domain walls from $CSO(2,0,2)$ gauge group

The simplest case is $CSO(2,0,2)$ gauge group with $\rho=\sigma=0$. In this case, $\phi_2'=\phi_3'=0$ and the remaining BPS equations considerably simplify

$$U'=\frac{1}{5}ge^{-2\phi_0+\phi_1},\qquad \phi_0'=\frac{1}{10}ge^{-2\phi_0+\phi_1},\qquad \phi_1'=-\frac{1}{2}ge^{-2\phi_0+\phi_1}\,. \quad (4.146)$$

Here, scalars ϕ_2 and ϕ_3 can be consistently truncated out. Thus, the solution for the remaining fields can be readily found

$$U=-\frac{1}{5}\phi_1,\quad \phi_0=-\frac{1}{5}\phi_1+C_0,\quad \phi_1=-\frac{5}{12}\ln\left[\frac{6}{5}(e^{-2C_0}gr-C)\right]. \quad (4.147)$$

(2) Domain walls from $SO(3,1)$ gauge group

In the case with $\sigma=-\rho=1$, the BPS equations give $\phi_2'=0$. Similar to the previous case, ϕ_2 does not appear in any BPS equations. After truncating out ϕ_2 and redefining r to $\tilde{r}$ by $\frac{d\tilde{r}}{dr}=e^{-2\phi_0-2\phi_1}$, we find a DW solution

$$\phi_1 = \frac{1}{2}\phi_3-\frac{1}{4}\ln\left[1+C_1(1+e^{4\phi_3})\right], \quad (4.148)$$

$$\phi_0 = C_0+\frac{1}{10}\phi_3-\frac{1}{10}\ln(1+e^{4\phi_3})+\frac{1}{20}\ln\left[1+C_1(1+e^{4\phi_3})\right], \quad (4.149)$$

$$\phi_3 = \frac{1}{2}\ln\tan(C_3-g\tilde{r}), \quad (4.150)$$

$$U = 2\phi_0. \quad (4.151)$$

(3) Domain walls from $CSO(3,0,1)$ and $CSO(2,1,1)$ gauge groups

In this case, we set $\rho=0$ and $\sigma=\pm1$ corresponding $CSO(3,0,1)$ and $CSO(2,1,1)$ gauge groups. All scalar fields are now non-vanishing and the DW solution is given by

$$U=2\phi_0, \quad (4.152)$$

$$\phi_0=\frac{1}{20}\ln\left[\frac{1}{4}g\tilde{r}\left(C_0-g^2\tilde{r}^2e^{4C_1}-4e^{4C_1+C_3}g^2\tilde{r}^2-4e^{4C_1+2C_3}g^2\tilde{r}^2\right)\right], \quad (4.153)$$

$$\phi_1 = C_1 - 5\phi_0 - \frac{1}{4}\ln(1 - e^{4\phi_2}) + \frac{1}{4}\ln(1 + 2e^{C_3} + e^{2C_3} - e^{2C_3+4\phi_2}), \quad (4.154)$$

$$\phi_2 = \frac{1}{4}\ln\left[\frac{4(1+e^{C_3})^2 + (1+2e^{C_3})^2 g^2\tilde{r}^2}{4e^{2C_3} + (1+2e^{C_3})^2 g^2\tilde{r}^2}\right], \quad (4.155)$$

$$\phi_3 = \frac{1}{4}\ln\left[\frac{(e^{2\phi_2} - 1)(1 + e^{C_3} + e^{C_3+2\phi_2})}{1 + e^{C_3} + e^{2\phi_2} - e^{C_3+4\phi_2}}\right] \quad (4.156)$$

with $\frac{d\tilde{r}}{dr} = e^{-2\phi_0 - 2\phi_1}$. In this solution, we have shifted the coordinate $\tilde{r}$ to $\tilde{r} + \frac{C}{g\sigma}$ with C being an integration constant in ϕ_2 solution.

(4) Domain walls from $SO(4)$ and $SO(2,2)$ gauge groups

In this case, we set $\rho = \sigma = \pm 1$ corresponding to $SO(4)$ and $SO(2,2)$ gauge groups. The DW solution can be obtained as in the previous case

$$U = 2\phi_0, \quad (4.157)$$

$$\phi_0 = C_0 + \frac{1}{10}\ln\left[1 + 4e^{2C_3} - e^{-2g\sigma\tilde{r}}\right] + \frac{1}{40}(4\sigma + e^{20C_1} + 4e^{20C_1+2C_3})g\tilde{r} - \frac{\sigma}{160}e^{20C_1 - 2g\sigma\tilde{r}}(16e^{4(C_3+g\sigma\tilde{r})} + 8e^{2C_3+4g\sigma\tilde{r}} + e^{4g\sigma\tilde{r}} - 1), \quad (4.158)$$

$$\phi_1 = 5C_1 - \frac{1}{2}\ln(1 - e^{4\phi_2}) + \frac{1}{4}\ln\left[e^{2C_3} - e^{4\phi_2} + e^{2C_3+4\phi_2}(e^{4\phi_2} - 2)\right], \quad (4.159)$$

$$\phi_2 = \frac{1}{4}\ln\left[\frac{1 - 2e^{g\sigma\tilde{r}} + e^{2g\sigma\tilde{r}} + 4e^{2C_3+2g\sigma\tilde{r}}}{1 + 2e^{g\sigma\tilde{r}} + e^{2g\sigma\tilde{r}} + 4e^{2C_3+2g\sigma\tilde{r}}}\right], \quad (4.160)$$

$$\phi_3 = \frac{1}{4}\ln\left[\frac{e^{2\phi_2} + e^{C_3+4\phi_2} - e^{C_3}}{e^{2\phi_2} + e^{C_3} - e^{C_3+4\phi_2}}\right] \quad (4.161)$$

with $\frac{d\tilde{r}}{dr} = e^{-2\phi_0 - 2\phi_1}$.

4.1.3 Gaugings in 15 and $\overline{\mathbf{40}}$ Representations

We close this flat DW section by looking for the solutions in $SO(2,1) \ltimes \mathbf{R}^4$ and $SO(2) \ltimes \mathbf{R}^4$ gauge groups. Besides, we also explicitly demonstrate that DWs from these gaugings in both **15** and $\overline{\mathbf{40}}$ representations are $\frac{1}{4}$-supersymmetric.

4.1.3.1 $\frac{1}{4}$-BPS Domain Wall from $SO(2,1) \ltimes \mathbf{R}^4$ Gauge Group

We start with $SO(2,1) \ltimes \mathbf{R}^4$ gauge group. In this case, we consider solutions that are invariant under the maximal compact subgroup $SO(2) \subset SO(2,1)$. Among the fourteen scalars in $SL(5)/SO(5)$ coset, there are four $SO(2)$ singlets corresponding to the following non-compact generators

$$\begin{aligned} \mathbf{Y}_1 &= 2e_{1,1} + 2e_{2,2} + 2e_{3,3} - 3e_{4,4} - 3e_{5,5}, \\ \mathbf{Y}_2 &= e_{1,1} + e_{2,2} - 2e_{3,3}, \\ \mathbf{Y}_3 &= e_{1,4} + e_{2,5} + e_{4,1} + e_{5,2}, \\ \mathbf{Y}_4 &= e_{1,5} - e_{2,4} - e_{4,2} + e_{5,1}\,. \end{aligned} \tag{4.162}$$

With the $SL(5)/SO(5)$ coset representative

$$\mathcal{V} = e^{\phi_0\mathbf{Y}_1+\phi_1\mathbf{Y}_2+\phi_2\mathbf{Y}_3+\phi_3\mathbf{Y}_4}, \tag{4.163}$$

we obtain the scalar potential

$$\mathbf{V} = \frac{g^2}{64}e^{-2(4\phi_0-\phi_1)}\left[6\cosh 2\phi_2\cosh 2\phi_3 + e^{6\phi_1}\right] \tag{4.164}$$

which does not admit any critical points.

Contrary to the previous cases, we need to impose two projection conditions on the Killing spinors (4.14) in order to obtain a consistent set of BPS equations in this case. This is because A_1 and A_2 matrices consist of two parts separately corresponding to Y_{MN} and $Z^{MN,P}$. While the latter comes with an extra $SO(5)$ gamma matrices Γ^3, the former does not. The resulting two projections are

$$\gamma^{\hat{6}}\epsilon^a_0 = -(\Gamma_3)^a{}_b\epsilon^b_0 = \epsilon^a_0, \tag{4.165}$$

which reduce the number of SUSY to 1/4 of the original amount or eight supercharges.

Following the same procedure as in the previous cases, we find the BPS equations

$$U' = \frac{g}{40}e^{-2(2\phi_0+\phi_1)}\left(3\cosh 2\phi_2\cosh 2\phi_3 - e^{6\phi_1}\right), \tag{4.166}$$

$$\phi_0' = \frac{g}{240}e^{-2(\phi_0+\phi_1)}\left(15\text{sech}2\phi_2\text{sech}2\phi_3 - 3\cosh 2\phi_2\cosh 2\phi_3 - 4e^{6\phi_1}\right), \tag{4.167}$$

$$\phi_1' = \frac{g}{48}e^{-2(\phi_0+\phi_1)}\left(3\text{sech}2\phi_2\text{sech}2\phi_3 + 3\cosh 2\phi_2\cosh 2\phi_3 + 4e^{6\phi_1}\right), \tag{4.168}$$

$$\phi_2' = -\frac{3g}{16}e^{-2(2\phi_0+\phi_1)}\sinh 2\phi_2\text{sech}2\phi_3, \tag{4.169}$$

$$\phi_3' = -\frac{3g}{16}e^{-2(2\phi_0+\phi_1)}\cosh 2\phi_2\sinh 2\phi_3\,. \tag{4.170}$$

Introducing a new radial coordinate $\tilde{r}$ via $\frac{d\tilde{r}}{dr} = e^{-4\phi_0-2\phi_1}$, we can find a DW solution to these equations

$$\begin{aligned}
\phi_0 &= C_0 + \frac{2}{45}(e^{3\phi_1}-3)\ln(1-e^{4\phi_2}) - \frac{1}{60}\ln(e^{2C_3} - e^{4\phi_2} + e^{8\phi_2+2C_3} - 2e^{4\phi_2+2C_3}) \\
&\quad -\frac{2}{45}\ln\left(1+e^{4\phi_2}+2\sqrt{e^{4\phi_2}-e^{2C_3}-e^{8\phi_2+2C_3}+2e^{4\phi_2+2C_3}}\right) \\
&\quad +\frac{1}{6}\ln(1+e^{4\phi_2}), \qquad (4.171)\\
\phi_1 &= C_1 - 5\phi_0 - \ln(1-e^{4\phi_2}) + \ln(1+e^{4\phi_2}), \qquad (4.172)\\
\phi_2 &= \frac{1}{4}\ln\left[\frac{1+4e^{2C_3}-2e^{\frac{3}{8}g\tilde{r}}+e^{\frac{3}{4}g\tilde{r}}}{1+4e^{2C_3}+2e^{\frac{3}{8}g\tilde{r}}+e^{\frac{3}{4}g\tilde{r}}}\right], \qquad (4.173)\\
\phi_3 &= \frac{1}{4}\ln\left[\frac{e^{2\phi_2}-e^{C_3}+e^{4\phi_2+C_3}}{e^{2\phi_2}+e^{C_3}-e^{4\phi_2+C_3}}\right], \qquad (4.174)\\
U &= \frac{1}{15}(e^{6\phi_1}-3)\ln(1-e^{4\phi_2}) + \frac{1}{10}\ln\left(e^{2C_3}-e^{4\phi_2}+e^{2C_3+8\phi_2}-2e^{2C_3+4\phi_2}\right) \\
&\quad -\frac{1}{15}e^{6\phi_1}\ln\left(2\sqrt{e^{4\phi_2}-e^{2C_3}-e^{8\phi_2+2C_3}+2e^{2C_3+4\phi_2}}+e^{4\phi_2}+1\right) \qquad (4.175)
\end{aligned}$$

in which C_i, $i = 0, 1, 3$, are integration constants for ϕ_i solutions. It should be noted that C_2 and another integration constant for U solution are neclected by shifting the radial coordinate $\tilde{r}$ and rescaling the flat coordinates x^m, respectively.

4.1.3.2 $\frac{1}{4}$-BPS Domain Wall from $SO(2)\ltimes\mathbf{R}^4$ Gauge Group

As the final case, we consider $SO(2) \ltimes \mathbf{R}^6$ gauge group with $\mathrm{Tr}Z^2 = -2$. As expressed before, this case admits a half-supersymmetric ($N = 2$) Minkowski vacuum, and the gauge group is reduced to $SO(2) \ltimes \mathbf{R}^4$. For definiteness, we take an explicit form of $Z_\alpha{}^\beta$ to be

$$Z_\alpha{}^\beta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}. \qquad (4.176)$$

There are four $SO(2)$ singlet scalars corresponding to the following $SL(5)$

non-compact generators

$$\begin{aligned}
\overline{Y}_1 &= 3e_{1,1}+3e_{2,2}-2e_{3,3}-2e_{4,4}-2e_{5,5},\\
\overline{Y}_2 &= e_{4,4}+e_{5,5}-2e_{3,3},\\
\overline{Y}_3 &= e_{1,4}+e_{2,5}+e_{4,1}+e_{5,2},\\
\overline{Y}_4 &= e_{1,5}-e_{2,4}-e_{4,2}+e_{5,1}\,. \qquad (4.177)
\end{aligned}$$

Using the parametrization of the $SL(5)/SO(5)$ coset representative in the form

$$\mathcal{V} = e^{\phi_0\overline{Y}_1+\phi_1\overline{Y}_2+\phi_2\overline{Y}_3+\phi_3\overline{Y}_4}, \qquad (4.178)$$

we find that the scalar potential vanishes identically. This is in agreement with $CSO(2,0,2)$ gauge group considered in the previous section.

With the projectors (4.165), we can derive the following BPS equations

$$\begin{aligned}
U' &= \frac{g}{10}e^{-6\phi_0}\cosh 2\phi_2\cosh 2\phi_3, \qquad (4.179)\\
\phi_0' &= \frac{g}{60}e^{-6\phi_0}\left(\cosh 2\phi_2\cosh 2\phi_3+5\text{sech}2\phi_2\text{sech}2\phi_3\right), \qquad (4.180)\\
\phi_1' &= \frac{g}{12}e^{-6\phi_0}\left(\cosh 2\phi_2\cosh 2\phi_3-\text{sech}2\phi_2\text{sech}2\phi_3\right), \qquad (4.181)\\
\phi_2' &= -\frac{g}{4}e^{-6\phi_0}\sinh 2\phi_2\text{sech}2\phi_3, \qquad (4.182)\\
\phi_3' &= -\frac{g}{4}e^{-6\phi_0}\cosh 2\phi_2\sinh 2\phi_3\,. \qquad (4.183)
\end{aligned}$$

By using a new radial coordinate $\tilde{r}$ defined by $\frac{d\tilde{r}}{dr}=e^{-6\phi_0}$, we find a DW solution to the above equations

$$\begin{aligned}
\phi_0 &= C_0-\frac{1}{5}\ln(1-e^{4\phi_2})+\frac{1}{6}\ln(1+e^{4\phi_2})\\
&\quad +\frac{1}{60}\ln\left[e^{2C_3}-e^{4\phi_2}+e^{2C_3+8\phi_2}-2e^{2C_3+4\phi_2}\right], \qquad (4.184)\\
\phi_1 &= C_1-\frac{1}{6}\ln(1+e^{4\phi_2})+\frac{1}{12}\ln\left[e^{2C_3}-e^{4\phi_2}+e^{2C_3+8\phi_2}-2e^{2C_3+4\phi_2}\right], \qquad (4.185)\\
\phi_2 &= \frac{1}{4}\ln\left[\frac{1+4e^{2C_3}-2e^{\frac{1}{2}g\tilde{r}}+e^{g\tilde{r}}}{1+4e^{2C_3}+2e^{\frac{1}{2}g\tilde{r}}+e^{g\tilde{r}}}\right], \qquad (4.186)\\
\phi_3 &= \frac{1}{4}\ln\left[\frac{e^{2\phi_2}+e^{4\phi_2+C_3}-e^{C_3}}{e^{2\phi_2}-e^{4\phi_2+C_3}+e^{C_3}}\right], \qquad (4.187)\\
U &= -\frac{1}{5}\ln(1-e^{4\phi_2})+\frac{1}{10}\ln\left[e^{2C_3}-e^{4\phi_2}+e^{2C_3+8\phi_2}-2e^{2C_3+4\phi_2}\right]. \qquad (4.188)
\end{aligned}$$

4.2 Charged Domain Wall Solutions

We subsequently extend the (flat) DW solutions found in the previous section by coupling them to non-vanishing modified three-forms $\mathcal{H}_{(3)M}$ in this section. Similar to Section 3.1, the solutions under consideration here also take the form of $AdS_3 \times S^3$-sliced DWs. In addition to $\mathcal{H}_{(3)M}$, it is also possible to further couple $SO(3)$ gauge fields to the solutions in some cases.

For $SO(5)$ gauge group with a supersymmetric AdS_7 vacuum, charged DW solutions should be interpreted as two-dimensional conformal defects within the dual $N = (2, 0)$ SCFT in six dimensions. For other gauge groups, their vacua take the form of supersymmetric flat DWs given in Section 4.1. We expect these charged DW solutions to describe surface defects in the dual six-dimensional $N = (2, 0)$ SQFTs. Moreover, as in the previous case, we will also give uplifted solutions only for $SO(5)$ and $CSO(4, 0, 1)$ gauge groups in which the complete truncation ansatze of eleven-dimensional supergravity on S^4 and type IIA theory on S^3 are known.

4.2.1 Gaugings in 15 Representation

As in the flat DW section, we begin with gaugings in **15** representation. To couple the modified three-forms to the DWs, we take the metric ansatz to be the $AdS_3 \times S^3$-sliced DW (3.1). In order to preserve some amount of SUSY on this metric, we use the ansatz for Killing spinors similar to (3.14)

$$\epsilon^a = e^{\frac{U(r)}{2}}\epsilon^a_0 \left[\cos\theta(r)\mathbf{1}_8 + \sin\theta(r)\gamma^{\hat{0}\hat{1}\hat{2}}\right]\epsilon^a_0 \tag{4.189}$$

in which ϵ^a_0 are constant SM spinors.

For the modified three-forms, we use the ansatz following (3.12)

$$\mathcal{H}_{\hat{m}\hat{n}\hat{p}M} = k_M(r)e^{-3U(r)}\varepsilon_{\hat{m}\hat{n}\hat{p}} \qquad \text{and} \qquad \mathcal{H}_{\hat{i}\hat{j}\hat{k}M} = l_M(r)e^{-3W(r)}\varepsilon_{\hat{i}\hat{j}\hat{k}} \tag{4.190}$$

or, equivalently, with vol_{AdS_3} and vol_{S^3} given in (3.13),

$$\mathcal{H}_{(3)M} = k_M\text{vol}_{AdS_3} + l_M\text{vol}_{S^3}\,. \tag{4.191}$$

As seen from their definitions, all tensor fields are involved in the modified three-forms. These contributions can come from either massive three-form or two-form fields depending on the non-vanishing components of the embedding tensor. Therefore, in order to identify which tensor fields determine non-vanishing $\mathcal{H}_{(3)M}$, we need to consider gauge group by gauge group.

4.2.1.1 $SO(4)$ Symmetric Charged Domain Walls

We first consider charged DW solutions with $SO(4)$ symmetry. To preserve $SO(4)$ symmetry, we keep only the following components of $\mathcal{H}_{(3)M}$ non-vanishing

$$\mathcal{H}_{\hat{m}\hat{n}\hat{p}5} = k(r)e^{-3U(r)}\varepsilon_{\hat{m}\hat{n}\hat{p}} \qquad \text{and} \qquad \mathcal{H}_{\hat{i}\hat{j}\hat{k}5} = l(r)e^{-3W(r)}\varepsilon_{\hat{i}\hat{j}\hat{k}}\,. \tag{4.192}$$

For $SO(5)$ and $SO(4,1)$ gauge groups corresponding to the non-degenerate Y_{MN} from (4.5), the field content of the gauged supergravity contains five massive three-form fields $S^M_{(3)}$. For vanishing gauge and two-form fields, the modified three-form is then given by

$$\mathcal{H}_{(3)5} = g\rho S^5_{(3)} \tag{4.193}$$

with $\rho = Y_{55} = \pm 1$. Therefore, the massive three-form field $S^5_{(3)}$ determines the $\mathcal{H}_{(3)5}$ in these gauge groups.

For $CSO(4,0,1)$ gauge group with $Y_{55} = 0$, the $S^5_{(3)}$ does not contribute to $\mathcal{H}_{(3)5}$, but there is a massless two-form field $B_{(2)5}$ with the field strength

$$\mathcal{H}_{(3)5} = DB_{(2)5}\,. \tag{4.194}$$

In this case, $k'(r) = l'(r) = 0$ is needed in order to satisfy the Bianchi's identity $D\mathcal{H}_{(3)M} = 0$. Taking this condition into account, we can write the ansatz for the two-form field as

$$B_{(2)5} = k\,\omega_{(2)} + l\,\tilde{\omega}_{(2)} \tag{4.195}$$

in which k and l are now constants. With the metrics given in (3.2) and (3.3), the explicit form of $\omega_{(2)}$ and $\tilde{\omega}_{(2)}$ is given by

$$\omega_{(2)} = -\frac{1}{\tau^3}\sinh x^1 dt \wedge dx^2 \qquad \text{and} \qquad \tilde{\omega}_{(2)} = -\frac{1}{\kappa^3}\sin x^5 dx^4 \wedge dx^6\,. \tag{4.196}$$

It can be readily verified that $\text{vol}_{AdS_3} = d\omega_{(2)}$ and $\text{vol}_{S^3} = d\tilde{\omega}_{(2)}$.

Using the scalar coset representative (4.7), and imposing two projection conditions

$$\gamma^{\hat{3}}\epsilon^a_0 = (\Gamma_5)^a{}_b\epsilon^b_0 = \epsilon^a_0, \quad (4.197)$$

we find the following BPS equations from the conditions $\delta\psi^a_\mu = 0$ and $\delta\chi^{abc} = 0$

$$\begin{aligned}
U' &= \frac{e^{V-2\phi}}{80\cos 2\theta}\left[g(8-\rho e^{10\phi}) + 3g\rho e^{10\phi}\cos 4\theta - 16\tau e^{2\phi-U}\sin 2\theta\right], && (4.198)\\
W' &= \frac{e^{V-2\phi}}{40\cos 2\theta}\left[g(4+2\rho e^{10\phi}) - g\rho e^{10\phi}\cos 4\theta - 8\tau e^{2\phi-U}\sin 2\theta\right], && (4.199)\\
\phi' &= \frac{e^{V-2\phi}}{80\cos 2\theta}\left[g(4-3\rho e^{10\phi}) - g\rho e^{10\phi}\cos 4\theta - 8\tau e^{2\phi-U}\sin 2\theta\right], && (4.200)\\
\theta' &= -\frac{1}{16}g\rho e^{V+8\phi}\sin 2\theta, && (4.201)\\
k &= \frac{1}{8}e^{2U-4\phi}(4\tau - g\rho e^{U+8\phi}\sin 2\theta), && (4.202)\\
l &= \frac{1}{8}e^{3W-6\phi}\left[g(\rho e^{10\phi}-2)\tan 2\theta + 4\tau e^{2\phi-U}\sec 2\theta\right] && (4.203)
\end{aligned}$$

together with an algebraic constraint

$$0 = e^{-W}\kappa - e^{-U}\tau\sec 2\theta + \frac{1}{2}ge^{-2\phi}\tan 2\theta\,. \quad (4.204)$$

Since the four-form field strengths do not enter the SUSY transformations of fermions, the functions $k(r)$ and $l(r)$ appear algebraically in the resulting BPS equations. This is in contrast to the pure $N = 2$ gauged supergravity considered in [62] and the matter-coupled $SO(4)$ theory in Section 3.1 in which the four-form field strength of the massive three-form field obviously appears in the fermionic SUSY transformations. In those cases, the BPS conditions result in differential equations for $k(r)$ and $l(r)$.

It should be noted here that the appearance of the $SO(5)$ gamma matrix Γ_5 in the projection conditions (4.197) is due to the non-vanishing $\mathcal{H}_{(3)5}$. We then consider various possible solutions to the BPS equations, these solutions are $\frac{1}{4}$-BPS since the Killing spinors are subject to two projectors (4.197).

4.2.1.1.1 $Mkw_3 \times \mathbb{R}^3$-Sliced Domain Walls

We start with a simple case of $Mkw_3 \times \mathbb{R}^3$-sliced DWs with vanishing τ and κ. Imposing $\tau = \kappa = 0$ into the constraint (4.204) gives

$$0 = \frac{1}{2} g e^{-2\phi} \tan 2\theta \,. \tag{4.205}$$

Setting $g = 0$ corresponds to ungauged $N = 4$ supergravity and gives rise to a supersymmetric $Mkw_3 \times \mathbb{R} \times \mathbb{R}^3 \sim Mkw_7$ background as expected.

Another possibility to satisfy the condition (4.205) is to set $\tan 2\theta = 0$ which implies $\theta = \frac{n\pi}{2}$, $n = 0, 1, 2, 3, \ldots$. For even n, we have $\sin\theta = 0$ and, from (4.189), the Killing spinors take the simple form given in (4.14). For odd n with $\cos\theta = 0$, the Killing spinors become

$$\epsilon^a = e^{\frac{U}{2}} \gamma^{\hat{0}\hat{1}\hat{2}} \epsilon^a_0 \,. \tag{4.206}$$

We can redefine ϵ^a_0 to $\tilde{\epsilon}^a_0 = \gamma^{\hat{0}\hat{1}\hat{2}} \epsilon^a_0$ satisfying the projection conditions

$$-\gamma^{\hat{3}} \epsilon^a_0 = (\Gamma_5)^a{}_b \epsilon^b_0 = \epsilon^a_0 \,. \tag{4.207}$$

This differs from the projectors in (4.197) only by a minus sign in the $\gamma^{\hat{3}}$ projector. Therefore, the two possibilities obtained from the condition $\tan 2\theta = 0$ are equivalent by flipping the sign of $\gamma^{\hat{3}}$ projector. We can accordingly choose $\theta = 0$ without losing generality.

With $\theta = 0$, the BPS equations (4.198) to (4.203) become

$$U' = W' = \frac{1}{40} g e^{V-2\phi}(4 + \rho e^{10\phi}), \tag{4.208}$$
$$\phi' = \frac{1}{20} g e^{V-2\phi}(1 - \rho e^{10\phi}), \tag{4.209}$$
$$k = l = 0 \,. \tag{4.210}$$

By choosing $V = -3\phi$, we find the following solution

$$U = W = 2\phi - \frac{1}{4} \ln\left[1 - \rho e^{10\phi}\right], \tag{4.211}$$
$$e^{5\phi} = \frac{1}{\sqrt{\rho}} \tanh\left[\frac{\sqrt{\rho}}{4}(gr + C)\right] \tag{4.212}$$

with an integration constant C. Since $k = l = \theta = 0$, the Γ_5 projection in (4.197) is not needed. This solution is then half-supersymmetric with vanishing three-form fluxes and is exactly the $SO(4)$ symmetric DW studied in Section 4.1.1.1. Therefore, the $Mkw_3 \times \mathbb{R}^3$-sliced solution is just the standard flat DW.

4.2.1.1.2 $\boldsymbol{Mkw_3 \times S^3}$-Sliced Domain Walls

We now look for DW solutions with a $Mkw_3 \times S^3$ slice. In this case, the follwing gauge choice is chosen as in [62]

$$e^{-V} = \frac{1}{16}e^{8\phi}. \tag{4.213}$$

For $\rho \neq 0$, we can solve the BPS equations (4.198) to (4.203) by setting $\tau = 0$. The resulting solution is given by

$$\begin{aligned}
U &= 2\phi - \ln(\sin 2\theta), && (4.214)\\
W &= 2\phi - \ln(\tan 2\theta), && (4.215)\\
e^{10\phi} &= 2C(\cos 4\theta - 3) + (4C + \rho)\sec^2 2\theta, && (4.216)\\
k &= -\frac{g}{8}\left(4\rho C + \csc^4 2\theta\right)\tan^2 2\theta, && (4.217)\\
l &= \frac{g}{16}\left[\rho C(\cos 8\theta + 3) - 2(2\rho C + 1)\cos 4\theta\right]\csc^2 2\theta, && (4.218)\\
\theta &= \arctan\left(e^{-2g\rho r}\right) && (4.219)
\end{aligned}$$

with $\kappa = -g/2$ and C being an integration constant in the solution for ϕ.

For $SO(5)$ gauge group with $\rho = 1$, the solution is locally asymptotic to the $N = 4$ supersymmetric AdS_7 in the limit $r \to \infty$ with

$$U \sim W \sim 2gr, \qquad \phi \sim \theta \sim 0\,. \tag{4.220}$$

In this limit, the main contribution to the solution is obtained from the scalar field, while the contribution from the modified three-forms is highly suppressed, as can be seen from (4.192). On the other hand, in the limit $r \to 0$, the solution is singular similar to the solution studied in [62].

For $SO(4,1)$ gauge group with $\rho = -1$, there is no AdS_7 asymptotic since this gauge group does not admit a supersymmetric AdS_7 vacuum. In this case, the solution is the $SO(4)$ symmetric flat DW studied in Section 4.1.1.1 with a dyonic profile of the three-form flux.

For $CSO(4,0,1)$ gauge group with $\rho = 0$, imposing $\tau = 0$ into the constraint (4.204) gives

$$\kappa = -\frac{1}{2} g e^{W-2\phi} \tan 2\theta, \qquad (4.221)$$

and the BPS equations from (4.198) to (4.203) with $\tau = 0$ become

$$U' = W' = \frac{1}{10} g e^{V-2\phi} \sec 2\theta, \qquad (4.222)$$

$$\phi' = \frac{1}{20} g e^{V-2\phi} \sec 2\theta, \qquad (4.223)$$

$$\theta' = k = 0, \qquad (4.224)$$

$$l = -\frac{1}{4} g e^{3W-6\phi} \tan 2\theta. \qquad (4.225)$$

The equation (4.224) implies that θ is constant. Note that these BPS equations will reduce to those of the $Mkw_3 \times \mathbb{R}^3$-sliced DW if $\theta = 0$.

In this case, the constraint (4.221) implies that θ cannot be zero since we keep $\kappa \neq 0$. Furthermore, a non-vanishing θ gives a non-trivial three-form flux according to (4.225) to support the S^3 part. For constant $\theta \neq 0$, we find the following solution, after choosing $V = 0$ gauge choice,

$$U = W = 2\phi, \qquad k = 0, \qquad (4.226)$$

$$l = -\frac{1}{4} g \tan 2\theta, \qquad (4.227)$$

$$e^{2\phi} = \frac{1}{10} g r \sec 2\theta + 2C \qquad (4.228)$$

with an integration constant C. The constant θ is given by

$$\theta = -\frac{1}{2} \tan^{-1} \frac{2\kappa}{g}\,. \qquad (4.229)$$

It can be verified that this charged DW solution is the $SO(4)$ symmetric flat DW of $CSO(4,0,1)$ gauge group given in Section 4.1.1.1 with a magnetic profile of a constant three-form flux obtained from a constant θ.

4.2.1.1.3 $AdS_3 \times S^3$-Sliced Domain Walls

We move on to more complicated solutions with an $AdS_3 \times S^3$ slice. As in [62], we begin with a more straightforward solution with $U = W$. From the BPS equations (4.198) to (4.203), imposing $U' = W'$ gives

$$\theta = 0, \qquad k = l, \qquad \kappa = \tau\,. \tag{4.230}$$

Setting $\theta = 0$, we find that the BPS equations become

$$U' = \frac{g}{40}e^{V-2\phi}(4+\rho e^{10\phi}), \tag{4.231}$$

$$\phi' = \frac{g}{20}e^{V-2\phi}(1-\rho e^{10\phi}), \tag{4.232}$$

$$k = \frac{1}{2}e^{2U-4\phi}\tau\,. \tag{4.233}$$

Choosing $V = -3\phi$, we obtain the following solution

$$U = 2\phi - \frac{1}{4}\ln\left(1-\rho e^{10\phi}\right), \tag{4.234}$$

$$e^{5\phi} = \frac{1}{\sqrt{\rho}}\tanh\left[\frac{\sqrt{\rho}}{4}(gr+C)\right], \tag{4.235}$$

$$k = \frac{1}{2}\tau\cosh\left[\frac{\sqrt{\rho}}{4}(gr+C)\right] \tag{4.236}$$

with an integration constant C. This solution is the $SO(4)$ symmetric flat DW coupled to a dyonic profile of the three-form flux.

For $SO(5)$ gauge group, the solution is locally asymptotic to the $N = 4$ supersymmetric AdS_7 vacuum. We expect this solution to describe a surface defect, corresponding to the AdS_3 part, in the six-dimensional $N = (2,0)$ SCFT. For $SO(4,1)$ and $CSO(4,0,1)$ gauge groups, we similarly interpret the solutions as conformal surface defects within six-dimensional $N = (2,0)$ SQFTs dual to flat DW vacua without the three-form flux.

In the more general case with $U \neq W$, we will separately find the solutions for the cases of $\rho = \pm 1$ and $\rho = 0$. With the same gauge choice given in (4.213), the BPS equations (4.198) to (4.203) for $\rho \neq 0$ are solved by

$$\begin{aligned} U &= 2\phi - \ln\left(\sin 2\theta\right), && (4.237)\\ W &= 2\phi - \ln\left(\tan 2\theta\right), && (4.238)\\ e^{10\phi} &= \frac{3gC + 2g\rho - 4\tau\rho + 4(\tau\rho - gC)\cos 4\theta + gC\cos 8\theta}{g(\cos 4\theta + 1)}, && (4.239)\\ k &= \frac{1}{8}\left(4\tau\csc^2 2\theta - g\csc^4 2\theta - 4g\rho C\right)\tan^2 2\theta, && (4.240)\\ l &= \frac{1}{8}\left(g\csc^2 2\theta - 2g\cot^2 2\theta - 4\tau + 4g\rho C\sin^2 2\theta\right), && (4.241)\\ \theta &= \arctan\left(e^{-2g\rho r}\right) && (4.242) \end{aligned}$$

together with the following relation obtained from the constraint (4.204)

$$\kappa = -\frac{g}{2} + \tau\,. \qquad (4.243)$$

Again, the solution is locally asymptotic to the AdS_7 vacuum for $SO(5)$ gauge group, and being a charged DW with a non-vanishing three-form flux for $SO(4,1)$ gauge group. In general, these solutions respectively describe holographic RG flows from an $N = (2,0)$ SCFT and an $N = (2,0)$ SQFT to a singularity at $r = 0$ except for a special case with $\tau = g(\rho C+1)/4$. This is very similar to the solutions of pure $N = 2$ gauged supergravity studied in [62].

As $r \to 0$ for $\tau = g(\rho C + 1)/4$, the scalar potential is constant and the solution turns out to be described by a locally $AdS_3 \times T^4$ geometry with the following leading profile

$$\begin{aligned} &e^{2U} \sim (\rho - 4C)^{\frac{2}{5}}, \qquad e^{2W} \sim 0, \qquad \phi \sim \frac{1}{10}\ln\left(\rho - 4C\right),\\ &\theta \sim \frac{\pi}{4}, \qquad k \sim \frac{g}{8}(4\rho C - 1), \qquad l \sim 0\,. \end{aligned} \qquad (4.244)$$

To obtain real solutions in $SO(5)$ and $SO(4,1)$ gauge groups, we respectively choose the integration constant $C < \frac{1}{4}$ and $C < -\frac{1}{4}$.

For $CSO(4,0,1)$ gauge group with $\rho = 0$, after setting $V = 0$, we find the following BPS solution

$$\begin{aligned} U &= W = 2\phi, \qquad k = \frac{1}{2}\tau, && (4.245)\\ l &= \frac{1}{4}(2\tau - g\sin 2\theta)\sec 2\theta, && (4.246)\\ e^{2\phi} &= \frac{1}{10}r\left(g\sec 2\theta - 2\tau\tan 2\theta\right) + 2C && (4.247) \end{aligned}$$

where the constant κ is given by

$$\kappa = \tau \sec 2\theta - \frac{1}{2} g \tan 2\theta. \tag{4.248}$$

In this case, θ is constant since the BPS equation (4.201) with $\rho = 0$ gives $\theta' = 0$.

4.2.1.1.4 Coupling to $SO(3)$ Gauge Fields

We extend our analysis by coupling the previously obtained solutions to $SO(3)$ gauge fields in this section. With the identity $\Gamma_1 \dots \Gamma_5 = \mathbf{1}_4$ and the projector $(\Gamma_5)^a{}_b \epsilon_0^b = \epsilon_0^a$, we turn on the gauge fields on the S^3 corresponding to the anti-self-dual $SO(3) \subset SO(4)$. The ansatz for these gauge fields is chosen to be

$$A^{23}_{(1)} = -A^{14}_{(1)} = e^{-W(r)} \frac{\kappa}{4} p(r) e^{\hat{4}}, \tag{4.249}$$
$$A^{31}_{(1)} = -A^{24}_{(1)} = e^{-W(r)} \frac{\kappa}{4} p(r) e^{\hat{5}}, \tag{4.250}$$
$$A^{12}_{(1)} = -A^{34}_{(1)} = e^{-W(r)} \frac{\kappa}{4} p(r) e^{\hat{6}}\,. \tag{4.251}$$

The function $p(r)$ is the magnetic charge depending on the radial coordinate. The corresponding two-form field strengths are given by

$$F^{23}_{(2)} = -F^{14}_{(2)} = e^{-V-W} \frac{\kappa}{4} p' e^{\hat{3}} \wedge e^{\hat{4}} + e^{-2W} \frac{\kappa^2}{8} p(2 - gp) e^{\hat{5}} \wedge e^{\hat{6}}, \tag{4.252}$$
$$F^{31}_{(2)} = -F^{24}_{(2)} = e^{-V-W} \frac{\kappa}{4} p' e^{\hat{3}} \wedge e^{\hat{5}} + e^{-2W} \frac{\kappa^2}{8} p(2 - gp) e^{\hat{6}} \wedge e^{\hat{4}}, \tag{4.253}$$
$$F^{12}_{(2)} = -F^{34}_{(2)} = e^{-V-W} \frac{\kappa}{4} p' e^{\hat{3}} \wedge e^{\hat{6}} + e^{-2W} \frac{\kappa^2}{8} p(2 - gp) e^{\hat{4}} \wedge e^{\hat{5}}\,. \tag{4.254}$$

Since $Z^{MN,P} = 0$ in the **15** representation, the modified two-forms are precisely the $SO(3)$ field strengths, $\mathcal{F}^{MN}_{(2)} = F^{MN}_{(2)}$, in this case.

To preserve some amount of SUSY, we need to impose additional projection conditions on the constant SM spinors ϵ_0^a as follow

$$\gamma^{\hat{4}\hat{5}} \epsilon_0^a = -(\Gamma_{12})^a{}_b \epsilon_0^b, \qquad \gamma^{\hat{5}\hat{6}} \epsilon_0^a = -(\Gamma_{23})^a{}_b \epsilon_0^b. \tag{4.255}$$

Therefore, together with the projectors given in (4.197), there are four independent projectors on ϵ_0^a, and the residual SUSY consists of two supercharges.

With all these, the resulting BPS equations for the $AdS_3 \times S^3$-sliced DW coupling to non-vanishing $SO(3)$ gauge fields are given by

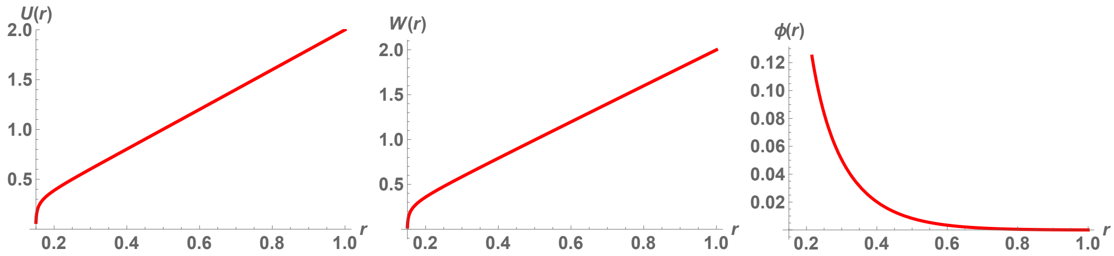
$$\begin{aligned} U' &= \frac{e^{V-2(W+\phi)}}{80\cos 2\theta}\left[e^{2W}\left(g(4+\rho e^{10\phi})(3\cos 4\theta-1)+32e^{2\phi-U}\tau\sin 2\theta\right)\right. \\ &\quad \left.+12e^{4\phi}\left(\kappa^2 p(gp-2)(\cos 4\theta-3)+2e^{W-2\phi}\kappa(gp-1)\sin 4\theta\right)\right], \qquad (4.256) \\ W' &= \frac{e^{V-2(W+\phi)}}{40\cos 2\theta}\left[e^{2W}\left(g(4+\rho e^{10\phi})(2-\cos 4\theta)+24e^{2\phi-U}\tau\sin 2\theta\right)\right. \\ &\quad \left.+4e^{4\phi}\left(\kappa^2 p(gp-2)(\cos 4\theta-8)-2e^{W-2\phi}\kappa(gp-1)\sin 4\theta\right)\right], \qquad (4.257) \\ \phi' &= \frac{e^{V-2(W+\phi)}}{80\cos 2\theta}\left[e^{2W}\left(g(6\cos 4\theta-2-\rho e^{10\phi}(\cos 4\theta+3))+16e^{2\phi-U}\tau\sin 2\theta\right)\right. \\ &\quad \left.+6e^{4\phi}\left(\kappa^2 p(gp-2)(3-\cos 4\theta)+2e^{W-2\phi}\kappa(gp-1)\sin 4\theta\right)\right], \qquad (4.258) \\ \theta' &= \frac{e^{V-2(W+\phi)}}{16}\left[24e^{W+2\phi}\left(e^{W-U}\tau+\kappa(gp-1)\cos 2\theta\right)\right. \\ &\quad \left.-\left(ge^{2W}(12+\rho e^{10\phi})-12e^{4\phi}\kappa^2 p(gp-2)\right)\sin 2\theta\right], \qquad (4.259) \\ k &= \frac{1}{8}e^{3U-4\phi}(4e^{-U}\tau-g\rho e^{8\phi}\sin 2\theta), \qquad (4.260) \\ l &= \frac{1}{8}e^{3W-6\phi}\left[g(4+\rho e^{10\phi})\tan 2\theta-8e^{2\phi-U}\tau\sec 2\theta\right. \\ &\quad \left.-12e^{4\phi-2W}\left(\kappa^2 p(gp-2)\tan 2\theta+e^{W-2\phi}\kappa(gp-1)\right)\right], \qquad (4.261) \\ p' &= \frac{e^{V-W-4\phi}}{2\kappa}\left[2e^{W+2\phi}\left(e^{W-U}\tau+\kappa(gp-1)\cos 2\theta\right)\right. \\ &\quad \left.-\left(ge^{2W}-e^{4\phi}\kappa^2 p(gp-2)\right)\sin 2\theta\right]. \qquad (4.262) \end{aligned}$$

It can be verified that these equations satisfy all the field equations (2.69) to (2.73) without imposing any constraint. Moreover, by setting $\tau = 0$, we obtain the BPS equations for the case with a $Mkw_3 \times S^3$ slice.

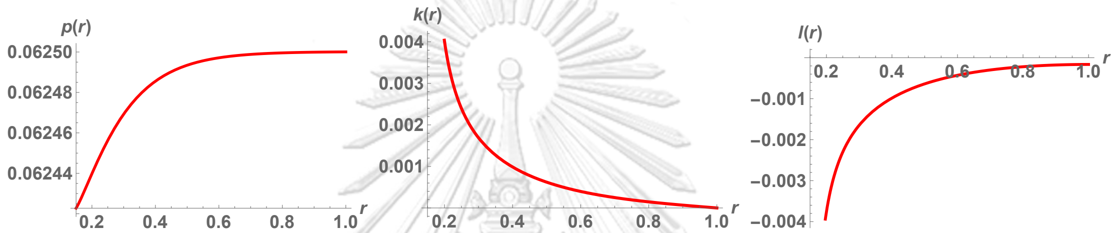
Since the BPS equations are much more complicated, we are not able to find analytic flow solutions in this case. Instead, we look for numerical solutions with some appropriate boundary conditions. We first consider the asymptotic AdS_7 vacuum in $SO(5)$ gauge group. With $\rho = 1$, the following locally AdS_7 configuration solves the above BPS equations at the leading order as $r \to \infty$

$$U \sim W \sim \frac{r}{L_{AdS_7}}, \qquad \phi \sim \theta \sim 0, \qquad p \sim \frac{1}{g}\left(1-\frac{\tau}{\kappa}\right) \qquad (4.263)$$

with $L_{AdS_7} = \frac{8}{g}$. Choosing $V = 0$ gauge choice, we find some examples of the BPS flow solutions from this locally AdS_7 geometry as $r \to \infty$ to the singularity at $r = 0$, as shown in Figures 4.1 and 4.2 for $g = 16$ and $\kappa = 2$. Note that, in these solutions, we have not imposed the boundary conditions on k and l since their BPS equations are algebraic. This is rather different from the solutions in [62] in which the BPS equations for k and l are differential.



(a) U solution (b) W solution (c) ϕ solution



(d) p solution (e) k solution (f) l solution

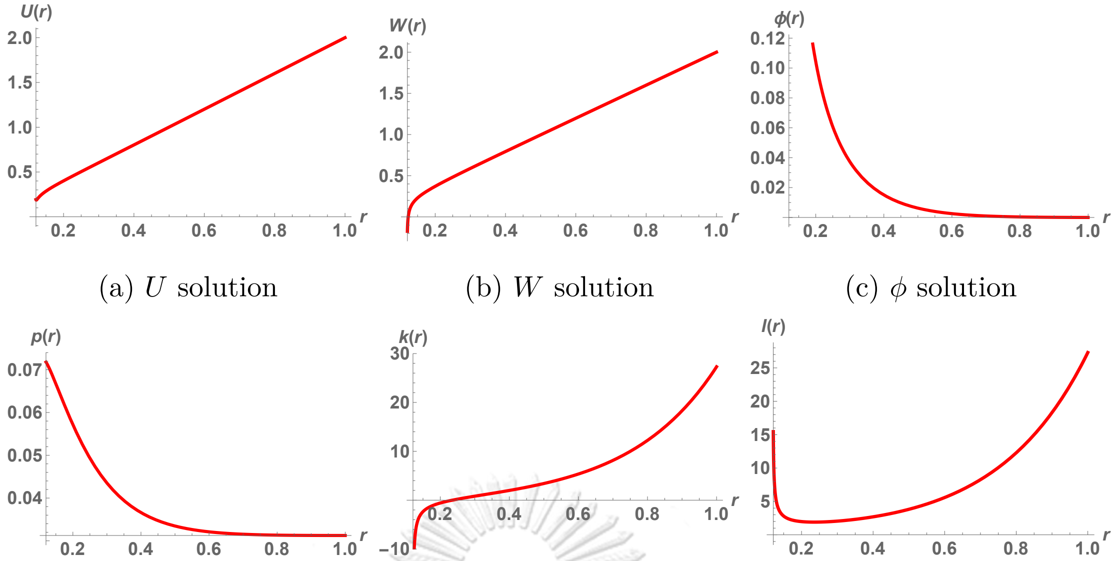
Figure 4.1: A numerical solution with $g = 16$, $\kappa = 2$, and $\tau = 0$ from the locally AdS_7 configuration in UV as $r \to 1$ to a singularity in the form of a $Mkw_3 \times S^3$-sliced DW in IR at $r = 0$ for SO(5) gauge group.

From the numerical solution in Figure 4.2, the solutions for k and l seem to be diverging as $k \sim e^{2U}$ and $l \sim e^{2W}$ when $r \to \infty$. However, the contribution from the three-form flux is highly suppressed in this limit since the terms involving $\mathcal{H}_{(3)5}$ in the BPS equations behave as $ke^{-3U} + le^{-3W}$.

We then look for numerical solutions of the BPS equations (4.256) to (4.262) in the form of a BPS flow from the charged DW without the $SO(3)$ gauge fields given previously to the singularity at $r = 0$. With the gauge choice $V = -3\phi$, we find the following behavior at the leading order when $gr \to C$, for a constant C,

$$\begin{aligned} U \sim W \sim \frac{2}{5}\ln(gr - C), \qquad \phi \sim \frac{1}{5}\ln(gr - C), \\ \theta \sim p \sim 0, \qquad \text{and} \qquad k \sim l \sim \frac{\tau}{2} \end{aligned} \tag{4.264}$$

with $\kappa = \tau$. It can be verified that this configuration solves the BPS equations (4.198) to (4.203), and (4.204) in the limit $gr \to C$ for all $SO(5)$, $SO(4,1)$, and



(a) U solution (b) W solution (c) ϕ solution

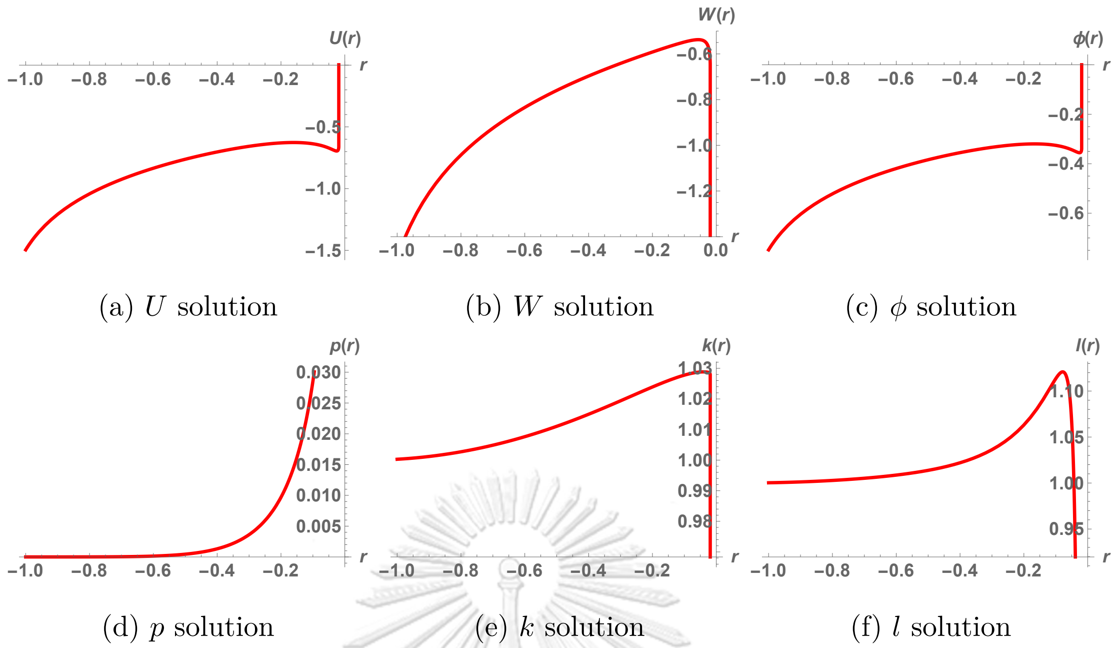
(d) p solution (e) k solution (f) l solution

Figure 4.2: A numerical solution with $g = 16$, $\kappa = 2$, and $\tau = 1$ from the locally AdS_7 configuration in UV as $r \to 1$ to a singularity in the form of an $AdS_3 \times S^3$-sliced DW in IR at $r = 0$ for SO(5) gauge group.

$CSO(4, 0, 1)$ gauge groups.

Examples of the BPS flows from the charged DW in (4.264) as $gr \to C$ to the singularity at $r = 0$ in $SO(5)$, $SO(4, 1)$, and $CSO(4, 0, 1)$ gauge groups are respectively given in Figures 4.3, 4.4, and 4.5 for $C = -1$, $g = 1$, and $\kappa = \tau = 2$. These solutions should describe surface defects within six-dimensional $N = (2, 0)$ SQFTs. For the solution in Figure 4.5, we can see that k is constant along the flow since the BPS equations (4.256) and (4.258) give constant $U - 2\phi$ when $\rho = 0$.

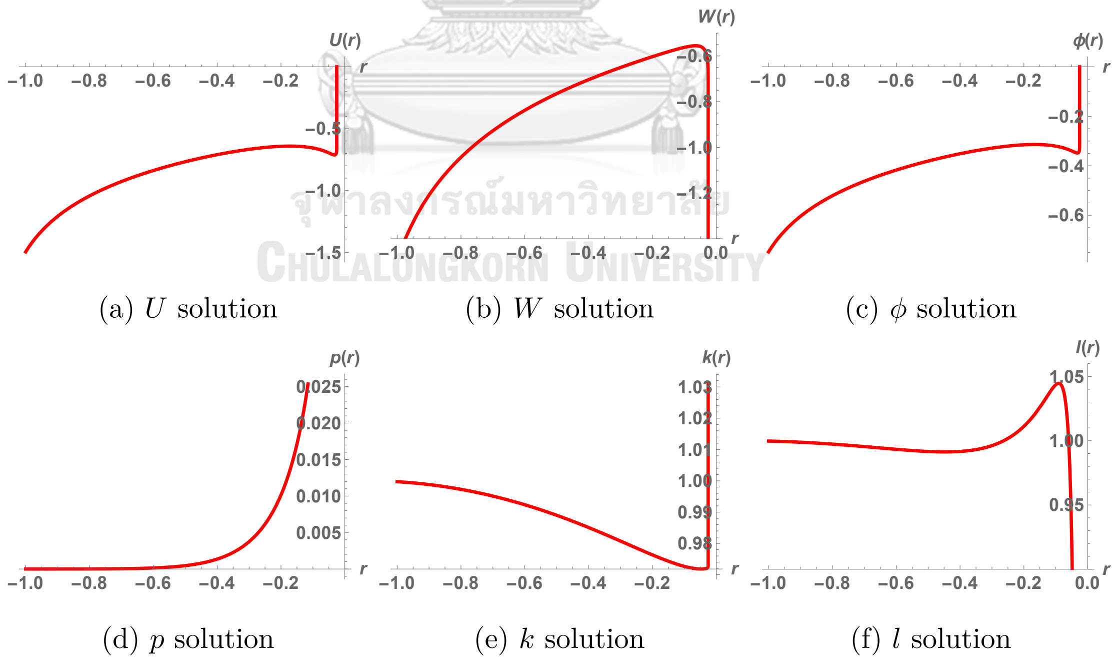
For $SO(5)$ gauge group, it is also possible to find BPS flow solutions interpolating between the asymptotically locally AdS_7 geometry and the charged DW configuration with an intermediate singularity in the presence of non-vanishing $SO(3)$ gauge fields at $r = 0$. With $C = -1$, $g = 1$, $\kappa = \tau = 2$, and the gauge fixing $V = -3\phi$, an example of these flow solutions is shown in Figure 4.6 in which the $SO(3)$ gauge fields vanish at both ends of the flow and become singular at $r = 0$.



(a) U solution (b) W solution (c) ϕ solution

(d) p solution (e) k solution (f) l solution

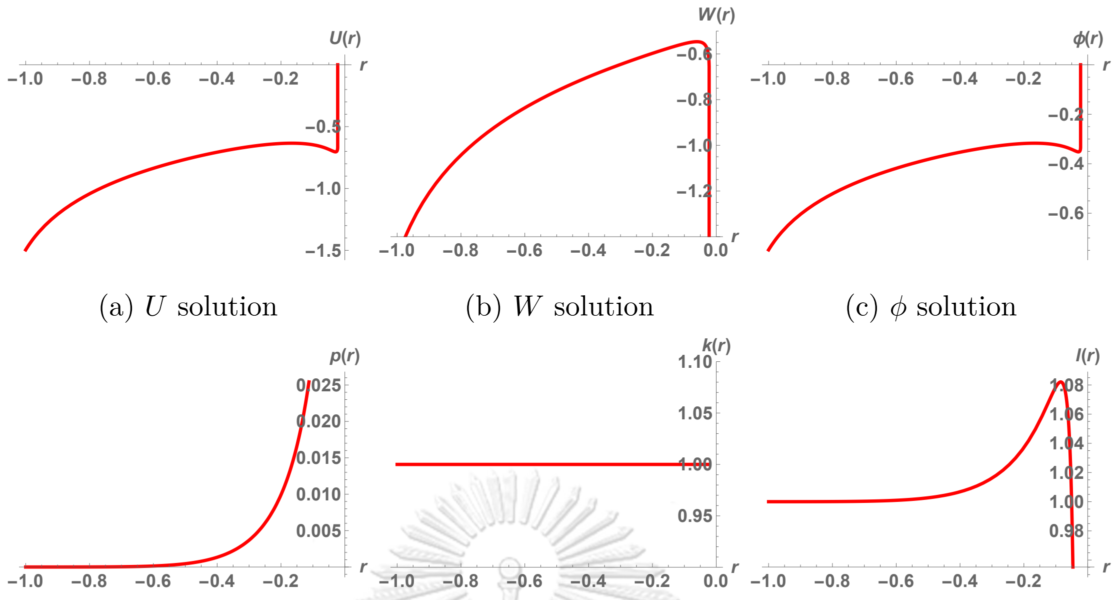
Figure 4.3: A numerical solution with $C = -1$, $g = 1$, $\kappa = \tau = 2$, and $V = -3\phi$ from a charged DW without the $SO(3)$ gauge fields at $r = -1$ to a singularity in the form of an $AdS_3 \times S^3$-sliced DW at $r = 0$ for $SO(5)$ gauge group.



(a) U solution (b) W solution (c) ϕ solution

(d) p solution (e) k solution (f) l solution

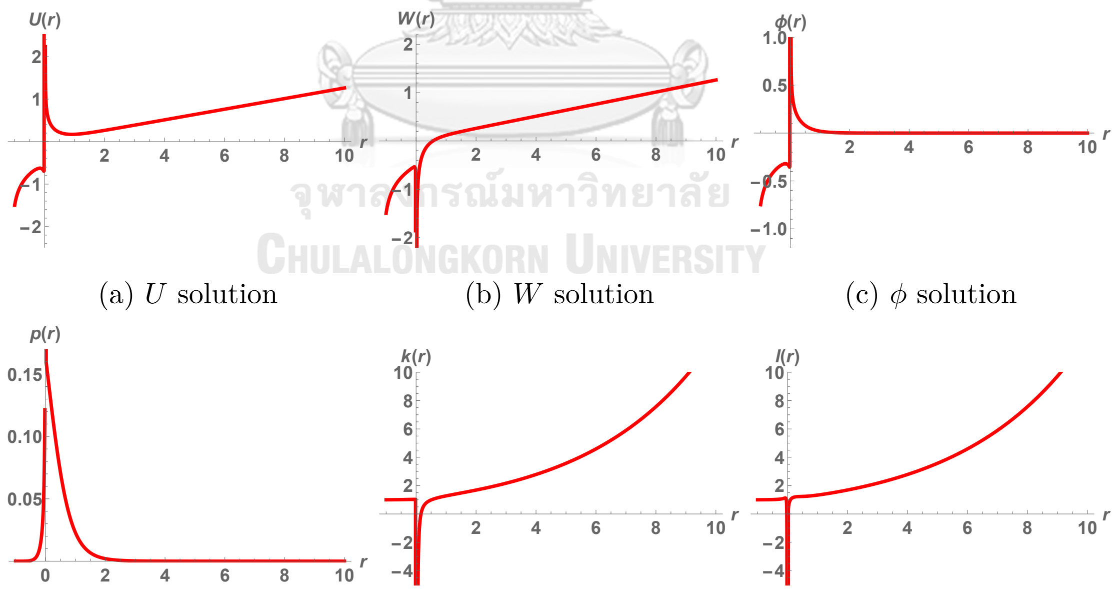
Figure 4.4: A numerical solution with $C = -1$, $g = 1$, $\kappa = \tau = 2$, and $V = -3\phi$ from a charged DW without the $SO(3)$ gauge fields at $r = -1$ to a singularity in the form of an $AdS_3 \times S^3$-sliced DW at $r = 0$ for $SO(4,1)$ gauge group.



(a) U solution (b) W solution (c) ϕ solution

(d) p solution (e) k solution (f) l solution

Figure 4.5: A numerical solution with $C = -1$, $g = 1$, $\kappa = \tau = 2$, and $V = -3\phi$ from a charged DW without the $SO(3)$ gauge fields at $r = -1$ to a singularity in the form of an $AdS_3 \times S^3$-sliced DW at $r = 0$ for $CSO(4, 0, 1)$ gauge group.



(a) U solution (b) W solution (c) ϕ solution

(d) p solution (e) k solution (f) l solution

Figure 4.6: A numerical solution with $C = -1$, $g = 1$, $\kappa = \tau = 2$, and $V = -3\phi$ between a charged DW without the $SO(3)$ gauge fields at $r = -1$ and the locally AdS_7 configuration as $r \to 10$ with a singularity at $r = 0$ for $SO(5)$ gauge group.

4.2.1.2 $SO(3)$ Symmetric Charged Domain Walls

By repeating the same procedure, we can look for charged DWs preserving $SO(3)$ residual symmetry. The $SO(3)$ residual symmetry allows only two non-vanishing $\mathcal{H}_{(3)4}$ and $\mathcal{H}_{(3)5}$. We will choose the following ansatz

$$\mathcal{H}_{\hat{m}\hat{n}\hat{p}4} = k_4(r)e^{-3U(r)}\varepsilon_{\hat{m}\hat{n}\hat{p}}, \qquad \mathcal{H}_{\hat{i}\hat{j}\hat{k}4} = l_4(r)e^{-3W(r)}\varepsilon_{\hat{i}\hat{j}\hat{k}}, \tag{4.265}$$

$$\mathcal{H}_{\hat{m}\hat{n}\hat{p}5} = k_5(r)e^{-3U(r)}\varepsilon_{\hat{m}\hat{n}\hat{p}}, \qquad \mathcal{H}_{\hat{i}\hat{j}\hat{k}5} = l_5(r)e^{-3W(r)}\varepsilon_{\hat{i}\hat{j}\hat{k}}\,. \tag{4.266}$$

With $\mathcal{H}_{(3)4}$ non-vanishing, the $SO(5)$ gamma matrix Γ_4 will appear in the BPS conditions. To evade a further projector, which will break more SUSY, we impose the following conditions

$$k_4(r) = k_5(r)\tanh\phi_2 \qquad \text{and} \qquad l_4(r) = l_5(r)\tanh\phi_2\,, \tag{4.267}$$

which make the coefficient of Γ_4 in the BPS equations vanish.

With the projection conditions (4.197), the coset representative (4.29), and the embedding tensor (4.27), we can find a consistent set of BPS equations if

$$\theta = 0 \qquad \text{and} \qquad \tau = e^{U-W}\kappa\,. \tag{4.268}$$

The latter forbids the possibility of setting $\tau = 0$ or $\kappa = 0$ without ending up with $\tau = \kappa = 0$. Thus, the solutions can be only $AdS_3 \times S^3$-sliced DWs in this case.

The resulting BPS equations take the form

$$U' = \frac{g}{40}e^{V+6\phi_1}\left(3e^{-10\phi_1} + (\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3 + (\rho-\sigma)\sinh 2\phi_3\right), \tag{4.269}$$

$$W' = \frac{g}{40}e^{V+6\phi_1}\left(3e^{-10\phi_1} + (\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3 + (\rho-\sigma)\sinh 2\phi_3\right), \tag{4.270}$$

$$\phi_1' = \frac{g}{40}e^{V+6\phi_1}\left(2e^{-10\phi_1} - (\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3 - (\rho-\sigma)\sinh 2\phi_3\right), \tag{4.271}$$

$$\phi_2' = -\frac{g}{8}e^{V+6\phi_1}(\rho+\sigma)\sinh 2\phi_2\ \text{sech}\ 2\phi_3, \tag{4.272}$$

$$\phi_3' = -\frac{g}{8}e^{V+6\phi_1}\left((\rho+\sigma)\cosh 2\phi_2\sinh 2\phi_3 + (\rho-\sigma)\cosh 2\phi_3\right), \tag{4.273}$$

$$k_5 = \frac{1}{2}e^{3U-W-3\phi_1-\phi_3}\cosh\phi_2\kappa, \tag{4.274}$$

$$l_5 = \frac{1}{2}e^{2W-3\phi_1-\phi_3}\cosh\phi_2\kappa. \tag{4.275}$$

However, these BPS equations are compatible with the field equations if and only if $\phi_2 = 0$ or $\phi_3 = 0$. It should be noted that setting $\phi_3 = 0$ is consistent with the BPS equation (4.273) only for $\rho = \sigma$, so solutions with vanishing ϕ_3 can be obtained only in $SO(5)$, $SO(3,2)$, and $CSO(3,0,2)$ gauge groups. We will find explicit solutions by separately considering various possible values of ρ and σ.

(1) Charged domain walls in $CSO(3,0,2)$ gauge group

For the simplest $CSO(3,0,2)$ gauge group with $\rho = \sigma = 0$, ϕ_2 and ϕ_3 can be consistently truncated out since $\phi_2' = \phi_3' = 0$ in this case. From (4.267), setting $\phi_2 = 0$ directly gives $k_4 = l_4 = 0$. By choosing $V = 0$ gauge choice, we find the following charged DW solution

$$U = W = \frac{3}{8}\ln\left[\frac{gr}{5} + C\right], \qquad \phi_1 = \frac{1}{4}\ln\left[\frac{gr}{5} + C\right], \qquad k_5 = l_5 = \frac{1}{2}\tau \tag{4.276}$$

with an integration constant C.

(2) Charged domain walls in $CSO(4,0,1)$ and $CSO(3,1,1)$ gauge groups

In this case with $\rho = 0$, we consider $CSO(4,0,1)$ and $CSO(3,1,1)$ gauge groups corresponding to $\sigma = 1, -1$, respectively. Choosing $V = -6\phi_1$, we find a charged DW solution, with $\phi_2 = 0$,

$$\phi_3 = \frac{1}{2}\ln\left[\frac{g\sigma r}{4} + C_1\right], \tag{4.277}$$

$$\phi_1 = -\frac{1}{5}\phi_3 + \frac{1}{10}\ln\left[C_2 + e^{4\phi_3}\right], \tag{4.278}$$

$$U = W = \frac{1}{5}\phi_3 + \frac{3}{20}\ln\left[C_2 + e^{4\phi_3}\right], \tag{4.279}$$

$$k_4 = l_4 = 0, \qquad \text{and} \qquad k_5 = l_5 = \frac{1}{2}\tau \tag{4.280}$$

where C_1 and C_2 are integration constants. As stated above, it is not possible to find solutions with $\phi_3 = 0$ in these gauge groups.

(3) Charged domain walls in $SO(4,1)$ gauge group

As in the previous case, it is also not possible to set $\phi_3 = 0$ in non-compact $SO(4,1)$ gauge group with $\sigma = -\rho = 1$. Therefore, we only consider BPS solutions with $\phi_2 = 0$ in this case. Using the same gauge choice $V = -6\phi_1$, we find the following solution

$$e^{2\phi_3} = \tan\left[\frac{gr}{4} + C_1\right], \quad (4.281)$$

$$\phi_1 = -\frac{1}{5}\phi_3 + \frac{1}{10}\ln\left[C_2(e^{4\phi_3}+1) - 1\right], \quad (4.282)$$

$$U = W = \frac{1}{5}\phi_3 - \frac{1}{4}\ln\left[e^{4\phi_3}+1\right] + \frac{3}{20}\ln\left[C_2(e^{4\phi_3}+1) - 1\right], \quad (4.283)$$

$$k_4 = l_4 = 0, \quad (4.284)$$

$$k_5 = l_5 = \frac{1}{2}\tau\cos\left[\frac{gr}{4} + C_1\right]. \quad (4.285)$$

(4) Charged domain walls in $SO(5)$ and $SO(3,2)$ gauge groups

We finally look at the last possibility with $\rho = \sigma = \pm 1$ corresponding to $SO(5)$ and $SO(3,2)$ gauge groups in which either $\phi_2 = 0$ or $\phi_3 = 0$ is possible. With $\phi_2 = 0$ and $V = -6\phi_1$, we find the following solution

$$\phi_3 = \frac{1}{2}\ln\left[\frac{e^{\frac{g\rho r}{2}} - C_1}{e^{\frac{g\rho r}{2}} + C_1}\right], \quad (4.286)$$

$$\phi_1 = -\frac{1}{5}\phi_3 + \frac{1}{10}\ln\left[C_2(e^{4\phi_3}-1) + 1\right], \quad (4.287)$$

$$U = W = \frac{1}{5}\phi_3 - \frac{1}{4}\ln\left[e^{4\phi_3}-1\right] + \frac{3}{20}\ln\left[C_2(e^{4\phi_3}-1) + 1\right] \quad (4.288)$$

together with

$$k_4 = l_4 = 0 \qquad \text{and} \qquad k_5 = l_5 = \frac{\tau}{2\sqrt{e^{4\phi_3}-1}}. \quad (4.289)$$

For $\phi_3 = 0$, we find the same solution as in (4.286) to (4.288) with ϕ_3 replaced by ϕ_2, but the solution for k_4, k_5, l_4 and l_5 is now given by

$$k_4 = l_4 = \frac{(e^{2\phi_2}-1)\tau}{4\sqrt{e^{4\phi_2}-1}} \qquad \text{and} \qquad k_5 = l_5 = \frac{(e^{2\phi_2}+1)\tau}{4\sqrt{e^{4\phi_2}-1}}. \quad (4.290)$$

Unlike the other cases, this solution has two non-vanishing three-form fluxes.

We end this section by commenting on solutions with non-vanishing $SO(3)$ gauge fields. Repeating the same procedure as in the $SO(4)$ symmetric case leads to a set of BPS equations together with the following two constraints

$$p' = 0 \qquad \text{and} \qquad p = \frac{\kappa - \tau e^{W-U}}{g\kappa}. \quad (4.291)$$

It turns out that the compatibility between the resulting BPS equations and the second-order field equations needs

$$\tau(e^{W}\tau - e^{U}\kappa) = 0\,. \tag{4.292}$$

For $\tau = 0$, the magnetic charge p is constant as required by the second condition in (4.291), but the three-form flux vanishes in this case due to (4.268). This $\tau = 0$ case corresponds to performing a topological twist along the S^3 part. We will return to this type of supersymmetric solutions in Section 4.3. On the other hand, setting $\tau = e^{U-W}\kappa$ leads to non-vanishing three-form fluxes, but equation (4.291) gives vanishing $SO(3)$ gauge fields. This $\tau \neq 0$ case corresponds to the charged DWs without the $SO(3)$ gauge fields given above. Similar to the result obtained in Section 3.1.3 for the matter-coupled $SO(4)$ gauged supergravity, there does not seem to be solutions with both non-vanishing $SO(3)$ gauge fields and three-form fluxes, at least for the ansatz considered here.

4.2.1.3 $SO(2) \times SO(2)$ Symmetric Charged Domain Walls

We finally consider charged DWs with $SO(2) \times SO(2)$ residual symmetry using the scalar coset representative (4.48) and the embedding tensor (4.49). As in the previous case, we find that a consistent set of BPS equations can be found if and only if $\theta = 0$ and $\tau = e^{U-W}\kappa$. With the three-form flux (4.192), which is manifestly invariant under $SO(2) \times SO(2)$ symmetry, and the projectors given in (4.197), the resulting BPS equations read

$$U' = W' = \frac{g}{40}e^{V}(2e^{-2\phi_1} + \rho e^{4(\phi_1+\phi_2)} + 2\sigma e^{-2\phi_2}), \tag{4.293}$$

$$\phi_1' = \frac{g}{20}e^{V}(3e^{-2\phi_1} - \rho e^{4(\phi_1+\phi_2)} - 2\sigma e^{-2\phi_2}), \tag{4.294}$$

$$\phi_2' = \frac{g}{20}e^{V}(3\sigma e^{-2\phi_2} - \rho e^{4(\phi_1+\phi_2)} - 2e^{-2\phi_1}), \tag{4.295}$$

$$k = \frac{1}{2}e^{2U-2(\phi_1+\phi_2)}\tau, \tag{4.296}$$

$$l = \frac{1}{2}e^{3W-U-2(\phi_1+\phi_2)}\tau\,. \tag{4.297}$$

By choosing $V = 2\phi_1$, we obtain the solution

$$\phi_1 = -\frac{1}{10}\ln\left[e^{C_1-\frac{gr}{2}}+\rho\right]-\frac{1}{5}\ln\left[e^{C_2-\frac{gr}{2}}+\sigma\right], \quad (4.298)$$

$$\phi_2 = -\frac{3}{2}\phi_1-\frac{1}{4}\ln\left[e^{C_1-\frac{gr}{2}}+\rho\right], \quad (4.299)$$

$$U = W = \frac{1}{8}gr+\frac{1}{20}\ln\left[e^{C_1-\frac{gr}{2}}+\rho\right]+\frac{1}{10}\ln\left[e^{C_2-\frac{gr}{2}}+\sigma\right], \quad (4.300)$$

$$k = l = \frac{1}{2}\tau e^{\frac{gr}{4}}\sqrt{e^{C_1-\frac{gr}{2}}+\rho} \quad (4.301)$$

with two integration constants C_1 and C_2. This solution is the $SO(2)\times SO(2)$ symmetric flat DW found in Section 4.1.1.4 including a dyonic profile for the three-form flux. Note that coupling the solution to $SO(3)$ gauge fields is not possible in this case due to the absence of any unbroken $SO(3)$ gauge symmetry.

4.2.1.4 Uplifted Solutions in Ten and Eleven Dimensions

In this section, we give the uplifted solutions for $SO(5)$ and $CSO(4,0,1)$ gauge groups using consistent truncations of eleven-dimensional supergravity on S^4 and type IIA theory on S^3, respectively. However, we will not consider uplifting of the solutions with non-vanishing $SO(3)$ gauge fields since the uplifted solutions are not really useful in this case due to the lack of analytic solutions.

4.2.1.4.1 Uplift to Eleven Dimensions

We first consider uplifting the seven-dimensional solutions in $SO(5)$ gauge group to eleven-dimensional supergravity. We start from the $SO(4)$ symmetric solution with the $SL(5)/SO(5)$ scalar matrix

$$\mathcal{M}_{MN} = \text{diag}(e^{2\phi}, e^{2\phi}, e^{2\phi}, e^{2\phi}, e^{-8\phi}), \quad (4.302)$$

and the coordinates on S^4 given by

$$\mu^M = (\mu^i, \mu^5) = (\sin\xi\hat{\mu}^i, \cos\xi), \qquad i = 1,2,3,4 \quad (4.303)$$

where $\hat{\mu}^i$ are S^3 coordinates satisfying $\hat{\mu}^i\hat{\mu}^i = 1$. With the formulae given in Appendix C.1, the eleven-dimensional metric and the four-form field strength are given by

$$\begin{aligned} d\hat{s}_{11}^2 &= \Delta^{\frac{1}{3}}\left(e^{2U(r)}ds^2_{M_3}+e^{2V(r)}dr^2+e^{2W(r)}ds^2_{S^3}\right) \\ &+\frac{16}{g^2}\Delta^{-\frac{2}{3}}\left[e^{-8\phi}\sin^2\xi d\xi^2+e^{2\phi}(\cos^2\xi d\xi^2+\sin^2\xi d\Omega^2_{(3)})\right], \qquad (4.304) \\ \hat{F}_{(4)} &= \frac{64}{g^3}\Delta^{-2}\sin^4\xi\left(\mathcal{U}\sin\xi d\xi-10e^{6\phi}\phi'\cos\xi dr\right)\wedge\epsilon_{(3)} \\ &-2\cos\xi e^{8\phi}\left(ke^{3W+V-3U}dr\wedge\text{vol}_{S^3}-le^{3U+V-3W}dr\wedge\text{vol}_{M_3}\right) \\ &-\frac{8}{g}\sin\xi(k\text{vol}_{M_3}+l\text{vol}_{S^3})\wedge d\xi \qquad (4.305) \end{aligned}$$

with $d\Omega^2_{(3)}=d\hat{\mu}^i d\hat{\mu}^i$ being the metric on a unit S^3 and

$$\Delta = e^{8\phi}\cos^2\xi+e^{-2\phi}\sin^2\xi, \qquad (4.306)$$

$$\mathcal{U} = (e^{16\phi}-4e^{6\phi})\cos^2\xi-(e^{6\phi}+2e^{-4\phi})\sin^2\xi, \qquad (4.307)$$

$$\epsilon_{(3)} = \frac{1}{3!}\varepsilon_{ijkl}\hat{\mu}^i d\hat{\mu}^j\wedge d\hat{\mu}^k\wedge d\hat{\mu}^l. \qquad (4.308)$$

We can see that the $SO(4)$ unbroken symmetry of the seven-dimensional solution is the isometry of the S^3 inside the S^4. In this case, the three-manifold M_3 can be either Mkw_3 or AdS_3. This solution should describe a bound state of M2- and M5-branes similar to the solutions considered in [62] due to the dyonic solution of the three-form field.

We can repeat a similar procedure for the $SO(3)$ symmetric solutions. With the index $M=(a,4,5)$, $a=1,2,3$, the $SL(5)/SO(5)$ scalar matrix is given by

$$\mathcal{M}=\begin{pmatrix} e^{4\phi_1}\mathbf{I}_3 & 0 \\ 0 & e^{-6\phi_1}M_2 \end{pmatrix} \qquad (4.309)$$

with the 2×2 matrix M_2 given by

$$M_2=\begin{pmatrix} e^{2\phi_3}\cosh^2\phi_2+\sinh^2\phi_2 & \sinh\phi_2\cosh\phi_2(1+e^{-2\phi_3}) \\ \sinh\phi_2\cosh\phi_2(1+e^{2\phi_3}) & e^{-2\phi_3}\cosh^2\phi_2+\sinh^2\phi_2 \end{pmatrix}. \qquad (4.310)$$

We now separately consider the uplifted solutions for the two cases with $\phi_2=0$ and $\phi_3=0$. We will also rename $k_5=k$ and $l_5=l$ together with $k_4=k\tanh\phi_2$ and $l_4=l\tanh\phi_2$. Recall also that we only have $M_3=AdS_3$ in this case.

For $\phi_2=0$ and the S^4 coordinates

$$\mu^M=(\cos\xi\hat{\mu}^a,\sin\xi\cos\psi,\sin\xi\sin\psi) \qquad (4.311)$$

with $\hat{\mu}^a\hat{\mu}^a = 1$, we find the eleven-dimensional metric

$$\begin{aligned} d\hat{s}_{11}^2 = \ & \Delta^{\frac{1}{3}}\left(e^{2U}ds^2_{AdS_3} + e^{2V}dr^2 + e^{2W}ds^2_{S^3}\right) + \frac{16}{g^2}\Delta^{-\frac{2}{3}}\left[e^{4\phi_1}(\sin^2\xi d\xi^2\right. \\ & + \cos^2\xi d\hat{\mu}^a d\hat{\mu}^a) + e^{-6\phi_1}\left\{\sin^2\xi(e^{2\phi_3}\sin^2\psi + e^{-2\phi_3}\cos^2\psi)d\psi^2\right. \\ & \left.\left. -\sin 2\psi\sin 2\xi\sinh 2\phi_3 d\xi d\psi + \cos^2\xi(e^{2\phi_3}\cos^2\psi + e^{-2\phi_3}\sin^2\psi)d\xi^2\right\}\right], \end{aligned} \tag{4.312}$$

and the four-form field strength

$$\begin{aligned} \hat{F}_{(4)} = \ & -2e^{6\phi_1+2\phi_3}\sin\xi\sin\psi dr\wedge(ke^{3W+V-3U}\text{vol}_{S^3} - le^{3U+V-3W}\text{vol}_{AdS_3}) \\ & +\frac{8}{g}(k\text{vol}_{AdS_3} + l\text{vol}_{S^3})\wedge(\cos\xi\sin\psi d\xi + \sin\xi\cos\psi d\psi) \\ & -\frac{64}{g^3}\Delta^{-2}\epsilon_{(2)}\wedge\left[\cos^2\xi\sin\xi\mathcal{U}d\xi\wedge d\psi + \phi_3' e^{12\phi_1}\sin^3\xi\cos^2\xi\sin 2\psi dr\wedge d\xi\right. \\ & -e^{2\phi_1-2\phi_3}\sin\xi\cos^3\xi dr\wedge\{(6\phi_1'\sin\xi + 2\phi_3'\sin\xi\cos\psi)d\psi - 2\phi_3'\cos\xi\times \\ & \sin\psi d\xi\} - 2\phi_1' e^{2\phi_1}\sin 2\xi\cos^2\xi dr\wedge\left\{(e^{-2\phi_3} - e^{2\phi_3})\sin\psi\cos\psi\cos\xi d\xi\right. \\ & \left.\left. +\sin\xi(e^{2\phi_3}\sin^2\psi + e^{-2\phi_3}\cos^2\psi)d\psi\right\}\right] \end{aligned} \tag{4.313}$$

in which

$$\Delta = e^{-4\phi_1}\cos^2\xi + e^{6\phi_1}\sin^2\xi(e^{-2\phi_3}\cos^2\psi + e^{2\phi_3}\sin^2\psi), \tag{4.314}$$

$$\begin{aligned} \mathcal{U} = \ & \frac{1}{2}e^{2\phi_1}\left[\sin^2\xi(1-e^{-4\phi_3})\{3e^{2\phi_3}\cos 2\psi - e^{10\phi_1}(1+\cos 2\psi - 2e^{4\phi_3}\sin^2\psi)\}\right. \\ & \left. +(\cos 2\xi - 5)\cosh 2\phi_3\right] - e^{-8\phi_1}\cos^2\xi, \end{aligned} \tag{4.315}$$

$$\epsilon_{(2)} = \frac{1}{2}\varepsilon_{abc}\hat{\mu}^a d\hat{\mu}^b\wedge\hat{\mu}^c. \tag{4.316}$$

For $\phi_3 = 0$, we obtain the eleven-dimensional metric

$$\begin{aligned} d\hat{s}_{11}^2 = \ & \Delta^{\frac{1}{3}}\left(e^{2U}ds^2_{AdS_3} + e^{2V}dr^2 + e^{2W}ds^2_{S^3}\right) + \frac{16}{g^2}\Delta^{-\frac{2}{3}}\left[e^{4\phi_1}(\sin^2\xi d\xi^2\right. \\ & + \cos^2\xi d\hat{\mu}^a d\hat{\mu}^a) + e^{-6\phi_1}\sinh 2\phi_2\{\sin 2\psi(\cos^2\xi d\xi^2 - \sin^2\xi d\psi^2) \\ & \left. +\sin 2\xi\cos 2\psi d\psi d\xi\} + e^{-6\phi_1}\cosh 2\phi_2(\cos^2\xi d\xi^2 + \sin^2\xi d\psi^2)\right] \end{aligned} \tag{4.317}$$

where

$$\Delta = e^{-4\phi_1}\cos^2\xi + e^{6\phi_1}\sin^2\xi(\cosh 2\phi_2 - \sin 2\psi\sinh 2\phi_2), \tag{4.318}$$

together with the four-form field strength

$$\begin{aligned}\hat{F}_{(4)} &= 2\sin\xi e^{6\phi_1+V}(\cos\psi\tanh\phi_2-\sin\psi)dr\wedge(ke^{3W-3U}\text{vol}_{S^3}-le^{3U-3W}\text{vol}_{AdS_3})\\ &\quad+\frac{8}{g}(k\text{vol}_{AdS_3}+l\text{vol}_{S^3})\wedge[(\tanh\phi_2\cos\psi+\sin\psi)\cos\xi d\xi\\ &\quad+\sin\xi(\cos\psi-\tanh\phi_2\sin\psi)]-\frac{64}{g^3}\mathcal{U}\Delta^{-2}\sin\xi\cos^2\xi\epsilon_{(2)}\wedge d\xi\wedge d\psi\\ &\quad+\frac{64}{g^3}\Delta^{-2}dr\wedge\epsilon_{(2)}\wedge\Big[\frac{1}{2}e^{12\phi_1}\phi_2'\sin\xi\sin^2 2\xi\cos 2\psi d\xi\\ &\quad+\frac{1}{2}e^{-4\phi_1}\cos^2\xi\sin 2\xi\Big\{\sin^2\xi\left(e^{6\phi_1}\cosh 2\phi_2\right)'d\psi\\ &\quad+(e^{6\phi_1}\sinh 2\phi_2)'(\cos\xi\cos 2\psi d\xi-\sin\xi\sin 2\psi d\psi)\Big\}\\ &\quad+2\phi_1'e^{2\phi_1}\cos^2\xi\sin 2\xi\left\{\sin\xi\cosh 2\phi_2 d\psi\right.\\ &\quad\left.-\sinh 2\phi_2(\sin\xi\sin 2\psi d\psi-\cos 2\psi d\xi)\right\}\Big] \qquad (4.319)\end{aligned}$$

in which

$$\begin{aligned}\mathcal{U} &= \sin^2\xi\left[3e^{2\phi_1}\sin 2\psi\sinh 2\phi_2+e^{12\phi_1}(6\cosh^2 2\phi_2-\sin 2\psi\sinh 4\phi_2)\right]\\ &\quad+(2e^{-4\phi_1}-3e^{-8\phi_1})\cos^2\xi+\frac{1}{2}e^{2\phi_1}\cosh 2\phi_2(\cos 2\xi-5). \qquad (4.320)\end{aligned}$$

These uplifted solutions should describe bound states of M2- and M5-branes with different transverse spaces and are expected to be dual to surface defects in the six-dimensional $N=(2,0)$ SCFT. The solution with $SO(2)\times SO(2)$ symmetry can similarly be uplifted, but we will not give them here due to their complexity.

4.2.1.4.2 Uplift to Type IIA Theory

We now provide a similar analysis for $CSO(4,0,1)$ gauge group in order to find uplifted solutions in type IIA supergravity. Relevant formulae are collected in Appendix C.2. In this case, gauge fields, massive three-forms, and axions vanish. The ten-dimensional fields are then only the metric, the dilaton, and the NS-NS two-form field. We expect the uplifted solutions to describe bound states of NS5-branes and the fundamental strings.

We begin with the solution with $SO(4)$ symmetry in which the $SL(4)/SO(4)$ scalar matrix is given by $\widetilde{\mathcal{M}}_{ij}=\delta_{ij}$. The ten-dimensional metric, NS-NS three-form flux, and the dilaton are given by

$$
\begin{aligned}
d\hat{s}^2_{10} &= e^{\frac{3}{2}\phi_0}\left(e^{2U}ds^2_{M_3}+e^{2V}dr^2+e^{2W}ds^2_{S^3}\right)+\frac{16}{g^2}e^{-\frac{5}{2}\phi_0}d\Omega^2_{(3)}, && (4.321)\\
\hat{H}_{(3)} &= \frac{128}{g^3}\epsilon_{(3)}+\frac{8}{g}(k\text{vol}_{M_3}+l\text{vol}_{S^3}), && (4.322)\\
\hat{\varphi} &= 5\phi_0\,. && (4.323)
\end{aligned}
$$

It should be noted that we have a constant NS-NS flux in this case.

For the solutions with $SO(3)$ unbroken symmetry, we parametrize the $SL(4)/SO(4)$ scalar matrix as

$$\widetilde{\mathcal{M}}_{ij}=\text{diag}(e^{2\phi},e^{2\phi},e^{2\phi},e^{-6\phi}), \qquad (4.324)$$

and choose the S^3 coordinates to be

$$\mu^i=(\sin\xi\hat{\mu}^a,\cos\xi), \qquad a=1,2,3 \qquad (4.325)$$

with $\hat{\mu}^a$ being the coordinates on S^2 subject to the condition $\hat{\mu}^a\hat{\mu}^a=1$. With all these ingredients and writing $k=k_5$ and $l=l_5$, we find that the ten-dimensional fields are given by

$$
\begin{aligned}
d\hat{s}^2_{10} &= \frac{16}{g^2}e^{-\frac{5}{2}\phi_0}\Delta^{-\frac{3}{4}}\left[\left(e^{-6\phi}\sin^2\xi+e^{2\phi}\cos^2\xi\right)d\xi^2+\sin^2\xi e^{2\phi}d\hat{\mu}^a d\hat{\mu}^a\right]\\
&\quad +e^{\frac{3}{2}\phi_0}\Delta^{\frac{1}{4}}\left(e^{2U}ds^2_{AdS_3}+e^{2V}dr^2+e^{2W}ds^2_{S^3}\right), && (4.326)\\
\hat{H}_{(3)} &= \frac{64}{g^3}\Delta^{-2}\sin^3\xi\left(\mathcal{U}\sin\xi d\xi+8e^{4\phi}\cos\xi\phi' dr\right)\wedge\epsilon_{(2)}\\
&\quad +\frac{8}{g}(k\text{vol}_{AdS_3}+l\text{vol}_{S^3}), && (4.327)\\
e^{2\hat{\varphi}} &= \Delta^{-1}e^{10\phi_0} && (4.328)
\end{aligned}
$$

in which

$$
\begin{aligned}
\Delta &= e^{6\phi}\cos^2\xi+e^{-2\phi}\sin^2\xi, \qquad \epsilon_{(2)}=\frac{1}{2}\varepsilon_{abc}\hat{\mu}^a d\hat{\mu}^b\wedge d\hat{\mu}^c,\\
\mathcal{U} &= e^{12\phi}\cos^2\xi-e^{-4\phi}\sin^2\xi-e^{4\phi}(\sin^2\xi+3\cos^2\xi). && (4.329)
\end{aligned}
$$

The solutions for ϕ_0 and ϕ are obtained from ϕ_1 and ϕ_3 respectively given in (4.278) and (4.277) with $\sigma=1$ by the following relations

$$\phi=\frac{1}{4}(5\phi_1-\phi_3) \qquad \text{and} \qquad \phi_0=-\frac{1}{4}(\phi_3+3\phi_1). \qquad (4.330)$$

4.2.2 Gaugings in $\overline{\mathbf{40}}$ Representation

We repeat the same analysis for gaugings from $\overline{\mathbf{40}}$ representation in this section. To find charged DW solutions in the following analyses, we will use the same ansatz as in the previous case. However, for gaugings in $\overline{\mathbf{40}}$ representation, there are no massive three-form fields $S^M_{(3)}$. The modified three-forms given in (4.190) correspond solely to the two-form fields $B_{(2)M}$ in this case.

4.2.2.1 *SO*(4) Symmetric Charged Domain Walls

Since only $SO(4)$ gauge group can accommodate $SO(4)$ residual symmetry, the embedding tensor component, in this case, takes the simple form of $w^{ij} = \delta^{ij}$ with $i, j = 1, \ldots, 4$. In the $SO(4)$ gauged theory, there are four massive two-form fields $B_{(2)i}$ and one massless two-form field $B_{(2)5}$ with the latter being an $SO(4)$ singlet. We take the ansatz for $B_{(2)5}$ as given in (4.195). With the projection conditions

$$\gamma^{\hat{3}}\epsilon^a_0 = -(\Gamma_5)^a{}_b\epsilon^b_0 = \epsilon^a_0, \tag{4.331}$$

and $\widetilde{\mathcal{M}}_{ij} = \delta_{ij}$, the BPS equations are given by

$$U' = W' = \frac{1}{5}e^V\left(2e^{-2\phi_0}g\sec 2\theta - e^{-U}\tau\tan 2\theta\right), \tag{4.332}$$

$$\phi_0' = \frac{1}{10}e^V\left(2e^{-2\phi_0}g\sec 2\theta - e^{-U}\tau\tan 2\theta\right), \tag{4.333}$$

$$k = -\frac{1}{2}e^{2U-4\phi_0}\tau, \qquad \theta' = 0, \tag{4.334}$$

$$l = -\frac{1}{2}e^{2U-4\phi_0}\tau\sec 2\theta + 3e^{3U-6\phi_0}g\tan 2\theta \tag{4.335}$$

together with an algebraic constraint

$$\kappa = \tau\sec 2\theta - 2e^{U-2\phi_0}g\tan 2\theta\,. \tag{4.336}$$

We find that θ is constant in this case. Choosing $V = 0$, we find the solution

$$U = W = 2\phi_0, \tag{4.337}$$

$$e^{2\phi_0} = \frac{2}{5}gr\sec 2\theta - \frac{1}{5}\tau r\tan 2\theta + C, \tag{4.338}$$

$$k = -\frac{1}{2}\tau, \tag{4.339}$$

$$l = -\frac{1}{2}\tau\sec 2\theta + g\tan 2\theta \tag{4.340}$$

with an integration constant C. For a particular value of $\theta = 0$, we find the following solution

$$U = W = 2\phi_0, \qquad e^{2\phi_0} = \frac{2}{5}gr + C, \qquad k = l = -\frac{1}{2}\tau\,. \tag{4.341}$$

4.2.2.1.1 Coupling to $SO(3)$ Gauge Fields

We now couple the charged DW solutions to $SO(3)$ gauge fields. From (4.331), the projector $(\Gamma_5)^a{}_b\epsilon_0^b = -\epsilon_0^a$ implies that the non-vanishing gauge fields correspond to the self-dual $SO(3) \subset SO(4)$ in this case. We then choose

$$A^{23}_{(1)} = A^{14}_{(1)} = \frac{\kappa}{16}p(r)e^{-W(r)}e^{\hat{4}}, \tag{4.342}$$
$$A^{31}_{(1)} = A^{24}_{(1)} = \frac{\kappa}{16}p(r)e^{-W(r)}e^{\hat{5}}, \tag{4.343}$$
$$A^{12}_{(1)} = A^{34}_{(1)} = \frac{\kappa}{16}p(r)e^{-W(r)}e^{\hat{6}} \tag{4.344}$$

with the two-form field strengths given by

$$F^{12}_{(2)} = F^{34}_{(2)} = e^{-V-W}\frac{\kappa}{16}p'e^{\hat{3}} \wedge e^{\hat{6}} + e^{-2W}\frac{\kappa^2}{32}p(2-gp)e^{\hat{4}} \wedge e^{\hat{5}}, \tag{4.345}$$
$$F^{23}_{(2)} = F^{14}_{(2)} = e^{-V-W}\frac{\kappa}{16}p'e^{\hat{3}} \wedge e^{\hat{4}} + e^{-2W}\frac{\kappa^2}{32}p(2-gp)e^{\hat{5}} \wedge e^{\hat{6}}, \tag{4.346}$$
$$F^{31}_{(2)} = F^{24}_{(2)} = e^{-V-W}\frac{\kappa}{16}p'e^{\hat{3}} \wedge e^{\hat{5}} + e^{-2W}\frac{\kappa^2}{32}p(2-gp)e^{\hat{6}} \wedge e^{\hat{4}}\,. \tag{4.347}$$

Since $Z^{ij,5}$ components vanish in this case, the two-form field $B_{(2)5}$ does not contribute to the modified two-forms so that $\mathcal{F}^{ij}_{(2)} = F^{ij}_{(2)}$.

Imposing the projection conditions (4.255) and (4.331), we find BPS equations of the form

$$\begin{aligned} U' = {} & \frac{e^{V-2(W+\phi_0)}}{80\cos 2\theta}\Big[16e^{2W}\left(g(3\cos 4\theta - 1) + 2e^{2\phi_0-U}\tau\sin 2\theta\right) \\ & -3e^{4\phi_0}\left(\kappa^2 p(gp-2)(\cos 4\theta - 3) - 8e^{W-2\phi_0}\kappa(gp-1)\sin 4\theta\right)\Big], \end{aligned} \tag{4.348}$$

$$\begin{aligned} W' = {} & \frac{e^{V-2(W+\phi_0)}}{40\cos 2\theta}\Big[8e^{2W}\left(2g(2-\cos 4\theta) - 3e^{2\phi_0-U}\tau\sin 2\theta\right) \\ & +e^{4\phi_0}\left(\kappa^2 p(gp-2)(\cos 4\theta - 8) - 8e^{W-2\phi_0}\kappa(gp-1)\sin 4\theta\right)\Big], \end{aligned} \tag{4.349}$$

$$\begin{aligned} \phi_0' = {} & \frac{e^{V-2(W+\phi_0)}}{160\cos 2\theta}\Big[16e^{2W}\left(g(3\cos 4\theta - 1) + 2e^{2\phi_0-U}\tau\sin 2\theta\right) \\ & +3e^{4\phi_0}\left(\kappa^2 p(gp-2)(3-\cos 4\theta) + 8e^{W-2\phi_0}\kappa(gp-1)\sin 4\theta\right)\Big], \end{aligned} \tag{4.350}$$

$$\theta' = \frac{e^{V-2(W+\phi_0)}}{16}\big[24e^{W+2\phi_0}\left(e^{W-U}\tau+\kappa(gp-1)\cos 2\theta\right)$$
$$-3\left(16ge^{2W}-e^{4\phi_0}\kappa^2 p(gp-2)\right)\sin 2\theta\big], \qquad (4.351)$$
$$k = -\frac{1}{2}e^{2U-4\phi_0}\tau, \qquad (4.352)$$
$$l = \frac{1}{8}e^{3W-6\phi_0}\big[-16g\tan 2\theta+8e^{2\phi_0-U}\tau\sec 2\theta$$
$$+3e^{4\phi_0-2W}\left(\kappa^2 p(gp-2)\tan 2\theta+4e^{W-2\phi_0}\kappa(gp-1)\right)\big], \qquad (4.353)$$
$$p' = \frac{e^{V-W-4\phi_0}}{2\kappa}\big[8e^{W+2\phi_0}\left(e^{W-U}\tau+\kappa(gp-1)\cos 2\theta\right)$$
$$-\left(16ge^{2W}-e^{4\phi_0}\kappa^2 p(gp-2)\right)\sin 2\theta\big]. \qquad (4.354)$$

It can be verified that these equations fully satisfy the field equations without any additional constraint.

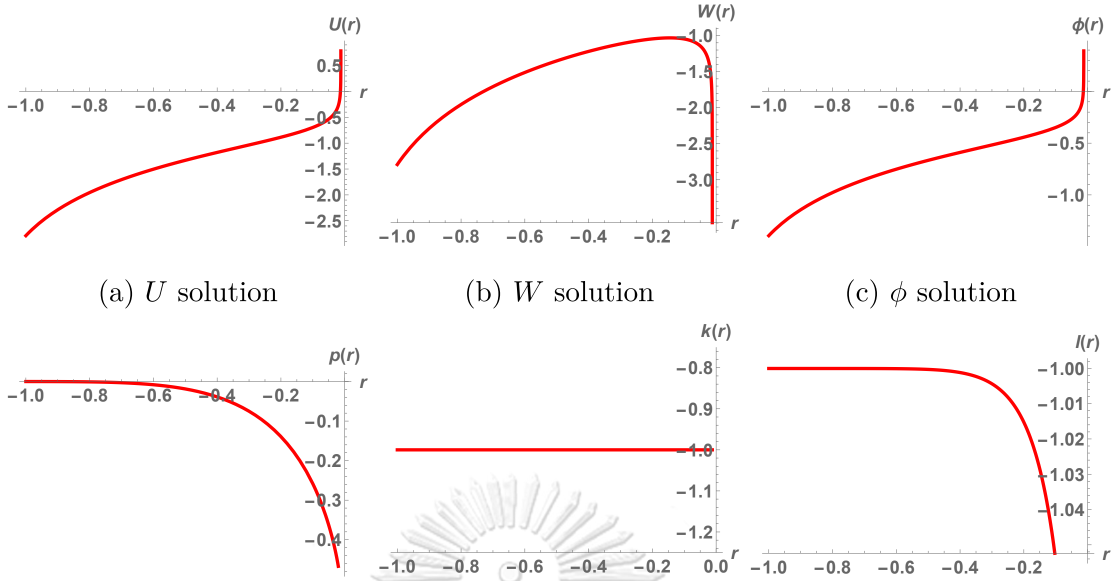
Since, in $SO(4)$ gauge group, there is no an asymptotically locally AdS_7 configuration, we will consider only flow solutions from the charged DW without $SO(3)$ gauge fields given in (4.337) to (4.340) to a singular solution with $SO(3)$ gauge fields non-vanishing. To find numerical solutions, we will consider the charged DW with $\theta = 0$ given in (4.341) for simplicity. As $r \to -\frac{5C}{2g}$, we impose the following boundary conditions

$$U \sim W \sim \ln\left[\frac{2gr}{5}+C\right], \qquad \phi \sim \frac{1}{2}\ln\left[\frac{2gr}{5}+C\right],$$
$$p \sim 0, \qquad k \sim l \sim -\frac{\tau}{2} \qquad (4.355)$$

with $\kappa = \tau$. An example of these BPS flows is shown in Figure 4.7. From this solution, we can see that k is constant along the flow since the above BPS equations give $U' = 2\phi_0'$ that implies the constancy of $U - 2\phi_0$. It should be noted that this solution is similar to that in $CSO(4,0,1)$ gauge group given in Figure 4.5. We also expect this solution to describe a conformal surface defect within a six-dimensional $N = (2,0)$ SQFT.

4.2.2.2 $SO(3)$ Symmetric Charged Domain Walls

We now look for more complicated solutions with $SO(3)$ symmetry. Gauge groups with an $SO(3)$ subgroup are $SO(4)$, $SO(3,1)$, and $CSO(3,0,1)$ corresponding to $\rho = 1, -1, 0$ in the embedding tensor w^{ij} from (4.118).



(a) U solution (b) W solution (c) ϕ solution

(d) p solution (e) k solution (f) l solution

Figure 4.7: A numerical solution with $g = 1$, $\kappa = \tau = 2$, $C = \frac{2}{5}$, and $V = 0$ from a charged DW without the $SO(3)$ gauge fields at $r = -1$ to a singularity at $r = 0$ for $SO(4)$ gauge group.

In this case, there are two two-form fields, $B_{(2)4}$ and $B_{(2)5}$, which are $SO(3)$ singlets. For $CSO(3,0,1)$ gauge group, both of them are massless. For the other two gauge groups, $B_{(2)4}$ is massive while $B_{(2)5}$ is massless. However, we are not able to consistently incorporate $B_{(2)4}$ in the BPS equations. We accordingly restrict ourselves to the solutions with only $\mathcal{H}_{(3)5}$ non-vanishing.

To find BPS equations, we use the same ansatze for the $SL(4)/SO(4)$ coset (4.119) and the modified three-forms (4.192), and impose the projection conditions (4.331). Consistency with the field equations also gives rise to the conditions given in (4.268). With all these, the resulting BPS equations are given by

$$U' = W' = \frac{g}{10} e^{V-6\phi-2\phi_0}(3e^{8\phi_1} + \rho), \tag{4.356}$$

$$\phi_0' = \frac{g}{20} e^{V-6\phi-2\phi_0}(3e^{8\phi_1} + \rho), \tag{4.357}$$

$$\phi' = -\frac{g}{4} e^{V-6\phi-2\phi_0}(3e^{8\phi_1} - \rho), \tag{4.358}$$

$$k = -\frac{1}{2} e^{3U-W-4\phi_0}\kappa, \tag{4.359}$$

$$l = -\frac{1}{2} e^{2W-4\phi_0}\kappa\,. \tag{4.360}$$

Setting $W = U$ and $V = 0$, we find the solutions for U and ϕ_0 as functions of ϕ together with the constant solutions for k and l

$$U = \frac{2}{5}\phi - \frac{1}{5}\ln\left(e^{8\phi} - \rho\right), \tag{4.361}$$

$$\phi_0 = \frac{1}{5}\phi - \frac{1}{10}\ln\left(e^{8\phi} - \rho\right) + C_0, \tag{4.362}$$

$$k = l = -\frac{1}{2}e^{-4C_0}\kappa \tag{4.363}$$

in which C_0 is an integration constant.

The solution for $\phi(r)$ is given by

$$\phi = -\frac{5}{16}\ln\left[\frac{4}{5}(e^{-2C_0}gr - C_1)\right] \tag{4.364}$$

for $\rho = 0$ and

$$4g\rho r(e^{8\phi} - \rho)^{1/5} = 5e^{2C_1 + \frac{32}{5}\phi}\left[4 - 3(1 - \rho e^{8\phi})^{1/5}{}_2F_1(\frac{1}{5}, \frac{4}{5}, \frac{9}{5}, \rho e^{8\phi})\right] \tag{4.365}$$

for $\rho = \pm 1$. This solution is again the flat DW found in Section 4.1.2.2 with a non-vanishing constant three-form flux.

As in the previous $SO(3)$ case in **15** representation, coupling this solution to $SO(3)$ gauge fields does not lead to new solutions. Consistency with the field equations also implies either vanishing two-form fields or vanishing gauge fields. Moreover, repeating the same analysis for finding $SO(2) \times SO(2)$ and $SO(2)$ symmetric solutions, we respectively obtain the flat DWs given in Sections 4.1.2.3 and 4.1.2.4 with a constant three-form flux

$$k = l = -\frac{1}{2}\tau\,. \tag{4.366}$$

To avoid a repetition, we will not give further detail for these cases.

4.2.3 Gaugings in 15 and $\overline{\mathbf{40}}$ Representations

We now consider charged DW solutions for gaugings in both **15** and $\overline{\mathbf{40}}$ representations. We start by finding the solutions with $SO(2)$ residual symmetry in $SO(2,1) \ltimes \mathbf{R}^4$ gauge group. From the gauge generators (2.103), we can see that the $SO(2)$ symmetry under consideration here is embedded diagonally along the

$1, 2, 4, 5$ directions. Therefore, only the modified three-form $\mathcal{H}_{(3)3}$ is singlet under the $SO(2)$ unbroken symmetry. With $Y_{33} \neq 0$, this $SO(2)$ singlet $\mathcal{H}_{(3)3}$ is then described by a massive three-form field $S_{(3)3}$ in $SO(2,1) \ltimes \mathbf{R}^4$ gauge group.

We take the ansatz for the modified three-forms to be

$$\mathcal{H}_{\hat{m}\hat{n}\hat{p}3} = k(r)e^{-3U(r)}\varepsilon_{\hat{m}\hat{n}\hat{p}} \qquad \text{and} \qquad \mathcal{H}_{\hat{i}\hat{j}\hat{k}3} = l(r)e^{-3W(r)}\varepsilon_{\hat{i}\hat{j}\hat{k}}\,. \tag{4.367}$$

After imposing the projection conditions

$$\gamma^{\hat{3}}\epsilon^a_0 = -(\Gamma_3)^a{}_b\epsilon^b_0 = \epsilon^a_0, \tag{4.368}$$

and using the $SL(5)/SO(5)$ coset representative (4.163), we find the following BPS equations

$$U' = W' = \frac{g}{40}e^{-2(2\phi_1+\phi_2)+V}\left(3\cosh 2\phi_3\cosh 2\phi_4 - e^{6\phi_2}\right), \tag{4.369}$$

$$\phi_1' = \frac{g}{240}e^{-2(\phi_1+\phi_2)+V}\left(15\,\text{sech}2\phi_3\,\text{sech}2\phi_4 - 3\cosh 2\phi_3\cosh 2\phi_4 - 4e^{6\phi_2}\right), \tag{4.370}$$

$$\phi_2' = \frac{g}{48}e^{-2(\phi_1+\phi_2)+V}\left(3\,\text{sech}2\phi_3\,\text{sech}2\phi_4 + 3\cosh 2\phi_3\cosh 2\phi_4 + 4e^{6\phi_2}\right), \tag{4.371}$$

$$\phi_3' = -\frac{3g}{16}e^{-2(2\phi_1+\phi_2)+V}\sinh 2\phi_3\,\text{sech}2\phi_4, \tag{4.372}$$

$$\phi_4' = -\frac{3g}{16}e^{-2(2\phi_1+\phi_2)+V}\cosh 2\phi_3\sinh 2\phi_4, \tag{4.373}$$

$$k = -\frac{1}{2}e^{2U+2\phi_1-2\phi_2}\tau, \tag{4.374}$$

$$l = -\frac{1}{2}e^{3W-U+2\phi_1-2\phi_2}\tau\,. \tag{4.375}$$

In these BPS equations, we have imposed the conditions (4.268) for consistency.

By taking $W = U$ and choosing $V = 4\phi_1 + 2\phi_2$, we obtain a charged DW solution

$$\phi_1 = \frac{2}{15}\phi_3 + \frac{1}{5}C_2 - \frac{1}{60}\ln\left[\frac{9}{16}(e^{2C_4} - e^{4\phi_3} - 2e^{2C_4+4\phi_3} + e^{2C_4+8\phi_3})\right] + \frac{1}{10}\ln\left[e^{4\phi_3}+1\right] - \frac{1}{5}\ln\left[e^{4\phi_3}-1\right], \tag{4.376}$$

$$\phi_2 = -5\phi_1 + C_2 + \ln\left[e^{3\phi_3}+1\right] - \ln\left[e^{3\phi_3}-1\right], \tag{4.377}$$

$$\phi_3 = \frac{1}{4}\ln\left[\frac{1+4e^{2C_4} - 2e^{\frac{3gr}{8}} + e^{\frac{3gr}{4}}}{1+4e^{2C_4} + 2e^{\frac{3gr}{8}} + e^{\frac{3gr}{4}}}\right], \tag{4.378}$$

$$\phi_4 = \frac{1}{4}\ln\left[\frac{e^{2\phi_3} - e^{C_4} + e^{C_4+4\phi_3}}{e^{2\phi_3} + e^{C_4} - e^{C_4+4\phi_3}}\right], \tag{4.379}$$

$$
\begin{aligned}
U &= -\frac{1}{5}\phi_3 - \frac{1}{20}C_2 + \frac{3}{20}\ln\left[e^{2C_4} - e^{4\phi_3} - 2e^{2C_4+4\phi_3} + e^{2C_4+8\phi_3}\right] \\
&\quad - \ln\left[\frac{16}{9}\right] - \frac{1}{5}\ln\left[e^{4\phi_3} - 1\right], \qquad (4.380) \\
k &= l = -\frac{e^{\frac{3}{10}(C_2+4\phi_3)}\tau}{2^{2/5}\times 3^{3/10}}\frac{\left(e^{2C_4} - e^{4\phi_3} - 2e^{2C_4+4\phi_3} + e^{2C_4+8\phi_3}\right)^{1/10}}{\left(e^{4\phi_3}-1\right)^{4/5}}. \qquad (4.381)
\end{aligned}
$$

This solution is the $\frac{1}{4}$-BPS DW obtained in Section 4.1.3.1 together with the running dyonic profile of the three-form flux. We emphasize here that, unlike charged DWs from gaugings only in **15** or $\overline{\mathbf{40}}$ representation, this solution with a non-vanishing three-form flux do not break SUSY any further. Therefore both charged and flat DWs are $\frac{1}{4}$-supersymmetric in $SO(2,1)\ltimes\mathbf{R}^4$ gauge group.

We finish this section by commenting on another case with $SO(2)\ltimes\mathbf{R}^4$ gauge group. Repeating the same procedure also leads to a $\frac{1}{4}$-supersymmetric charged DW given by the flat DW solution found in Section 4.1.3.2 and a constant three-form flux given in (4.366). In contrast to $SO(2,1)\ltimes\mathbf{R}^4$ gauge group, the three-form flux is due to the massless two-form field $B_{(2)3}$ in this case since we have $Y_{33}=0$ for $SO(2)\ltimes\mathbf{R}^4$ gauge group. We will not give the full detail of this analysis here as it closely follows that of the previous cases.

4.3 Twisted Solutions

In this section, we are interested in supersymmetric solutions of the maximal gauged supergravity in the form of $AdS_n\times\Sigma^{7-n}$ geometries with Σ^{7-n} being a $(7-n)$-dimensional compact manifold for $n=2,3,4,5$. This type of solutions can be obtained by the twist procedure in the same way as those found in Section 3.2 for the matter-coupled $SO(4)$ gauged theory. By the AdS/CFT correspondence, these $AdS_n\times\Sigma^{7-n}$ solutions describe conformal fixed points corresponding to $(n-1)$-dimensional SCFTs. For $SO(5)$ gauge group with the supersymmetric AdS_7 vacuum, these fixed points are dual to $(n-1)$-dimensional SCFTs obtained from twisted compactifications of the six-dimensional $N=(2,0)$ SCFT on Σ^{7-n}. For other gauge groups, their vacua are the flat DWs given in Section 4.1 DW/QFT dual to $N=(2,0)$ SQFTs in six dimensions. We accordingly interpret the resulting $AdS_n \times \Sigma^{7-n}$ solutions as conformal fixed points in lower-dimensions of these $N = (2,0)$ SQFTs.

In the following sections, we will study this type of supersymmetric solutions in $CSO(p,q,5-p-q)$ and $CSO(p,q,4-p-q)$ gauge groups. Besides, various possible RG flows from both conformal and non-conformal six-dimensional field theories to SCFTs in lower dimensions, as well as to non-conformal ones, are also considered. The final results will extend the previously known solutions in $SO(5)$ gauge group mentioned before.

4.3.1 Gaugings in 15 Representation

We again start from considering supersymmetric $AdS_n \times \Sigma^{7-n}$ solutions with $CSO(p,q,5-p-q)$ gauge group obtained from gaugings in **15** representation. From $n = 5$ to $n = 2$, we look for the solutions preserving different unbroken symmetries.

4.3.1.1 Supersymmetric $AdS_5 \times \Sigma^2$ Solutions with $SO(2) \times SO(2)$ Symmetry

As the first case, we consider $AdS_5 \times \Sigma^2$ solutions preserving $SO(2) \times SO(2)$ symmetry in this section. The coset representative for the two $SO(2) \times SO(2)$ singlet scalars are given in (4.50), while the embedding tensor for gauge groups containing an $SO(2) \times SO(2)$ subgroup can be found from (4.49). For the seven-dimensional metric, we take the ansatz of the form

$$ds_7^2 = e^{2U(r)}dx_{1,3}^2 + dr^2 + e^{2V(r)}ds^2_{\Sigma_k^2}\,. \qquad (4.382)$$

In this ansatz, $dx_{1,3}^2 = \eta_{mn}dx^m dx^n$ with $m,n = 0,..,3$ is the metric on the four-dimensional flat spacetime and Σ_k^2 is a Riemann surface whose metric is given in (3.79). Recall that $\Sigma_k^2 = S^2, \mathbb{R}^2, H^2$ corresponding to $k = 1, 0, -1$, respectively.

With the following choice of vielbein

$$e^{\hat{m}} = e^U dx^m, \qquad e^{\hat{4}} = dr, \qquad e^{\hat{5}} = e^V d\theta, \qquad e^{\hat{6}} = e^V f_k(\theta)d\varphi, \qquad (4.383)$$

we find the following non-vanishing components of the spin connection

$$\omega^{\hat{m}\hat{4}}_{(1)} = U'e^{\hat{m}}, \qquad \omega^{\hat{i}\hat{4}}_{(1)} = V'e^{\hat{i}}, \quad \hat{i} = \hat{5}, \hat{6}, \qquad \omega^{\hat{6}\hat{5}}_{(1)} = \frac{f'_k(\theta)}{f_k(\theta)}e^{-V}e^{\hat{6}}\,. \tag{4.384}$$

The function $f_k(\theta)$ is given in (3.80) with $f'_k(\theta) = df_k(\theta)/d\theta$.

To perform the twist on Σ^2_k, we turn on the following $SO(2) \times SO(2)$ gauge fields

$$A^{12}_{(1)} = -e^{-V}\frac{p_1}{k}\frac{f'_k(\theta)}{f_k(\theta)}e^{\hat{6}} \qquad \text{and} \qquad A^{34}_{(1)} = -e^{-V}\frac{p_2}{k}\frac{f'_k(\theta)}{f_k(\theta)}e^{\hat{6}} \tag{4.385}$$

with p_1 and p_2 being constant magnetic charges, and set all other tensor fields to zero. By imposing the twist condition

$$g(p_1 + \sigma p_2) = k \tag{4.386}$$

together with the following projection conditions on the Killing spinors (4.14)

$$\gamma^{\hat{5}\hat{6}}\epsilon^a = -(\Gamma_{12})^a{}_b\epsilon^b = -(\Gamma_{34})^a{}_b\epsilon^b \tag{4.387}$$

and

$$\gamma^r\epsilon^a = \epsilon^a, \tag{4.388}$$

we can derive the BPS equations

$$U' = \frac{g}{40}(2e^{-2\phi_1} + \rho e^{4(\phi_1+\phi_2)} + 2\sigma e^{-2\phi_2}) - \frac{2}{5}e^{-2V}(e^{2\phi_1}p_1 + e^{2\phi_2}p_2), \tag{4.389}$$

$$V' = \frac{g}{40}(2e^{-2\phi_1} + \rho e^{4(\phi_1+\phi_2)} + 2\sigma e^{-2\phi_2}) + \frac{8}{5}e^{-2V}(e^{2\phi_1}p_1 + e^{2\phi_2}p_2), \tag{4.390}$$

$$\phi'_1 = \frac{g}{20}(3e^{-2\phi_1} - \rho e^{4(\phi_1+\phi_2)} - 2\sigma e^{-2\phi_2}) - \frac{2}{5}e^{-2V}(3e^{2\phi_1}p_1 - 2e^{2\phi_2}p_2), \tag{4.391}$$

$$\phi'_2 = \frac{g}{20}(3\sigma e^{-2\phi_2} - \rho e^{4(\phi_1+\phi_2)} - 2e^{-2\phi_1}) + \frac{2}{5}e^{-2V}(2e^{2\phi_1}p_1 - 3e^{2\phi_2}p_2). \tag{4.392}$$

It can be verified that these BPS equations together with the twist condition (4.386) imply the second-ordered field equations.

Imposing the conditions $V' = \phi'_1 = \phi'_2 = 0$ and $U' = \frac{1}{L_{\text{AdS}_5}}$ on the BPS equations, we find a class of AdS_5 fixed point solutions given by

$$e^{2V} = -\frac{8(e^{4\phi_1}p_1 + 2e^{2(\phi_1+\phi_2)}p_2)}{g}, \tag{4.393}$$

$$e^{10\phi_1} = \frac{12p_1^2 - 24\sigma p_1^2 p_2 + 22\sigma^2 p_1 p_2^2 - 8\sigma^3 p_2^3 - 2K}{3p_1^3\rho\sigma^2}, \tag{4.394}$$

$$e^{2\phi_2} = \frac{6p_1^3 - 15\sigma p_1^2 p_2 + 13\sigma^2 p_1 p_2^2 - 4\sigma^3 p_2^3 + K}{p_2(9\sigma p_1 p_2 - 6p_1^2 - 4\sigma^2 p_2^2)} e^{2\phi_1}, \quad (4.395)$$

$$L_{\text{AdS}_5} = \frac{4(e^{4\phi_1}p_1 + 2e^{2(\phi_1+\phi_2)}p_2)}{g\,(e^{2\phi_1}p_1 + e^{2\phi_2}p_2)} \quad (4.396)$$

where

$$K = (6p_1^2 - 9\sigma p_1 p_2 + 4p_2^2\sigma^2)\sqrt{(p_1^2 - \sigma p_1 p_2 + \sigma^2 p_2^2)}\,. \quad (4.397)$$

It turns out that good AdS_5 fixed points exist only in $SO(5)$ and $SO(3,2)$ gauge groups with $\rho = \sigma = 1$ and $\rho = -\sigma = 1$, respectively.

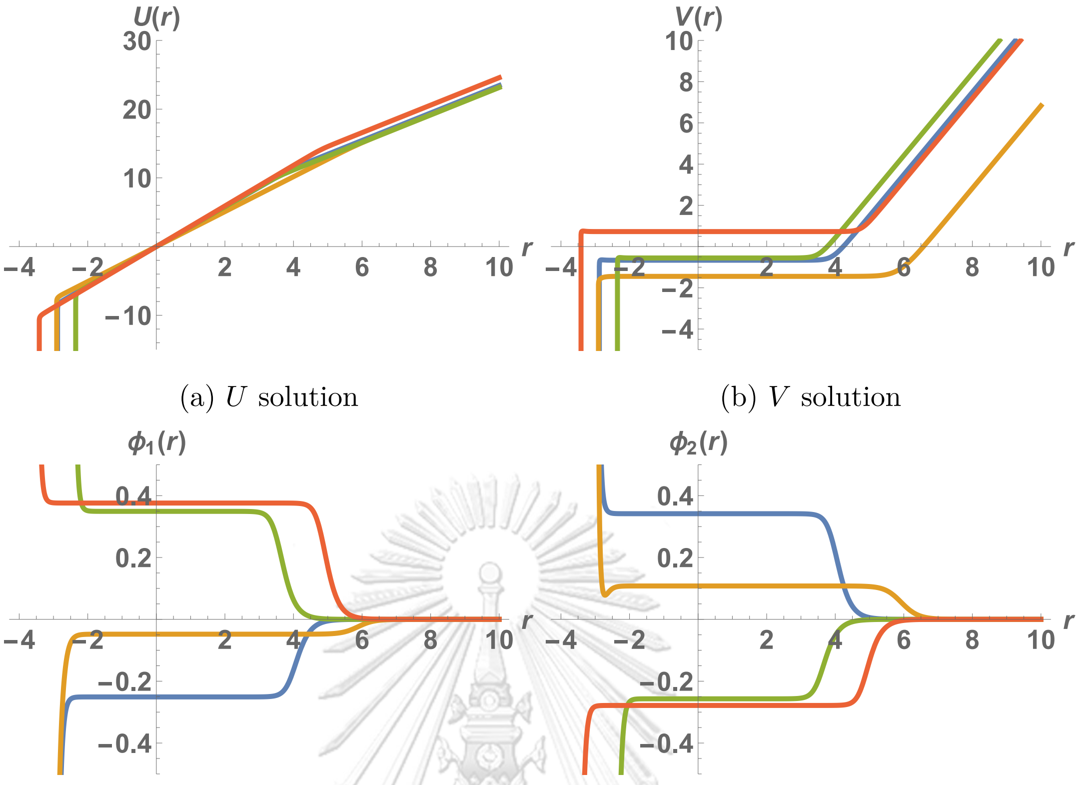
Since one of the magnetic charges is fixed by the twist condition (4.386), we will choose p_2 to characterize the solutions. In $SO(5)$ gauge group, there exist good AdS_5 fixed points when

$$gp_2 \neq -1, 0, \qquad gp_2 \neq 0, \qquad \text{and} \qquad gp_2 < 0 \;\cup\; gp_2 > 1, \quad (4.398)$$

for $g > 0$ and $\Sigma^2 = H^2, \mathbb{R}^2, S^2$, respectively. We find that the $AdS_5 \times \mathbb{R}^2$ fixed point preserves sixteen supercharges while the others preserve only eight. This is because there is no spin connection on $\mathbb{R}^2$ so that the $\gamma^{\hat{5}\hat{6}}$ projector is not needed. Note that all of these $AdS_5 \times \Sigma^2$ fixed points and their RG flows from the supersymmetric AdS_7 vacuum have been previously discussed in [57] in the context of four-dimensional SCFTs from M5-branes.

In this work, we consider more general RG flows from the AdS_7 critical point to these $AdS_5 \times \Sigma^2$ fixed points and then to singularities in the form of curved DWs with $Mkw_4 \times \Sigma^2$ slices. According to the usual holographic interpretation, these singular geometries should correspond to SQFTs in four dimensions obtained through the RG flows from four-dimensional SCFTs dual to $AdS_5 \times \Sigma^2$ fixed points. The latter are, in turn, obtained from twisted compactifications of the six-dimensional $N = (2,0)$ SCFT dual to the AdS_7 vacuum. For $AdS_5 \times H^2$, $AdS_5 \times \mathbb{R}^2$, and $AdS_5 \times S^2$ fixed points, examples of these RG flows are given in figures 4.8, 4.9, and 4.10, respectively. In these numerical solutions, we have chosen the position of the $AdS_5 \times \Sigma^2$ fixed points to be $r = 0$ and set $g = 16$.

From the eleven-dimensional metric ansatz given in (C.4), we can determine



(a) U solution (b) V solution

(c) ϕ_1 solution (d) ϕ_2 solution

Figure 4.8: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to $AdS_5 \times H^2$ fixed points at $r = 0$ and then to singularities in the form of $Mkw_4 \times H^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $p_2 = -\frac{1}{4}, -\frac{1}{24}, \frac{1}{4}, 4$.

whether these IR singularities are physical by examining the (00)-component

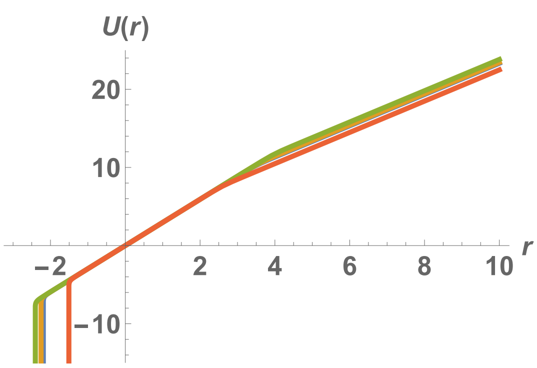
$$\hat{g}_{00} = \Delta^{\frac{1}{3}} g_{00} \,. \tag{4.399}$$

In this case, the warped factor Δ is given by

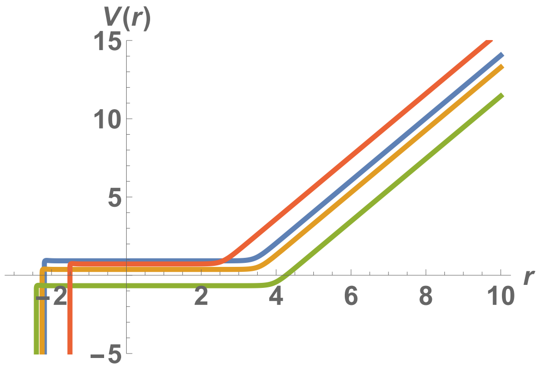
$$\Delta = \mathcal{M}^{MN} \delta_{MP} \delta_{NQ} \mu^P \mu^Q \tag{4.400}$$

in which μ^M with $M = 1, \ldots, 5$ are the S^4 coordinates satisfying $\mu^M \mu^M = 1$. Using the coset representative given in (4.50) and the S^4 coordinates

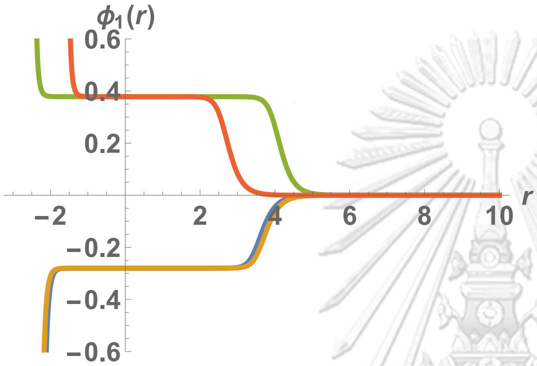
$$\mu^M = (\cos\xi, \sin\xi\cos\psi\cos\alpha, \sin\xi\cos\psi\sin\alpha, \sin\xi\sin\psi\cos\beta, \sin\xi\sin\psi\sin\beta) \,, \tag{4.401}$$



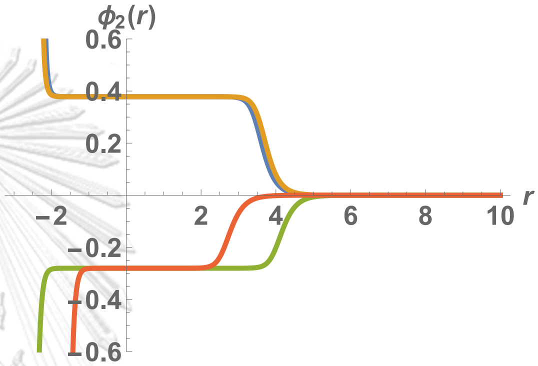
(a) U solution



(b) V solution



(c) ϕ_1 solution



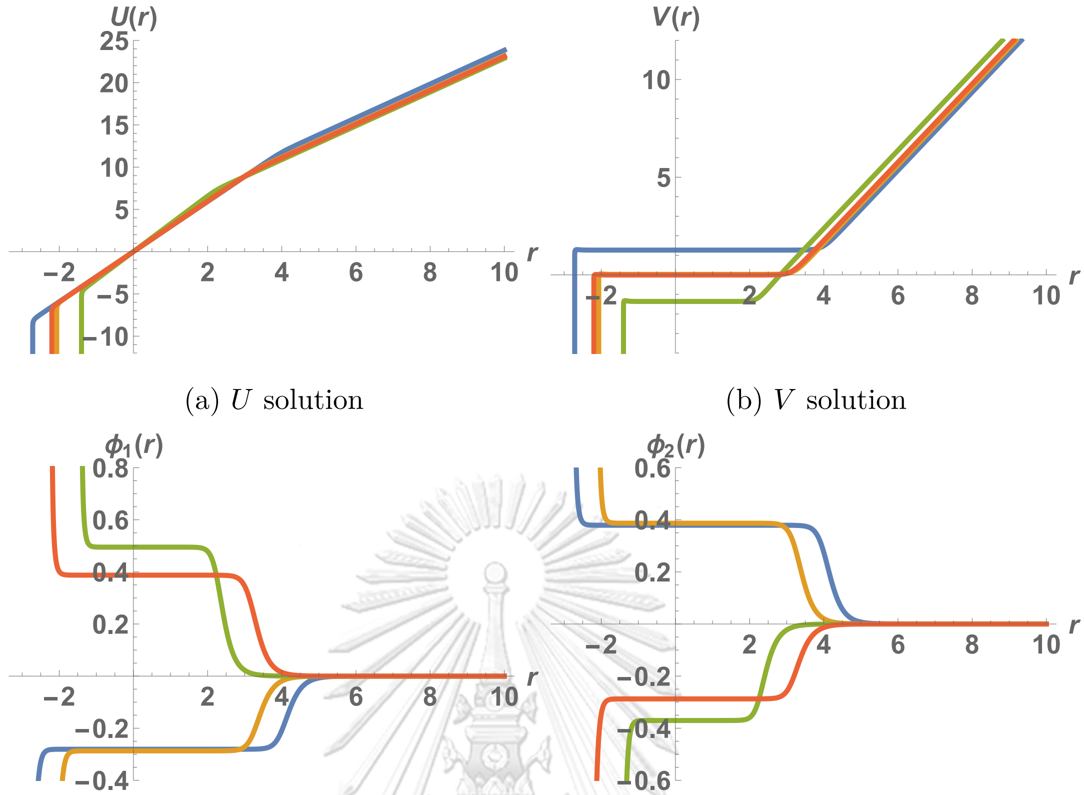
(d) ϕ_2 solution

Figure 4.9: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to $AdS_5 \times \mathbb{R}^2$ fixed points at $r = 0$ and then to singularities in the form of $Mkw_4 \times \mathbb{R}^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $p_2 = -6, -2, \frac{1}{4}, 4$.

we find the behavior of $\hat{g}_{00}$ along the flows given in Figure 4.11. Since $\hat{g}_{00} \to 0$ near the singularities, as can be seen from Figure 4.11, these IR singularities are all physical according to the criterion given in [86]. Therefore, the singularities can be interpreted as holographic duals of non-conformal phases of the four-dimensional SCFTs obtained from twisted compactifications of the six-dimensional $N = (2, 0)$ SCFT on Σ^2.

For $SO(3, 2)$ gauge group, we find new $AdS_5 \times S^2$ fixed points in a small range, with $g > 0$,

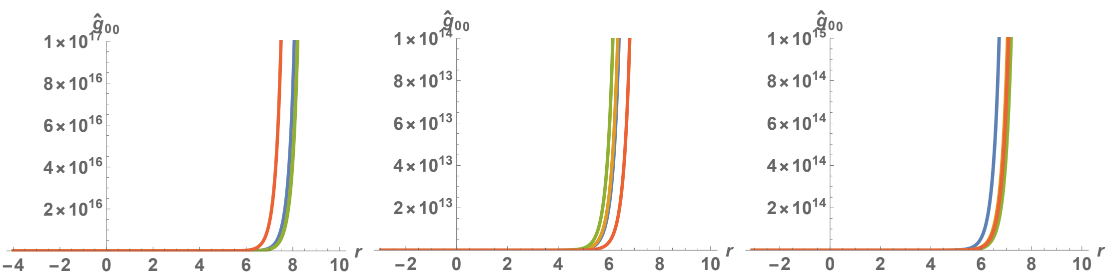
$$-\frac{1}{2} < gp_2 < 0\,. \tag{4.402}$$



(a) U solution (b) V solution

(c) ϕ_1 solution (d) ϕ_2 solution

Figure 4.10: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to $AdS_5 \times S^2$ fixed points at $r = 0$ and then to singularities in the form of $Mkw_4 \times S^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $p_2 = -12, -1, \frac{1}{8}, 1$.



(a) $\Sigma^2 = H^2$ (b) $\Sigma^2 = \mathbb{R}^2$ (c) $\Sigma^2 = S^2$

Figure 4.11: The behavior of $\hat{g}_{00}$ for RG flows given in Figures 4.8, 4.9, and 4.10, respectively, where $\hat{g}_{00} \to 0$ in the region $r < 0$ for every case.

These $AdS_5 \times S^2$ solutions preserve eight supercharges and are dual to $N = 1$ SCFTs in four dimensions. Since the vacuum solution in $SO(3,2)$ gauge group is given by a half-supersymmetric flat DW in Section 4.1.1.4 DW/QFT dual to an $N = (2,0)$ SQFT in six dimensions, the above $AdS_5 \times S^2$ fixed points can be regarded as conformal fixed points in four dimensions arising from twisted compactifications of the six-dimensional $N = (2,0)$ SQFT on S^2. In Figure 4.12, we give examples of RG flows between the $AdS_5 \times S^2$ fixed points and curved DWs with the world-volume given by $Mkw_4 \times S^2$. The latter should describe non-conformal phases of the four-dimensional $N = 1$ SCFTs. The two ends of the flows in Figure 4.12 represent two possible non-conformal phases with ($\phi_1 \to +\infty, \phi_2 \to -\infty$) and ($\phi_1 \to -\infty, \phi_2 \to +\infty$). In all of these flow solutions, we have set $g = 16$.

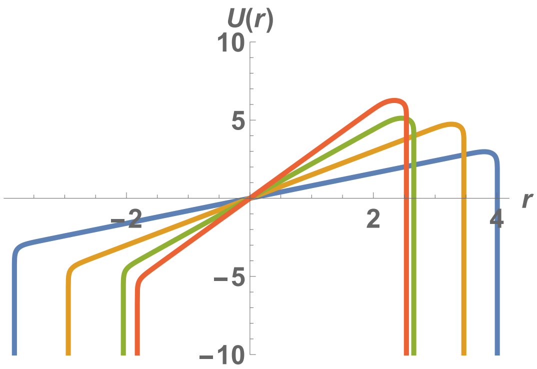
The behavior of the eleven-dimensional metric component $\hat{g}_{00}$ along the flows are given in Figure 4.13. This can be obtained by using the consistent truncation of eleven-dimensional supergravity on $H^{p,q}$ given in [64]. The explicit form of $\hat{g}_{00}$ is similar to that given in (4.399) but with the warped factor

$$\Delta = \mathcal{M}^{MN}\eta_{MP}\eta_{NQ}\mu^P\mu^Q\,. \tag{4.403}$$

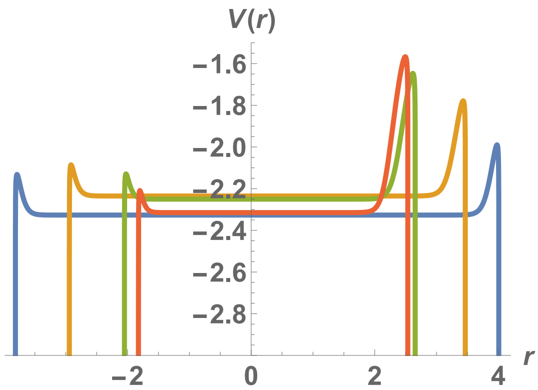
The tensor $\eta_{MN} = \text{diag}(1,\ldots,1,-1,\ldots,-1)$ is the $SO(p,q)$ invariant tensor, and μ^M are $H^{p,q}$ coordinates satisfying $\mu^M\mu^N\eta_{MN} = 1$. For $SO(3,2)$ gauged theory, we have a consistent truncation of eleven-dimensional supergravity on $H^{3,2}$ with $\eta_{MN} = \text{diag}(1,1,1,-1,-1)$. We can see from Figure 4.13 that $\hat{g}_{00} \to 0$ on both sides of the flows. Therefore, all of these singularities are physically acceptable. We accordingly interpret these solutions as RG flows between $N = 1$ SCFTs and SQFTs in four dimensions obtained from twisted compactifications of the six-dimensional $N = (2,0)$ SQFT on S^2.

4.3.1.2 Supersymmetric $AdS_4 \times \Sigma^3$ Solutions with $SO(3)$ Symmetry

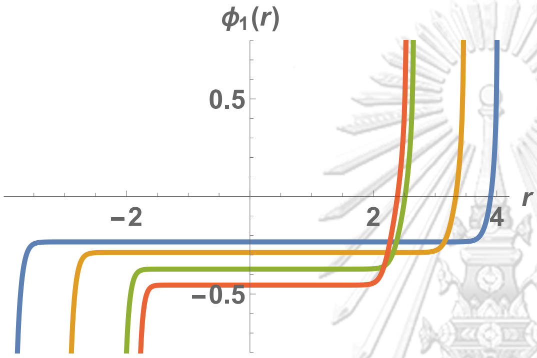
We now carry on our analysis for $AdS_4 \times \Sigma^3$ solutions with Σ^3 being a three-manifold with constant curvature. In this case, the ansatz for the seven-dimensional



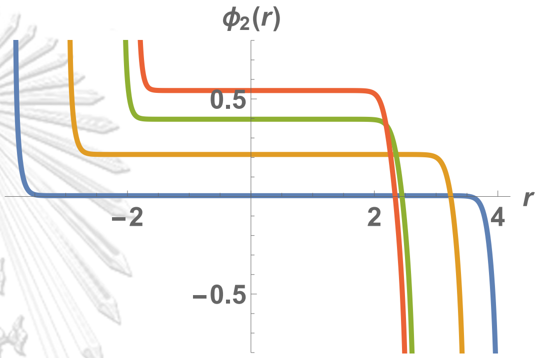
(a) U solution



(b) V solution



(c) ϕ_1 solution



(d) ϕ_2 solution

Figure 4.12: Numerical flows for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(3,2)$ gauge group. The flows start from $AdS_5 \times S^2$ fixed points at $r = 0$ to singularities in the form of $Mkw_4 \times S^2$-sliced DWs on both $r \neq 0$ sides. The blue, orange, green, and red curves refer to $p_2 = -\frac{1}{36}, -\frac{1}{48}, -\frac{1}{64}, -\frac{1}{86}$.

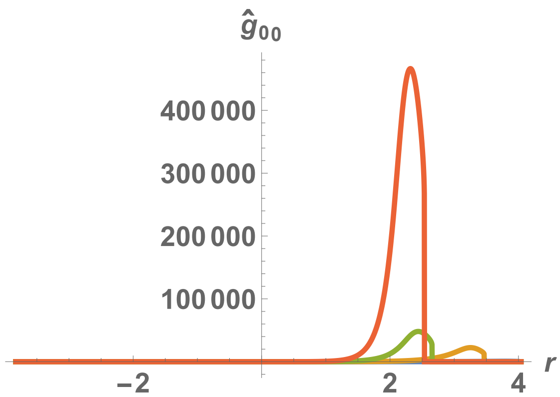


Figure 4.13: The behavior of $\hat{g}_{00}$ for RG flows given in Figure 4.12 where $\hat{g}_{00} \to 0$ on both sides.

metric takes the form of

$$ds_7^2 = e^{2U(r)}dx_{1,2}^2 + dr^2 + e^{2V(r)}ds^2_{\Sigma^3_k} \tag{4.404}$$

where $dx_{1,2}^2 = \eta_{mn}dx^m dx^n$, $m, n = 0, 1, 2$, is the metric on the three-dimensional flat spacetime. The metric on Σ^3_k is given by

$$ds^2_{\Sigma^3_k} = d\psi^2 + f_k(\psi)^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{4.405}$$

with the function $f_k(\psi)$ defined similarly in (3.80). As a Riemann surface in the previous case, this three-manifold Σ^3_k can be a three-dimensional sphere S^3, a flat space $\mathbb{R}^3$, or a hyperbolic space H^3 depending on $k = 1, 0, -1$, respectively.

Using the vielbein

$$\begin{aligned} e^{\hat{m}} &= e^U dx^m, \qquad e^{\hat{3}} = dr, \qquad e^{\hat{4}} = e^V d\psi, \\ e^{\hat{5}} &= e^V f_k(\psi)d\theta, \qquad e^{\hat{6}} = e^V f_k(\psi)\sin\theta d\varphi, \end{aligned} \tag{4.406}$$

we find non-vanishing components of the spin connection as follow

$$\begin{aligned} \omega_{(1)}^{\hat{m}\hat{3}} &= U'e^{\hat{m}}, \qquad \omega_{(1)}^{\hat{i}\hat{3}} = V'e^{\hat{i}}, \quad \hat{i} = \hat{4}, \hat{5}, \hat{6}, \qquad \omega_{(1)}^{\hat{5}\hat{4}} = \frac{f_k'(\psi)}{f_k(\psi)}e^{-V}e^{\hat{5}}, \\ \omega_{(1)}^{\hat{6}\hat{4}} &= \frac{f_k'(\psi)}{f_k(\psi)}e^{-V}e^{\hat{6}}, \qquad \omega_{(1)}^{\hat{6}\hat{5}} = \frac{\cot\theta}{f_k(\psi)}e^{-V}e^{\hat{6}} \end{aligned} \tag{4.407}$$

We will perform the twist on Σ^3_k using gauge fields corresponding to $SO(3) \subset SO(3) \times SO(2) \subset SO(5)_R$ and $SO(3)_+ \subset SO(3)_+ \times SO(3)_- \sim SO(4) \subset SO(5)_R$ with $SO(5)_R$ denoting the R-symmetry.

4.3.1.2.1 Solutions with $SO(3)$ Twists

We first consider twisted solutions with $SO(3) \subset SO(3) \times SO(2) \subset SO(5)_R$ residual symmetry by turning on the following $SO(3)$ gauge fields

$$A_{(1)}^{12} = -e^{-V}\frac{p}{k}\frac{f_k'(\psi)}{f_k(\psi)}e^{\hat{5}}, \qquad A_{(1)}^{13} = -e^{-V}\frac{p}{k}\frac{f_k'(\psi)}{f_k(\psi)}e^{\hat{6}}, \qquad A_{(1)}^{23} = -e^{-V}\frac{p}{k}\frac{\cot\theta}{f_k(\psi)}e^{\hat{6}}. \tag{4.408}$$

In this case, we also use the three $SO(3)$ singlet scalars corresponding to the $SL(5)$ non-compact generators (4.28), and the embedding tensor (4.27) for gauge groups with an $SO(3)$ subgroup.

We now impose a simple twist condition

$$gp = k, \tag{4.409}$$

together with the following projectors on the Killing spinors (4.14)

$$\gamma^{\hat{4}\hat{5}}\epsilon^a = -(\Gamma_{12})^a{}_b\epsilon^b \qquad \text{and} \qquad \gamma^{\hat{5}\hat{6}}\epsilon^a = -(\Gamma_{23})^a{}_b\epsilon^b\,. \tag{4.410}$$

With all these and the γ^r projector given in (4.388), we obtain the BPS equations

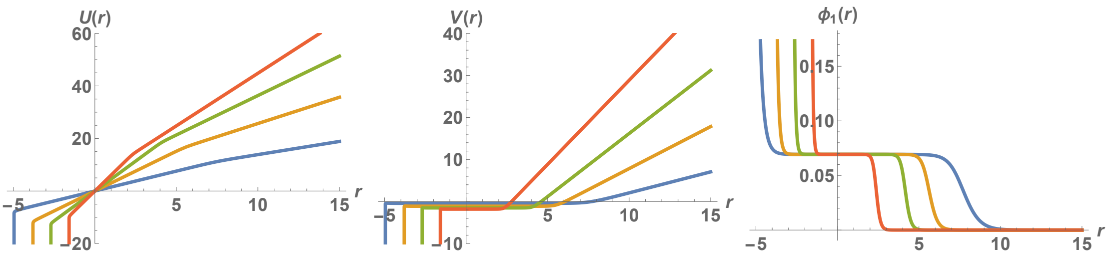
$$\begin{aligned}
U' &= \frac{g}{40}e^{6\phi_1}\left[3e^{-10\phi_1} + (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3 + (\rho-\sigma)\sinh 2\phi_3\right] \\
&\quad -\frac{6}{5}e^{-2(V-2\phi_1)}p, && (4.411)\\
V' &= \frac{g}{40}e^{6\phi_1}\left[3e^{-10\phi_1} + (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3 + (\rho-\sigma)\sinh 2\phi_3\right] \\
&\quad +\frac{14}{5}e^{-2(V-2\phi_1)}p, && (4.412)\\
\phi_1' &= \frac{g}{40}e^{6\phi_1}\left[2e^{-10\phi_1} + (\rho-\sigma)\sinh 2\phi_3 - (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3\right] \\
&\quad -\frac{4}{5}e^{-2(V-2\phi_1)}p, && (4.413)\\
\phi_2' &= -\frac{g}{8}e^{6\phi_1}(\rho+\sigma)\sinh 2\phi_2 \text{sech} 2\phi_3, && (4.414)\\
\phi_3' &= -\frac{g}{8}e^{6\phi_1}\left((\rho+\sigma)\cosh 2\phi_2 \sinh 2\phi_3 + (\rho-\sigma)\cosh 2\phi_3\right). && (4.415)
\end{aligned}$$

From these BPS equations, we find an $AdS_4 \times H^3$ fixed point only for $SO(5)$ gauge group with $\rho = \sigma = 1$ given by

$$\phi_1 = \frac{1}{10}\ln 2, \quad \phi_2 = \phi_3 = 0, \quad V = \ln\left[\frac{16^{3/5}}{g}\right], \quad L_{AdS_4} = \frac{4\times 2^{2/5}}{g}\,. \tag{4.416}$$

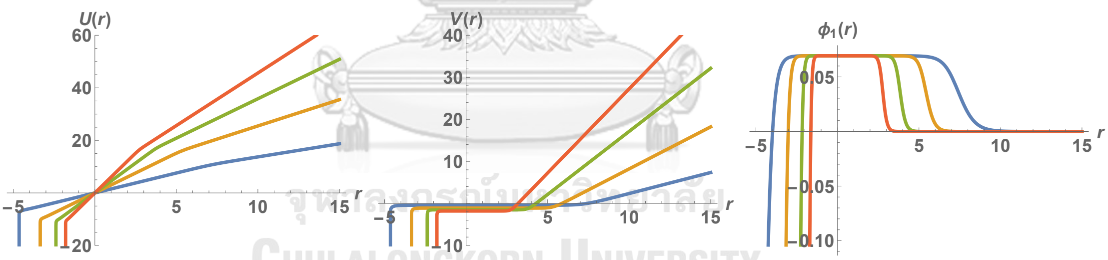
This is the same solution studied in [53]. The $AdS_4 \times H^3$ fixed point preserves eight supercharges and corresponds to an $N = 2$ SCFT in three dimensions. As in the previous case, we also consider general RG flows from the supersymmetric AdS_7 vacuum to this $AdS_4 \times H^3$ fixed point and then to curved DWs with a $Mkw_3 \times H^3$ slice dual to three-dimensional $N = 2$ SQFTs.

When $\phi_2 = \phi_3 = 0$ along the flows, we find examples of the RG flows in which $\phi_1 \to +\infty$ and $\phi_1 \to -\infty$ in the IR as respectively shown in Figures 4.14 and 4.15. Both types of singularities are physically acceptable as can be seen from the behavior of $\hat{g}_{00}$ in Figure 4.16. These singular geometries are then dual to three-dimensional $N = 2$ SQFTs obtained from twisted compactifications of the $N = (2, 0)$ SCFT in six dimensions on H^3.



(a) U solution (b) V solution (c) ϕ_1 solution

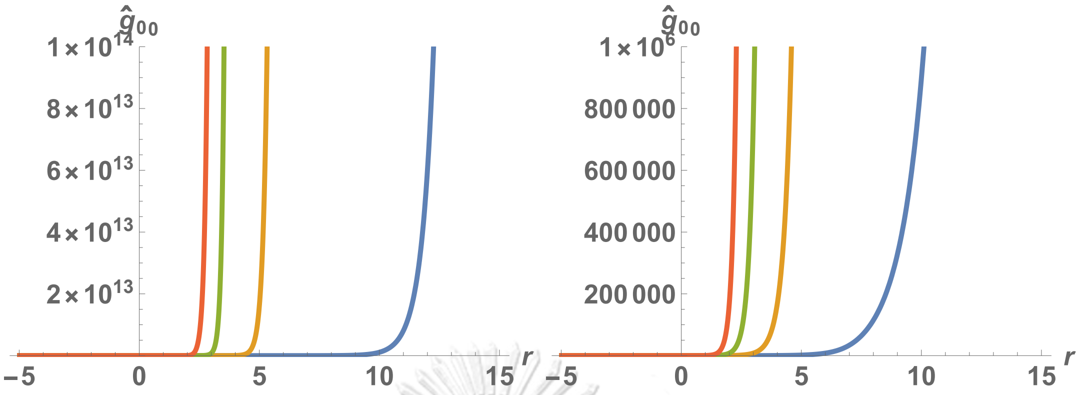
Figure 4.14: Numerical solutions for $SO(3)$ twists in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 15$ to the $AdS_4 \times H^3$ fixed point at $r = 0$ and then to singularities in the form of $Mkw_3 \times H^3$-sliced DWs with $\phi_1 \to +\infty$ in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.



(a) U solution (b) V solution (c) ϕ_1 solution

Figure 4.15: Numerical solutions for $SO(3)$ twists in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 15$ to the $AdS_4 \times H^3$ fixed point at $r = 0$ and then to singularities in the form of $Mkw_3 \times H^3$-sliced DWs with $\phi_1 \to -\infty$ in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.

Although ϕ_2 and ϕ_3 vanish at both AdS_7 vacuum and $AdS_4 \times H^3$ fixed point, we can consider the RG flows to curved DWs with non-vanishing ϕ_2 and ϕ_3. Various examples of these RG flows are given in Figure 4.17. However, the behavior of $\hat{g}_{00}$ near the singularities, $\hat{g}_{00} \to +\infty$, indicates that these singularities are unphysical by the criterion of [86].



(a) To $r<0$ singularities with $\phi_1 \to +\infty$ (b) To $r<0$ singularities with $\phi_1 \to -\infty$

Figure 4.16: The behavior of $\hat{g}_{00}$ for RG flows given in Figures 4.14 and 4.15 in which $\hat{g}_{00} \to 0$ in the region $r<0$ for both cases.

4.3.1.2.2 Solutions with $SO(3)_+$ Twists

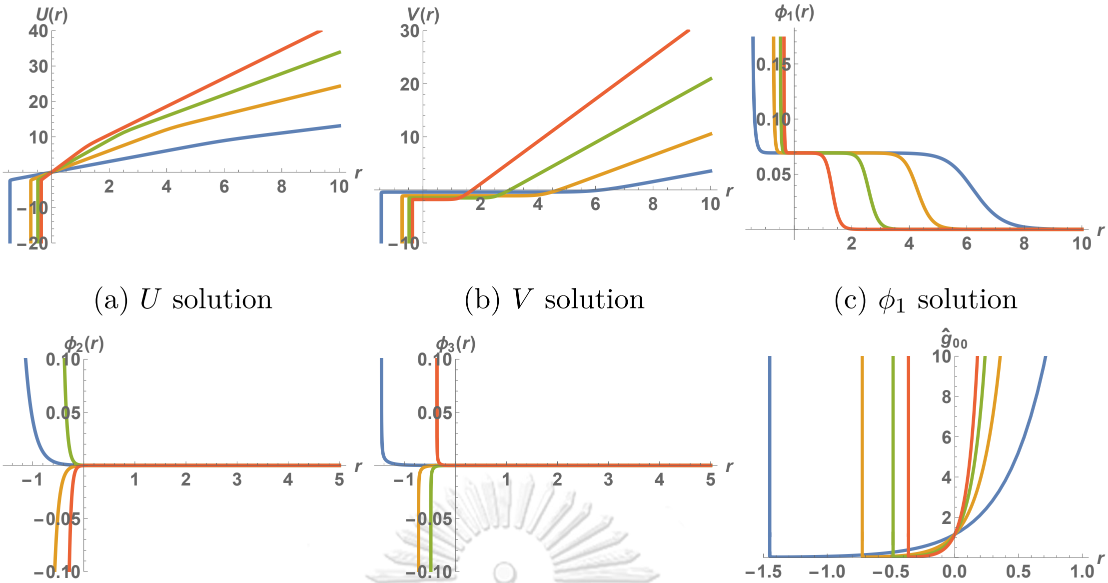
We now consider another twist by turning on the following $SO(3)_+$ gauge fields

$$\begin{aligned} A^{12}_{(1)} &= A^{34}_{(1)} = -e^{-V}\frac{p}{2k}\frac{f_k'(\psi)}{f_k(\psi)}e^{\hat{5}}, \\ A^{13}_{(1)} &= A^{24}_{(1)} = -e^{-V}\frac{p}{2k}\frac{f_k'(\psi)}{f_k(\psi)}e^{\hat{6}}, \\ A^{23}_{(1)} &= A^{14}_{(1)} = -e^{-V}\frac{p}{2k}\frac{\cot\theta}{f_k(\psi)}e^{\hat{6}}. \end{aligned} \tag{4.417}$$

In this case, the $SO(3)_+$ is identified with the self-dual $SO(3)$ subgroup of $SO(4) \sim SO(3)_+ \times SO(3)_- \subset SO(5)$. Therefore, the gauge groups containing $SO(3)_+$ are given by $SO(5)$, $SO(4,1)$, and $CSO(4,0,1)$ with the embedding tensor (4.5). There is only one $SO(3)_+$ singlet scalar corresponding to the non-compact generator (4.6) that is also invariant under a larger symmetry $SO(4)$.

To implement the twist, we impose the projection conditions given in (4.410) and

$$(\Gamma_{12})^a{}_b\epsilon^b = (\Gamma_{34})^a{}_b\epsilon^b\,. \tag{4.418}$$



(a) U solution (b) V solution (c) ϕ_1 solution

(d) ϕ_2 solution (e) ϕ_3 solution (f) Profiles of $\hat{g}_{00}$

Figure 4.17: Numerical solutions for $SO(3)$ twists in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to the $AdS_4 \times H^3$ fixed point at $r = 0$ and then to unphysical singularities in the form of $Mkw_3 \times H^3$-sliced DWs with ϕ_1, ϕ_2, and ϕ_3 non-vanishing in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.

Together with the twist condition (4.409) and the γ^r projection condition (4.388), we find the following BPS equations

$$U' = \frac{g}{40}(4e^{-2\phi} + \rho e^{8\phi}) - \frac{6}{5}e^{-2(V-\phi)}p, \qquad (4.419)$$

$$V' = \frac{g}{40}(4e^{-2\phi} + \rho e^{8\phi}) + \frac{14}{5}e^{-2(V-\phi)}p, \qquad (4.420)$$

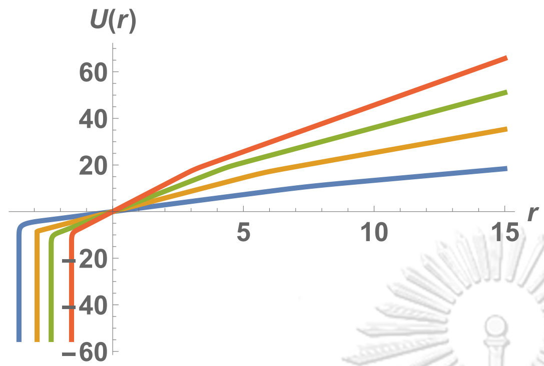
$$\phi' = \frac{g}{20}(e^{-2\phi} - \rho e^{8\phi}) - \frac{3}{5}e^{-2(V-\phi)}p\,. \qquad (4.421)$$

These equations admit an $AdS_4 \times H^3$ fixed point only for $SO(5)$ gauge group with $\rho = 1$. This $AdS_4 \times H^3$ vacuum is given by

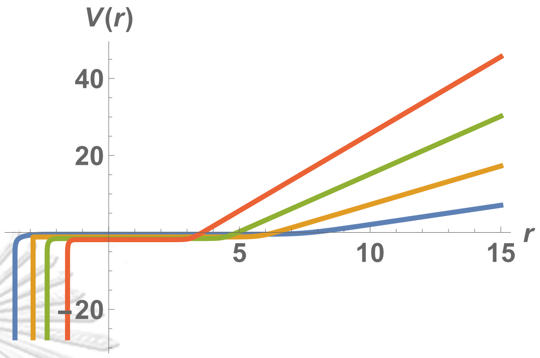
$$V = \frac{1}{2}\ln\left[\frac{8 \times 2^{1/5} \times 5^{3/5}}{g^2}\right], \quad \phi = \frac{1}{10}\ln\left[\frac{8}{5}\right], \quad L_{\text{AdS}_4} = \frac{2^{3/5} \times 5^{4/5}}{g} \qquad (4.422)$$

which does not seem to appear in the previously known results.

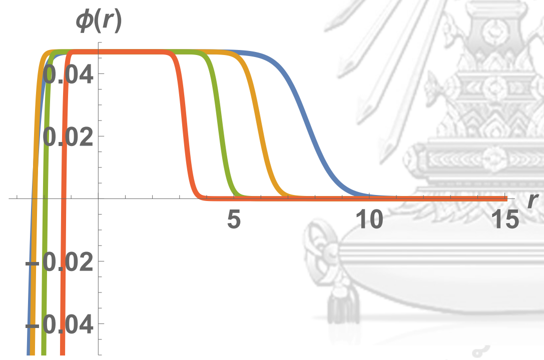
Unlike the $SO(3)$ twist, this $AdS_4 \times H^3$ fixed point preserves only four supercharges and corresponds to a three-dimensional $N = 1$ SCFT. Examples of general RG flows, from the supersymmetric AdS_7 vacuum to this $AdS_4 \times H^3$ fixed point and curved DWs dual to $N = 1$ SQFTs in three dimensions, are given in Figure 4.18. In these solutions, the IR singularities are physical, as can be seen from $\hat{g}_{00} \to 0$ near the singularities.



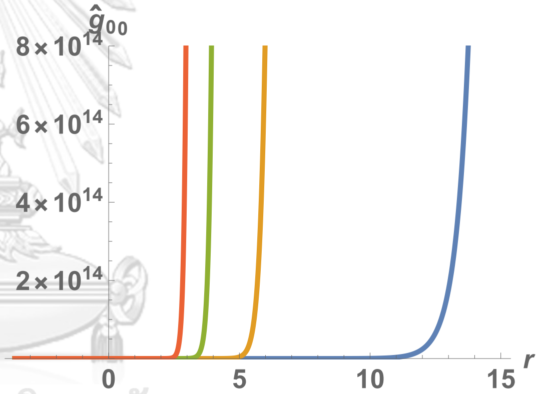
(a) U solution



(b) V solution



(c) ϕ solution



(d) Profiles of $\hat{g}_{00}$

Figure 4.18: Numerical solutions for $SO(3)_+$ twists in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 15$ to the $AdS_4 \times H^3$ fixed point at $r = 0$ and then to physical singularities in the form of $Mkw_3 \times H^3$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.

When $\rho = 0$, we can analytically solve the BPS equations for $CSO(4,0,1)$ gauge group. With the new radial coordinate $\tilde{r}$ defined by $\frac{d\tilde{r}}{dr} = e^{-V}$, the resulting solution is given by

$$\phi = C_0 + \frac{g}{160p}(4p\tilde{r} + C_1)^2 - \frac{3}{20}\ln(4p\tilde{r} + C_1), \tag{4.423}$$

$$V = 2\phi + \ln(4p\tilde{r} + C_1), \tag{4.424}$$

$$U = V - \ln(4p\tilde{r} + C_1) + C_2. \tag{4.425}$$

The integration constants C_1 and C_2 can be neglected by shifting the radial coordinate $\tilde{r}$ and rescaling the coordinates x^m on Mkw_3, respectively.

Setting $C_1 = C_2 = 0$, we find the following leading behavior of the solution at large $\tilde{r}$

$$\phi \sim \tilde{r}^2 \qquad \text{and} \qquad U \sim V \sim 2\phi\,. \tag{4.426}$$

Due to $V \to \infty$, the contribution from the gauge fields to the BPS equations is highly suppressed in this limit. The asymptotic behavior is then identified with the flat DW found in Section 4.1.1.1. Similar to the case of solutions with an asymptotically locally AdS_7 space, we will call this limit an asymptotically locally flat DW.

On the other hand, as $\tilde{r} \to 0$, we find an IR singularity with

$$\phi \sim -\frac{3}{20}\ln(4p\tilde{r}), \qquad V \sim \frac{7}{10}\ln(4p\tilde{r}), \qquad U \sim -\frac{3}{10}\ln(4p\tilde{r}). \tag{4.427}$$

This solution can be embedded in type IIA supergravity using the complete truncation ansatz collected in Appendix C.2. However, in this section, we are only interested in the time component of the ten-dimensional metric given by

$$\hat{g}_{00} = e^{2U + \frac{3}{2}\phi}\,. \tag{4.428}$$

Using this result, we find that $\hat{g}_{00} \to \infty$, as $\tilde{r} \to 0$, so the IR singularity, in this case, is unphysical.

4.3.1.3 Supersymmetric $AdS_3 \times M^4$ Solutions with $SO(4)$ Symmetry

In this section, we move on to the analysis of $AdS_3 \times M^4$ solutions. In this case, the internal space is a Riemannian four-manifold M^4 with constant curvature. Labeled by $k = 1, -1, 0$, M^4_k can be a four-dimensional sphere S^4, a flat space $\mathbb{R}^4$, or a hyperbolic space H^4, respectively.

With the embedding tensor (4.5) and the coset representative (4.7), we will consider $SO(4)$ symmetric solutions for $SO(5)$, $SO(4,1)$, and $CSO(4,0,1)$ gauge groups. To find $AdS_3 \times M^4_k$ solutions, we use the following ansatz for the seven-dimensional metric

$$ds^2_7 = e^{2U(r)}dx^2_{1,1} + dr^2 + e^{2V(r)}ds^2_{M^4_k} \tag{4.429}$$

with $dx^2_{1,1} = \eta_{mn}dx^m dx^n$ for $m, n = 0, 1$ being the metric on two-dimensional Minkowski space. The explicit form of the metric on M^4_k is given by

$$ds^2_{\Sigma^4_k} = d\chi^2 + f_k(\chi)^2\left[d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\varphi^2)\right] \tag{4.430}$$

in which $\chi, \psi, \theta \in [0, \frac{\pi}{2}]$, $\varphi \in [0, 2\pi]$, and $f_k(\chi)$ is the function defined in (3.80).

With the vielbein basis of the form

$$\begin{aligned} e^{\hat{m}} &= e^U dx^m, \qquad e^{\hat{2}} = dr, \qquad e^{\hat{3}} = e^V d\chi, \qquad e^{\hat{4}} = e^V f_k(\chi)d\psi, \\ e^{\hat{5}} &= e^V f_k(\chi)\sin\psi d\theta, \qquad e^{\hat{6}} = e^V f_k(\chi)\sin\psi\sin\theta d\varphi, \end{aligned} \tag{4.431}$$

we obtain the following non-vanishing components of the spin connection

$$\begin{aligned} \omega^{\hat{m}\hat{2}}_{(1)} &= U'e^{\hat{m}}, \qquad \omega^{\hat{i}\hat{2}}_{(1)} = V'e^{\hat{i}}, \;\; \hat{i} = \hat{3}, ..., \hat{6}, \qquad \omega^{\hat{4}\hat{3}}_{(1)} = \frac{f'_k(\chi)}{f_k(\chi)}e^{-V}e^{\hat{4}}, \\ \omega^{\hat{5}\hat{3}}_{(1)} &= \frac{f'_k(\chi)}{f_k(\chi)}e^{-V}e^{\hat{5}}, \qquad \omega^{\hat{6}\hat{3}}_{(1)} = \frac{f'_k(\chi)}{f_k(\chi)}e^{-V}e^{\hat{6}}, \qquad \omega^{\hat{5}\hat{4}}_{(1)} = \frac{\cot\psi}{f_k(\chi)}e^{-V}e^{\hat{5}}, \\ \omega^{\hat{6}\hat{4}}_{(1)} &= \frac{\cot\psi}{f_k(\chi)}e^{-V}e^{\hat{6}}, \qquad \omega^{\hat{6}\hat{5}}_{(1)} = \frac{\cot\theta}{f_k(\chi)\sin\psi}e^{-V}e^{\hat{6}}. \end{aligned} \tag{4.432}$$

To cancel the spin connection $\omega^{\hat{i}\hat{j}}_{(1)}$, we perform the twist on M^4_k by turning on $SO(4)$ gauge fields as follow

$$A^{IJ}_{(1)} = -\frac{p}{k}\delta^{I+2}_{[\hat{i}}\delta^{J+2}_{\hat{j}]}\omega^{\hat{i}\hat{j}}_{(1)}, \qquad I, J = 1, ..., 4. \tag{4.433}$$

The corresponding modified two-forms are exactly the $SO(4)$ gauge field strengths given by

$$\mathcal{F}^{IJ}_{\hat{i}\hat{j}} = F^{IJ}_{\hat{i}\hat{j}} = 2\delta^{I+2}_{[\hat{i}}\delta^{J+2}_{\hat{j}]}e^{-2V}p\,. \tag{4.434}$$

In this case, the modified three-forms cannot vanish in order to satisfy the Bianchi's identity since the above $SO(4)$ gauge fields lead to non-vanishing $\epsilon_{MNPQR}\mathcal{F}^{NP}_{(2)} \wedge \mathcal{F}^{QR}_{(2)}$ terms in (2.63). To preserve the residual $SO(4)$ symmetry, only $\mathcal{H}_{(3)5}$ is allowed. We also note that for $SO(5)$ and $SO(4,1)$ gauge groups, their embedding tensor Y_{MN} is non-degenerate. For these gauge groups, there are in total five massive three-form fields $S^M_{(3)}$, so $\mathcal{H}_{(3)5}$ is obtained by turning on the massive three-form field $S^5_{(3)}$. On the other hand, we have $Y_{55} = 0$ for $CSO(4, 0, 1)$ gauge group, so the contribution to $\mathcal{H}_{(3)5}$ comes from the massless two-form field $B_{(2)5}$ in this case. However, we are not able to determine a suitable ansatz for $B_{(2)5}$ in order to find a consistent set of BPS equations that are compatible with the second-ordered field equations. Accordingly, in the following analysis, we will not consider the non-semisimple $CSO(4, 0, 1)$ gauge group.

For $SO(5)$ and $SO(4, 1)$ gauge groups, the appropriate ansatz for the modified three-form is given by

$$\mathcal{H}_{\hat{m}\hat{n}\hat{2}5} = -\frac{96}{g}\rho e^{-4(V+2\phi)}p^2\varepsilon_{\hat{m}\hat{n}}\,. \tag{4.435}$$

Imposing the twist condition (4.409) and the projector in (4.388) together with additional projectors of the form

$$\gamma^{\hat{3}\hat{4}}\epsilon^a = -(\Gamma_{12})^a{}_b\epsilon^b, \qquad \gamma^{\hat{4}\hat{5}}\epsilon^a = -(\Gamma_{23})^a{}_b\epsilon^b, \qquad \gamma^{\hat{5}\hat{6}}\epsilon^a = -(\Gamma_{34})^a{}_b\epsilon^b, \tag{4.436}$$

we find the BPS equations

$$U' = \frac{g}{40}(4e^{-2\phi} + \rho e^{8\phi}) - \frac{12}{5}e^{-2V+2\phi}p + \frac{288}{5g}\rho e^{-4(V+\phi)}p^2, \tag{4.437}$$

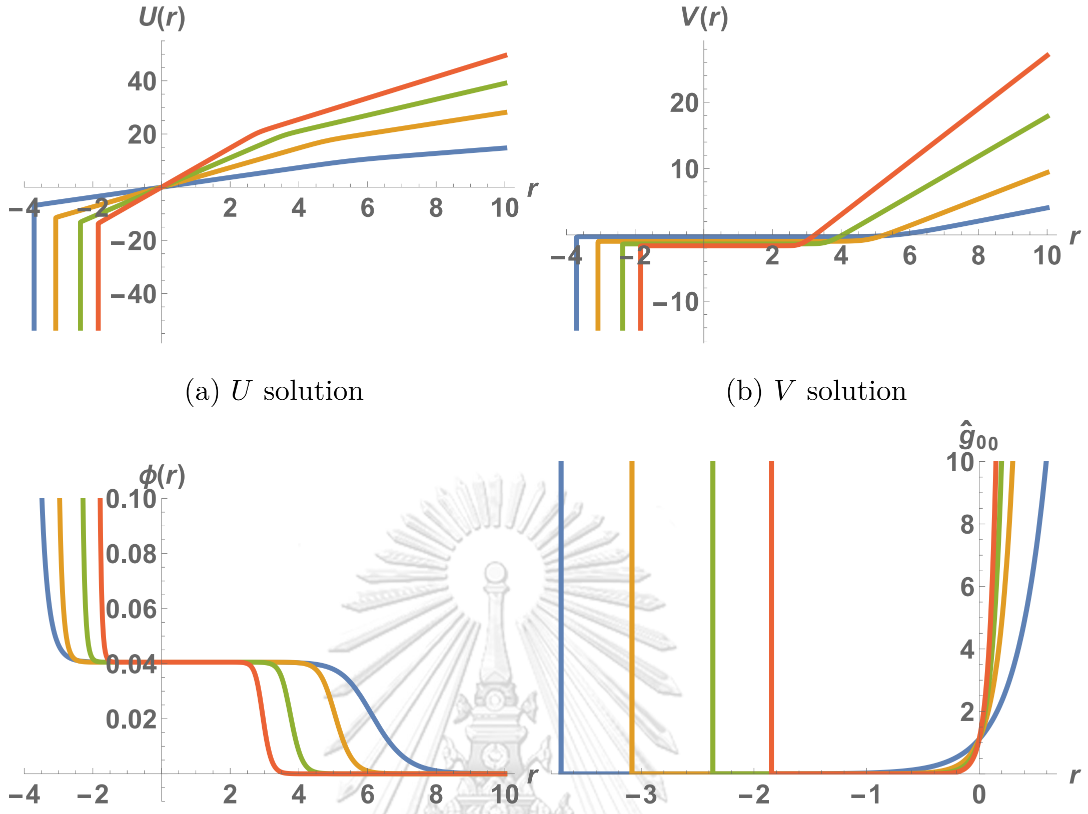
$$V' = \frac{g}{40}(4e^{-2\phi} + \rho e^{8\phi}) + \frac{18}{5}e^{-2V+2\phi}p - \frac{192}{5g}\rho e^{-4(V+\phi)}p^2, \tag{4.438}$$

$$\phi' = \frac{g}{20}(e^{-2\phi} - \rho e^{8\phi}) - \frac{6}{5}e^{-2V+2\phi}p - \frac{96}{5g}\rho e^{-4(V+\phi)}p^2\,. \tag{4.439}$$

From these BPS equations, we find an AdS_3 fixed point only for $k = -1$ and $\rho = 1$. The resulting $AdS_3 \times H^4$ solution is given by

$$V = \frac{1}{2}\ln\left[\frac{16\times 2^{3/5}\times 3^{2/5}}{g^2}\right], \quad \phi = \frac{1}{10}\ln\left[\frac{3}{2}\right], \quad L_{\text{AdS}_3} = \frac{2^{9/5}\times 3^{1/5}}{g}\,. \tag{4.440}$$

This is the $AdS_3 \times H^4$ fixed point given in [53] for the maximal $SO(5)$ gauged supergravity. The solution preserves four supercharges and corresponds to a two-dimensional $N = (1, 1)$ SCFT with $SO(4)$ symmetry. As in the previous cases, we also consider general RG flows from the supersymmetric AdS_7 vacuum to this $AdS_3 \times H^4$ fixed point and then to curved DWs. Examples of these RG flows are given in Figure 4.19. Unlike the previous cases, the IR singularities are unphysical in this case due to the behavior $\hat{g}_{00} \to \infty$ near the singularities.



(a) U solution (b) V solution

(c) ϕ solution (d) Profiles of $\hat{g}_{00}$

Figure 4.19: Numerical solutions for $SO(4)$ twists in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to the $AdS_3 \times H^4$ fixed point at $r = 0$ and then to unphysical singularities in the form of $Mkw_2 \times H^4$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.

4.3.1.4 Supersymmetric $AdS_3 \times \Sigma^2 \times \Sigma^2$ Solutions with $SO(2) \times SO(2)$ Symmetry

We now consider the internal four-manifold as a product of two Riemann surfaces $\Sigma^2 \times \Sigma^2$. The ansatz for the seven-dimensional metric takes the form of

$$ds_7^2 = e^{2U(r)}dx_{1,1}^2 + dr^2 + e^{2V(r)}ds^2_{\Sigma^2_{k_1}} + e^{2W(r)}ds^2_{\Sigma^2_{k_2}} \tag{4.441}$$

in which the metrics on the Riemann surfaces $\Sigma^2_{k_1}$ and $\Sigma^2_{k_2}$ are given in (3.79).

Using the following choice for the vielbein

$$e^{\hat{m}} = e^U dx^m, \qquad e^{\hat{2}} = dr, \qquad e^{\hat{3}} = e^V d\theta_1,$$
$$e^{\hat{4}} = e^V f_{k_1}(\theta_1) d\varphi_1, \qquad e^{\hat{5}} = e^W d\theta_2, \qquad e^{\hat{6}} = e^W f_{k_2}(\theta_2) d\varphi_2, \qquad (4.442)$$

we obtain all non-vanishing components of the spin connection as follow

$$\omega_{(1)}^{\hat{m}\hat{2}} = U' e^{\hat{m}}, \qquad \omega_{(1)}^{\hat{i}_1\hat{2}} = V' e^{\hat{i}_1}, \qquad \omega_{(1)}^{\hat{i}_2\hat{2}} = W' e^{\hat{i}_2},$$
$$\omega_{(1)}^{\hat{4}\hat{3}} = \frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)} e^{-V} e^{\hat{4}}, \quad \omega_{(1)}^{\hat{6}\hat{5}} = \frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)} e^{-W} e^{\hat{6}} \qquad (4.443)$$

with $\hat{i}_1 = \hat{3}, \hat{4}$ and $\hat{i}_2 = \hat{5}, \hat{6}$ being flat indices on $\Sigma^2_{k_1}$ and $\Sigma^2_{k_2}$, respectively.

As in other cases, we consider gauge groups of the form $CSO(p, q, 5-p-q)$ with an $SO(2) \times SO(2)$ subgroup. These gauge groups are obtained from the embedding tensor given in (4.49). To perform the twist, we turn on the following $SO(2) \times SO(2)$ gauge fields

$$A_{(1)}^{12} = -\frac{p_{11}}{k_1} \frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)} e^{-V} e^{\hat{4}} - \frac{p_{12}}{k_2} \frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)} e^{-W} e^{\hat{6}},$$
$$A_{(1)}^{34} = -\frac{p_{21}}{k_1} \frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)} e^{-V} e^{\hat{4}} - \frac{p_{22}}{k_2} \frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)} e^{-W} e^{\hat{6}}. \qquad (4.444)$$

The corresponding modified two-forms are given by

$$\mathcal{F}_{(2)}^{12} = F_{(2)}^{12} = e^{-2V} p_{11} e^{\hat{3}} \wedge e^{\hat{4}} + e^{-2W} p_{12} e^{\hat{5}} \wedge e^{\hat{6}}, \qquad (4.445)$$
$$\mathcal{F}_{(2)}^{34} = F_{(2)}^{34} = e^{-2V} p_{21} e^{\hat{3}} \wedge e^{\hat{4}} + e^{-2W} p_{22} e^{\hat{5}} \wedge e^{\hat{6}}. \qquad (4.446)$$

We also need to turn on the modified three-form

$$\mathcal{H}_{\hat{m}\hat{n}\hat{2}5} = \alpha e^{-2(V+W+2\phi_1+2\phi_2)} \varepsilon_{\hat{m}\hat{n}} \qquad (4.447)$$

where α is a constant related to the magnetic charges by the relation

$$g\rho\alpha = -32(p_{12}p_{21} + p_{11}p_{22}). \qquad (4.448)$$

For $CSO(2,2,1)$ and $CSO(4,0,1)$ gauge groups with $\rho = 0$, we need to impose an additional relation on the magnetic charges

$$p_{12}p_{21} + p_{11}p_{22} = 0 \qquad (4.449)$$

in order to ensure that the resulting BPS equations are compatible with all the second-ordered field equations.

Using the projection conditions (4.388) and

$$\gamma^{\hat{3}\hat{4}}\epsilon^a = \gamma^{\hat{5}\hat{6}}\epsilon^a = -(\Gamma_{12})^a{}_b\epsilon^b = -(\Gamma_{34})^a{}_b\epsilon^b \qquad (4.450)$$

together with the twist conditions

$$g(p_{11}+\sigma p_{21}) = k_1 \qquad \text{and} \qquad g(p_{12}+\sigma p_{22}) = k_2, \qquad (4.451)$$

we obtain the following BPS equations

$$\begin{aligned}
U' &= \frac{g}{40}(2e^{-2\phi_1}+\rho e^{4(\phi_1+\phi_2)}+2\sigma e^{-2\phi_2}) - \frac{3\alpha}{5g}e^{-2(V+W+\phi_1+\phi_2)} \\
&\quad -\frac{2}{5}\left[e^{-2V}(e^{2\phi_1}p_{11}+e^{2\phi_2}p_{21})+e^{-2W}(e^{2\phi_1}p_{12}+e^{2\phi_2}p_{22})\right], && (4.452)\\
V' &= \frac{g}{40}(2e^{-2\phi_1}+\rho e^{4(\phi_1+\phi_2)}+2\sigma e^{-2\phi_2}) + \frac{2\alpha}{5g}e^{-2(V+W+\phi_1+\phi_2)} \\
&\quad +\frac{2}{5}\left[4e^{-2V}(e^{2\phi_1}p_{11}+e^{2\phi_2}p_{21})-e^{-2W}(e^{2\phi_1}p_{12}+e^{2\phi_2}p_{22})\right], && (4.453)\\
W' &= \frac{g}{40}(2e^{-2\phi_1}+\rho e^{4(\phi_1+\phi_2)}+2\sigma e^{-2\phi_2}) + \frac{2\alpha}{5g}e^{-2(V+W+\phi_1+\phi_2)} \\
&\quad -\frac{2}{5}\left[e^{-2V}(e^{2\phi_1}p_{11}+e^{2\phi_2}p_{21})-4e^{-2W}(e^{2\phi_1}p_{12}+e^{2\phi_2}p_{22})\right], && (4.454)\\
\phi_1' &= \frac{g}{20}(3e^{-2\phi_1}-\rho e^{4(\phi_1+\phi_2)}-2\sigma e^{-2\phi_2}) + \frac{\alpha}{5g}e^{-2(V+W+\phi_1+\phi_2)} \\
&\quad -\frac{2}{5}\left[3e^{2\phi_1}(e^{-2V}p_{11}+e^{-2W}p_{12})-2e^{2\phi_2}(e^{-2V}p_{21}+e^{-2W}p_{22})\right], && (4.455)\\
\phi_2' &= \frac{g}{20}(3\sigma e^{-2\phi_2}-\rho e^{4(\phi_1+\phi_2)}-2e^{-2\phi_1}) + \frac{\alpha}{5g}e^{-2(V+W+\phi_1+\phi_2)} \\
&\quad +\frac{2}{5}\left[2e^{2\phi_1}(e^{-2V}p_{11}+e^{-2W}p_{12})-3e^{2\phi_2}(e^{-2V}p_{21}+e^{-2W}p_{22})\right]. && (4.456)
\end{aligned}$$

In deriving these BPS equations, we have used the coset representative given in (4.50) for $SO(2)\times SO(2)$ singlet scalars.

We find a class of $AdS_3\times\Sigma^2\times\Sigma^2$ fixed point solutions from the BPS equations

$$\begin{aligned}
e^{2V} &= -\frac{16(e^{2\phi_1}p_{11}+e^{2\phi_2}p_{21})}{ge^{4(\phi_1+\phi_2)}\rho}, && (4.457)\\
e^{2W} &= -\frac{16(e^{2\phi_1}p_{12}+e^{2\phi_2}p_{22})}{ge^{4(\phi_1+\phi_2)}\rho}, && (4.458)\\
e^{10\phi_1} &= \frac{64\Theta\left[32(p_{12}p_{21}+p_{11}p_{22})-g\rho\alpha-64\sigma p_{21}p_{22}\right]^2}{\left[32\sigma(p_{12}p_{21}+p_{11}p_{22})-g\rho\sigma\alpha-64p_{11}p_{12}\right]^3}, && (4.459)
\end{aligned}$$

$$e^{10\phi_2} = \frac{64\Theta\left[32\sigma(p_{12}p_{21}+p_{11}p_{22}) - g\rho\sigma\alpha - 64p_{11}p_{12}\right]^2}{\left[32(p_{12}p_{21}+p_{11}p_{22}) - g\rho\alpha - 64\sigma p_{21}p_{22}\right]^3}, \qquad (4.460)$$

$$L_{\text{AdS}_3} = \frac{8e^{2(\phi_1+\phi_2)}}{g(e^{2\phi_2}+e^{2\phi_1}\sigma)} \qquad (4.461)$$

with

$$\Theta = \frac{\Xi\left[32\left(p_{12}p_{22}(p_{11}+p_{21}\sigma) - p_{12}^2p_{21} - p_{11}p_{22}^2\sigma\right) + g\rho\alpha(p_{12}+p_{22}\sigma)\right]}{\rho\left[1024p_{12}^2p_{21}^2 + (32p_{11}p_{22}-g\rho\alpha)^2 - 64p_{12}p_{21}(32p_{11}p_{22}+g\rho\alpha)\right]}, \qquad (4.462)$$

$$\Xi = 32\left[p_{11}p_{21}(p_{12}+p_{22}\sigma) - p_{11}^2p_{22} - p_{12}p_{21}^2\sigma\right] + g\rho\alpha(p_{11}+p_{21}\sigma). \qquad (4.463)$$

It turns out that good $AdS_3 \times \Sigma^2 \times \Sigma^2$ solutions are possible only for $SO(5)$ and $SO(3,2)$ gauge groups with $\rho = \sigma = 1$ and $\rho = -\sigma = 1$, respectively. For $SO(5)$ gauge group, the solutions have been extensively studied in [58]. For $SO(3,2)$ gauge group, all the $AdS_3 \times \Sigma^2 \times \Sigma^2$ fixed points given here are new.

Following [58], we define the following two parameters to characterize the $AdS_3 \times \Sigma^2 \times \Sigma^2$ solutions

$$z_1 = g(p_{11} - \sigma p_{21}) \qquad \text{and} \qquad z_2 = g(p_{12} - \sigma p_{22}) \qquad (4.464)$$

in which we have set $\rho = 1$. In order for good AdS_3 fixed points to exist in $SO(5)$ gauge group with $\sigma = 1$, one of the Riemann surfaces needs to be negatively curved, and $AdS_3 \times H^2 \times \Sigma^2$ solutions can be found within the regions in the parameter space (z_1, z_2) shown in Figure 4.20. These regions are the same as those given in [58] and Figure 3.1. The $AdS_3 \times H^2 \times \Sigma^2$ fixed points preserve four supercharges and correspond to two-dimensional $N = (2,0)$ SCFTs with $SO(2) \times SO(2)$ symmetry. Examples of RG flows with $g = 16$, from the supersymmetric AdS_7 vacuum to $AdS_3 \times H^2 \times \Sigma^2$ fixed points and curved DWs in the IR, are given in Figures 4.21, 4.22, and 4.23 for $\Sigma^2 = H^2, \mathbb{R}^2$, and S^2, respectively. All the IR singularities are physical since $\hat{g}_{00} \to 0$ near the singularities.

For $SO(3,2)$ gauge group, we find good AdS_3 fixed points only for at least one of the two Riemann surfaces is positively curved. Using the parameters z_1 and z_2 defined in (4.464) with $\sigma = -1$, we find regions in the parameter space (z_1, z_2) for good AdS_3 vacua to exist in the $SO(3,2)$ gauged supergravity as shown in Figure 4.24.

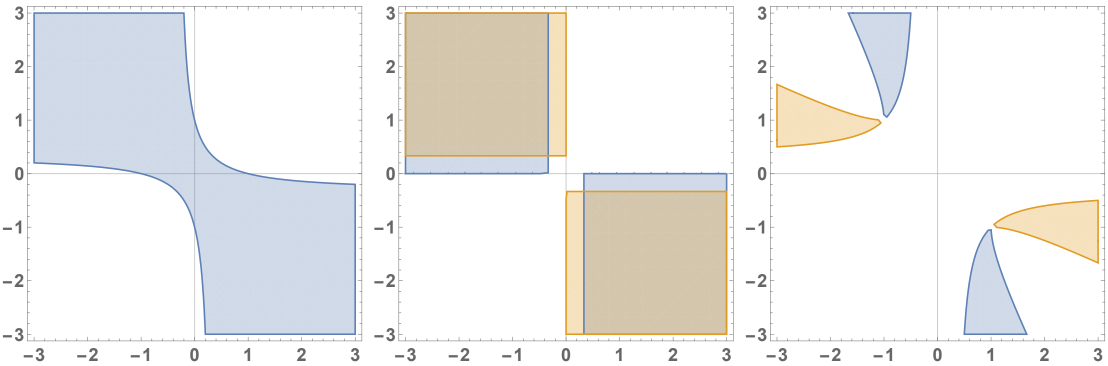
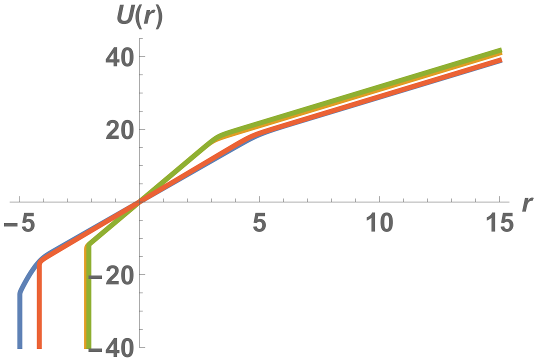


Figure 4.20: Regions (blue) in the parameter space (z_1, z_2) where good AdS_3 vacua exist in $SO(5)$ gauge group for $g = 16$. From left to right, these figures correspond to the cases of $(k_1 = k_2 = -1)$, $(k_1 = -1, k_2 = 0)$, and $(k_1 = -k_2 = -1)$, respectively. The orange regions are obtained from interchanging k_1 and k_2.

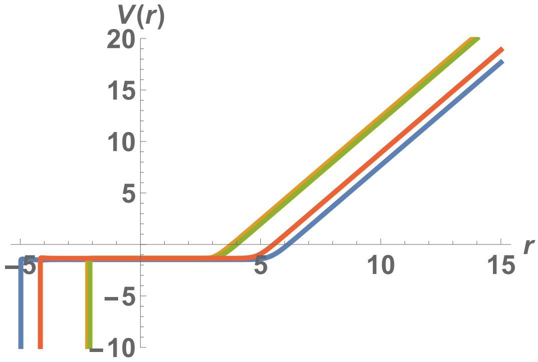
We will consider RG flows between the $AdS_3 \times S^2 \times \Sigma^2$ fixed points and curved DWs with $Mkw_2 \times S^2 \times \Sigma^2$ slices in this case since there is no asymptotically locally AdS_7 geometry for $SO(3,2)$ gauge group. Examples of these RG flows with $g = 16$ and different values of z_1 and z_2 are given in Figures 4.25, 4.26, and 4.27 for $\Sigma^2 = H^2, \mathbb{R}^2$, and S^2, respectively. We see that all singularities in the flows from $AdS_3 \times S^2 \times \mathbb{R}^2$ fixed points are unphysical, while only the singularities on the left (right) with $\phi_1 \to -\infty$ and $\phi_2 \to +\infty$ ($\phi_1 \to +\infty$ and $\phi_2 \to -\infty$) of the flows from $AdS_3 \times S^2 \times H^2$ ($AdS_3 \times S^2 \times S^2$) fixed points are physical. These singularities are expected to describe two-dimensional SQFTs with $SO(2) \times SO(2)$ symmetry obtained from twisted compactifications of the six-dimensional $N = (2,0)$ SQFT.

4.3.1.5 Supersymmetric $AdS_3 \times K^4$ Solutions

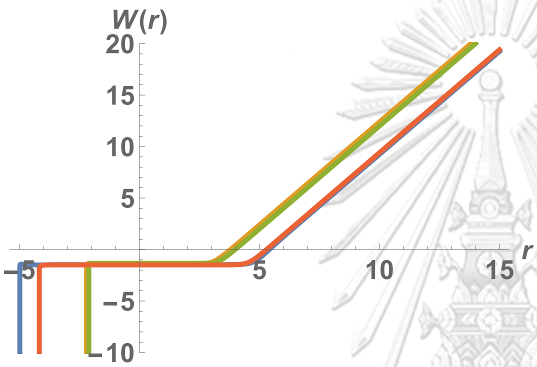
Apart from M^4 and $\Sigma^2 \times \Sigma^2$, we are also interested in supersymmetric solutions with AdS_3 vacua in the case of the internal four-manifold being a Kahler four-cycle K^4. As expressed in Section 3.2.2, we can perform a topological twist on K^4 using either $SO(2) \sim U(1)$ or $SO(3) \sim SU(2)$ gauge fields in order to cancel the $U(1)$ or $SU(2)$ parts of the $U(2)$ spin connection.



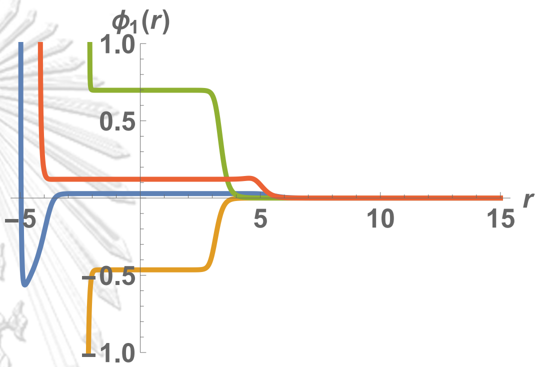
(a) U solution



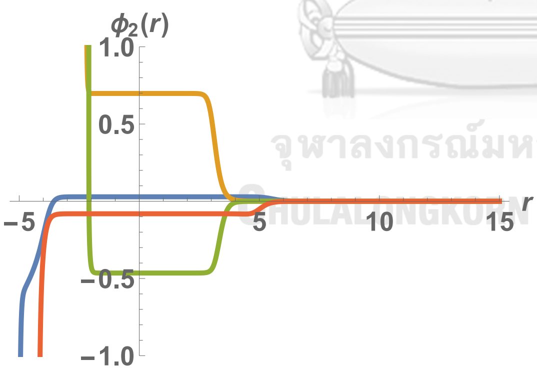
(b) V solution



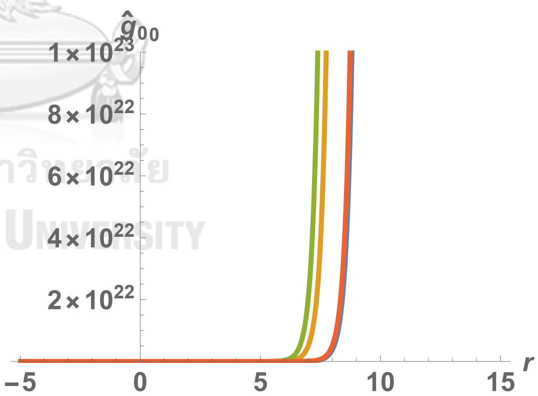
(c) W solution



(d) ϕ_1 solution

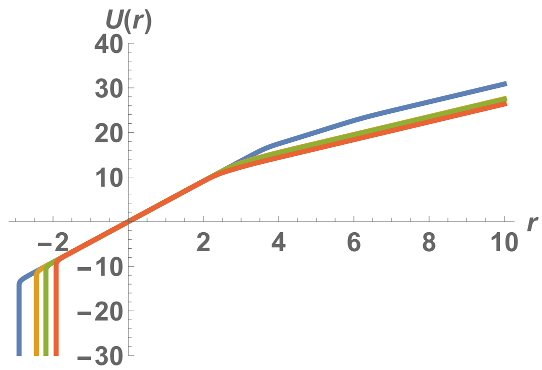


(e) ϕ_2 solution

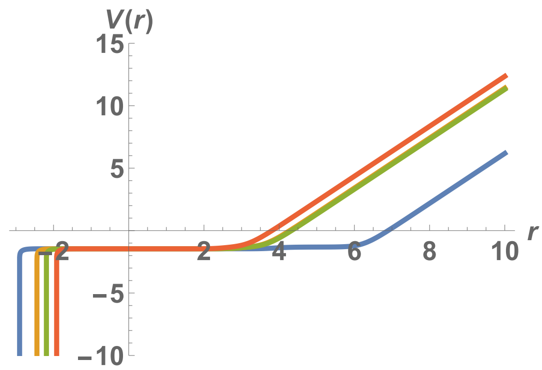


(f) Profiles of $\hat{g}_{00}$

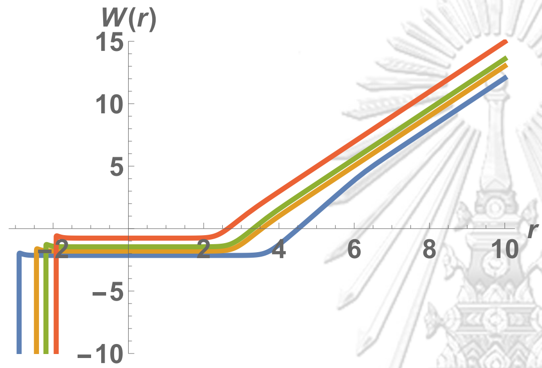
Figure 4.21: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 15$ to $AdS_3 \times H^2 \times H^2$ fixed points at $r = 0$ and then to physical singularities in the form of $Mkw_2 \times H^2 \times H^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $(z_1, z_2) = (0, 0), (0.3, 0.3), (-0.3, -0.3), (-1, 0.5)$.



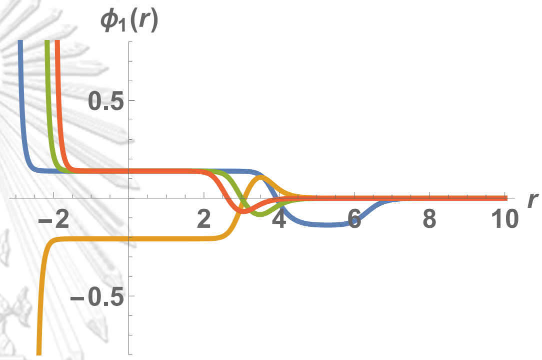
(a) U solution



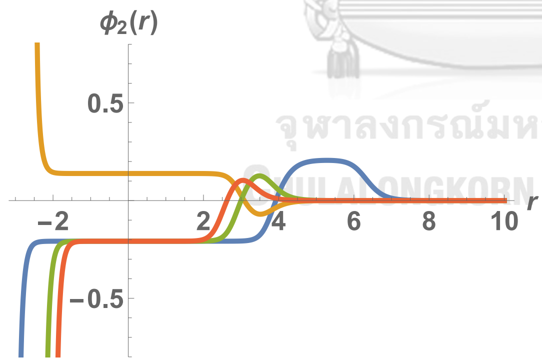
(b) V solution



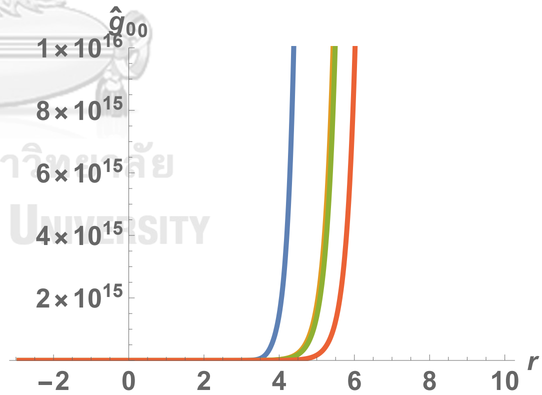
(c) W solution



(d) ϕ_1 solution

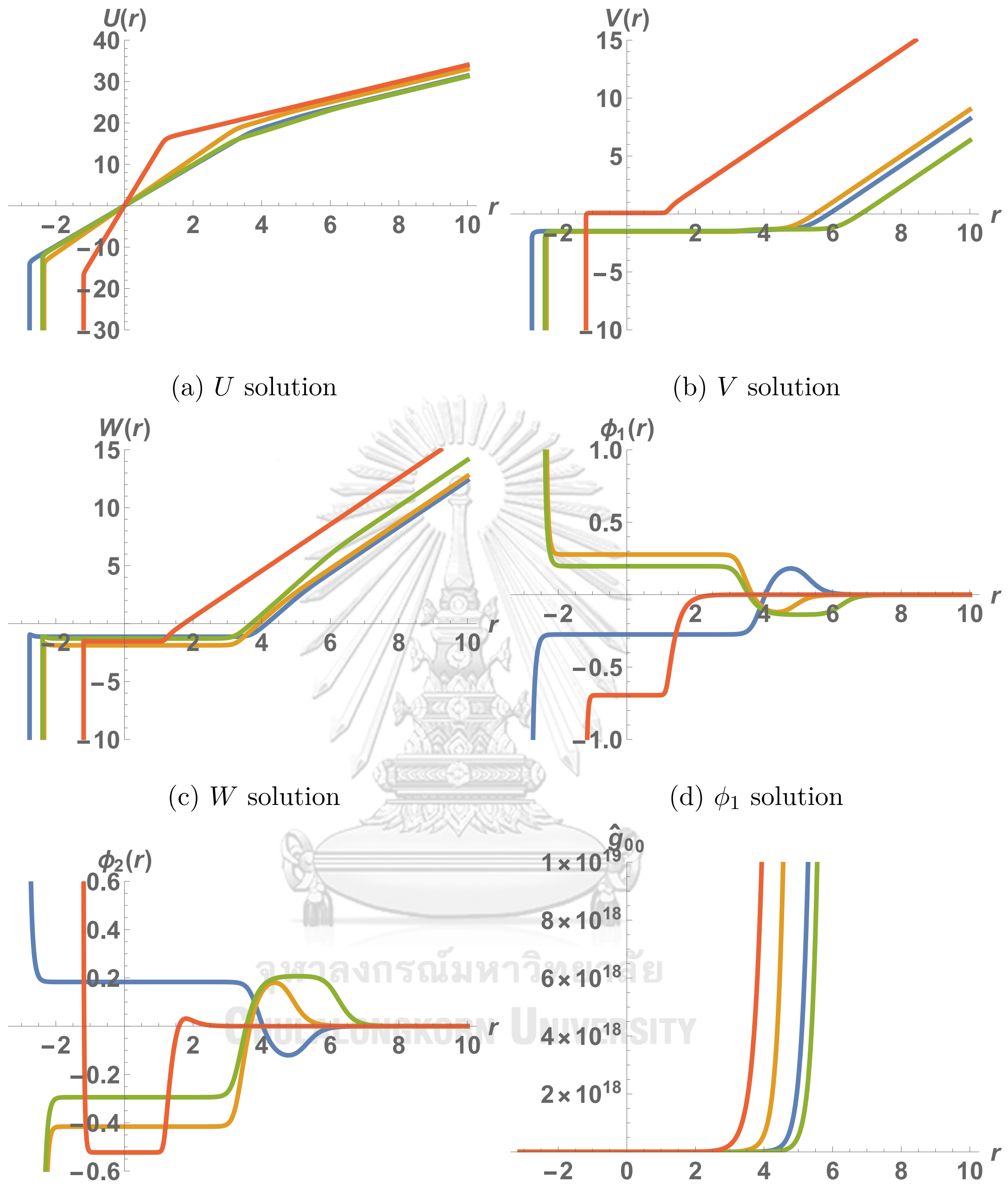


(e) ϕ_2 solution



(f) Profiles of $\hat{g}_{00}$

Figure 4.22: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to $AdS_3 \times H^2 \times \mathbb{R}^2$ fixed points at $r = 0$ and then to physical singularities in the form of $Mkw_2 \times H^2 \times \mathbb{R}^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $(z_1, z_2) = (1, -0.5), (-1, 1), (1, -2), (-8, 1)$.



(a) U solution (b) V solution

(c) W solution (d) ϕ_1 solution

(e) ϕ_2 solution (f) Profiles of $\hat{g}_{00}$

Figure 4.23: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to $AdS_3 \times H^2 \times S^2$ fixed points at $r = 0$ and then to physical singularities in the form of $Mkw_2 \times H^2 \times S^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $(z_1, z_2) = (-1, 5), (1, -2), (1, -4), (-3, 8)$.

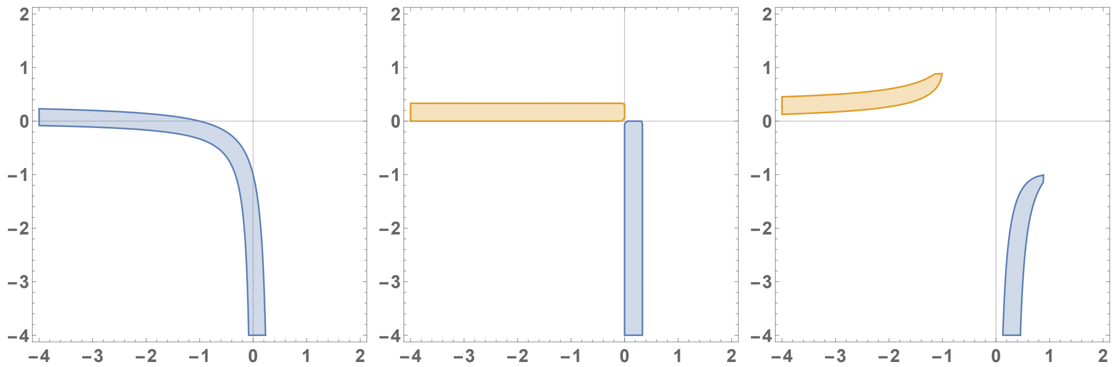


Figure 4.24: Regions (blue) in the parameter space $(z_1,\ z_2)$ where good AdS_3 vacua exist in $SO(3,2)$ gauge group for $g = 16$. From left to right, these figures correspond to the cases of $(k_1 = k_2 = 1)$, $(k_1 = 1,\ k_2 = 0)$, and $(k_1 = -k_2 = 1)$, respectively. The orange regions are obtained from interchanging k_1 and k_2.

A general ansatz for the seven-dimensional metric in this section takes the form of

$$ds_7^2 = e^{2U(r)}dx_{1,1}^2 + dr^2 + e^{2V(r)}ds_{K_k^4}^2 \qquad (4.465)$$

in which the explicit form for the metric on the Kahler four-cycle K_k^4 will be specified separately in each case.

4.3.1.5.1 Solutions with $SO(3)$ Twists

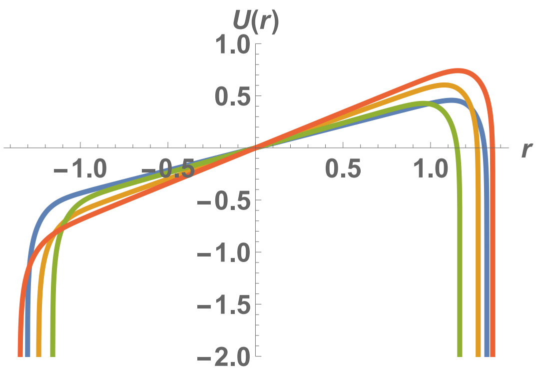
We begin with performing the twist along the $SU(2) \sim SO(3)$ part of the spin connection by choosing the metric on K_k^4 given in (3.165). With the following choice of vielbein

$$\begin{aligned} e^{\hat{m}} &= e^U dx^m, \qquad e^{\hat{2}} = dr, \qquad e^{\hat{3}} = e^V f_k(\psi)\tau_1, \\ e^{\hat{4}} &= e^V f_k(\psi)\tau_2, \qquad e^{\hat{5}} = e^V f_k(\psi)\tau_3, \qquad e^{\hat{6}} = e^V d\psi, \end{aligned} \qquad (4.466)$$

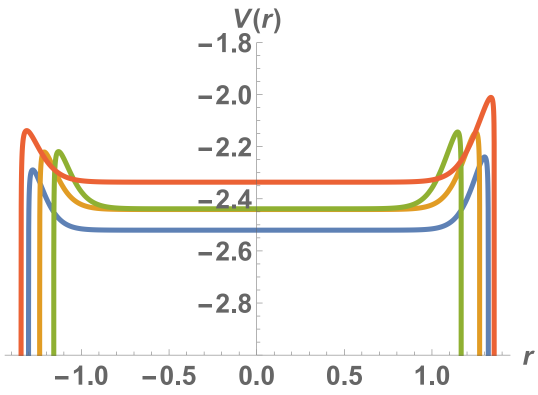
we find non-vanishing components of the spin connection

$$\begin{aligned} \omega_{(1)}^{\hat{m}\hat{2}} &= U'e^{\hat{m}}, \qquad \omega_{(1)}^{\hat{i}\hat{2}} = V'e^{\hat{i}}, \qquad \omega_{(1)}^{\hat{3}\hat{6}} = f_k'(\psi)\tau_1, \qquad \omega_{(1)}^{\hat{4}\hat{5}} = \tau_1, \\ \omega_{(1)}^{\hat{4}\hat{6}} &= f_k'(\psi)\tau_2, \qquad \omega_{(1)}^{\hat{5}\hat{3}} = \tau_2, \qquad \omega_{(1)}^{\hat{5}\hat{6}} = f_k'(\psi)\tau_3, \qquad \omega_{(1)}^{\hat{3}\hat{4}} = \tau_3 \end{aligned} \qquad (4.467)$$

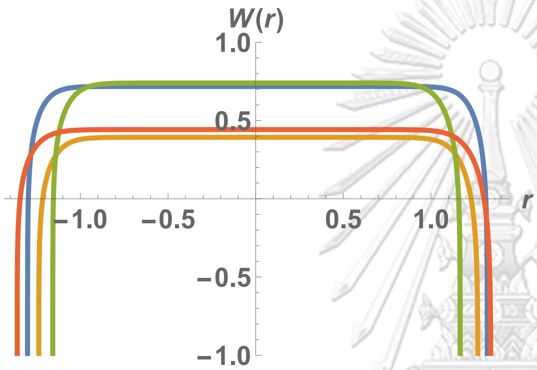
where $\hat{i} = \hat{3}, ..., \hat{6}$ is a flat index on K_k^4, and $\omega_{(1)}^{\hat{i}\hat{j}}$ are the $SU(2)$ parts of the spin connection.



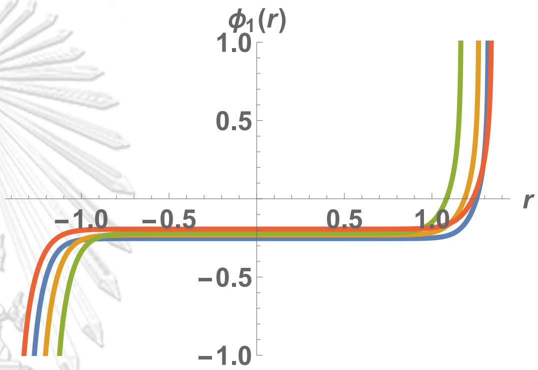
(a) U solution



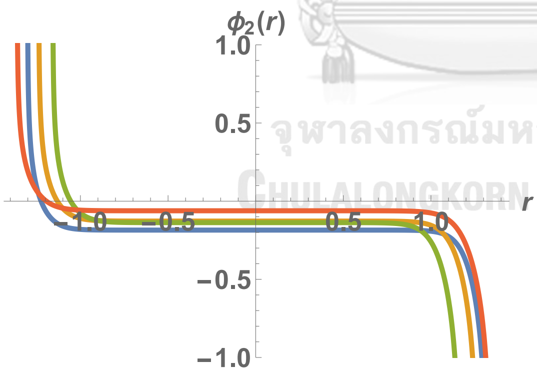
(b) V solution



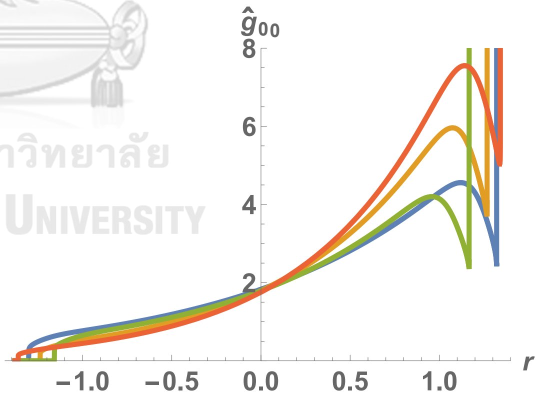
(c) W solution



(d) ϕ_1 solution

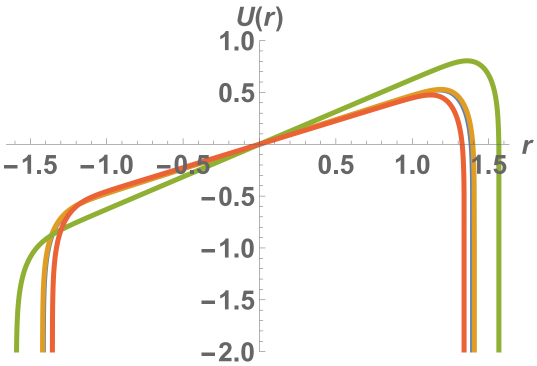


(e) ϕ_2 solution

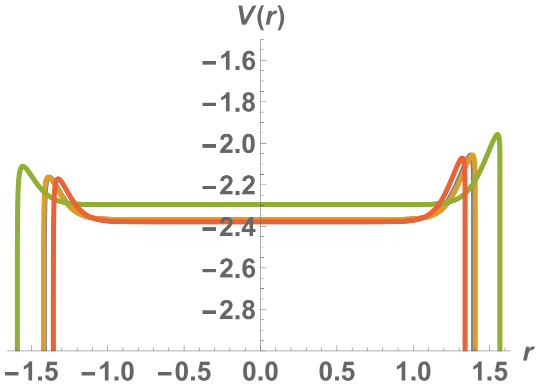


(f) Profiles of $\hat{g}_{00}$

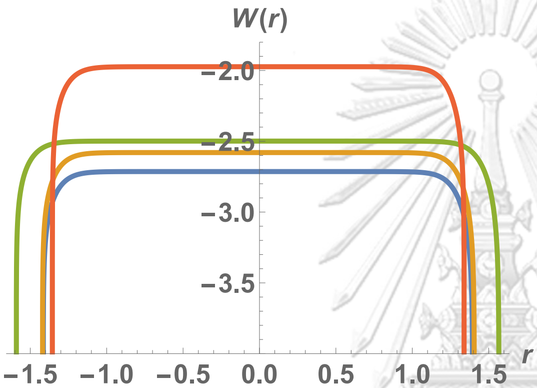
Figure 4.25: Numerical flows for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(3,2)$ gauge group. The flows start from $AdS_3 \times S^2 \times H^2$ fixed points at $r = 0$ to (un)physical singularities in the form of $Mkw_2 \times S^2 \times H^2$-sliced DWs in the region $r < 0$ $(r > 0)$. The blue, orange, green, and red curves refer to $(z_1, z_2) = (\frac{1}{24}, -18), (\frac{1}{16}, -12), (\frac{1}{24}, -24), (\frac{1}{16}, -24)$.



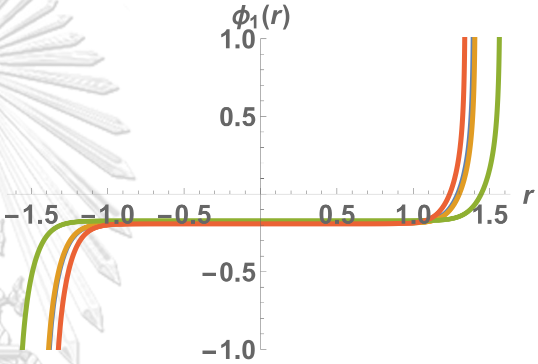
(a) U solution



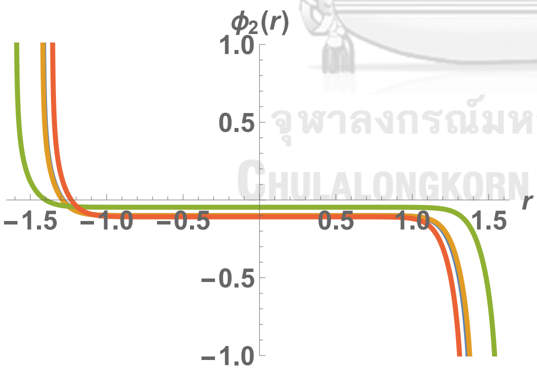
(b) V solution



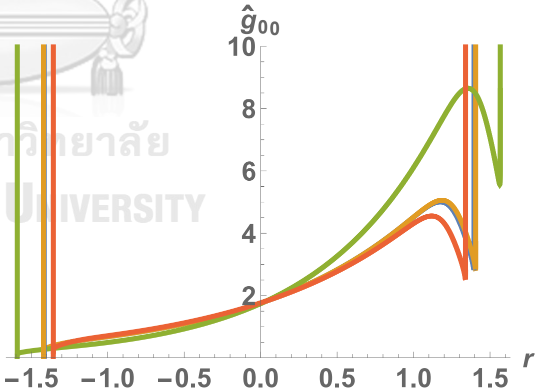
(c) W solution



(d) ϕ_1 solution

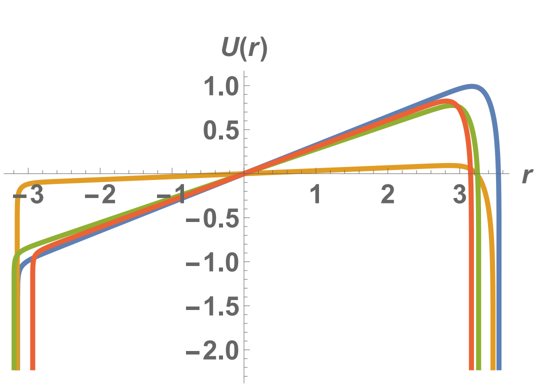


(e) ϕ_2 solution

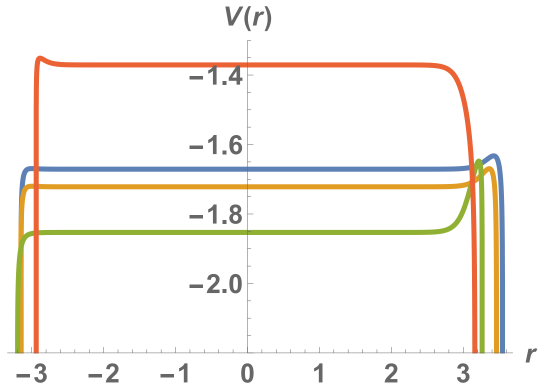


(f) Profiles of $\hat{g}_{00}$

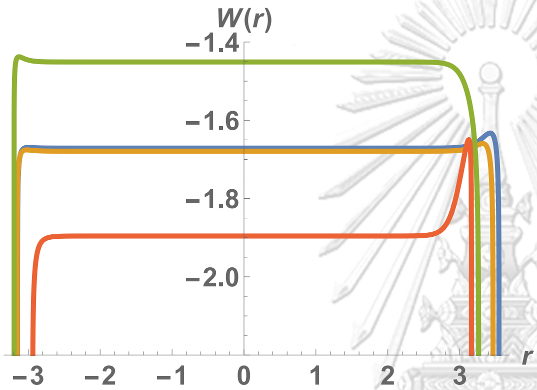
Figure 4.26: Numerical flows for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(3,2)$ gauge group. The flows start from $AdS_3 \times S^2 \times \mathbb{R}^2$ fixed points at $r = 0$ to unphysical singularities in the form of $Mkw_2 \times S^2 \times \mathbb{R}^2$-sliced DWs on both $r \neq 0$ sides. The blue, orange, green, and red curves refer to $(z_1, z_2) = (\frac{1}{34}, -\frac{1}{34}), (\frac{1}{34}, -\frac{1}{26}), (\frac{1}{24}, -\frac{1}{18}), (\frac{1}{36}, -\frac{1}{8})$.



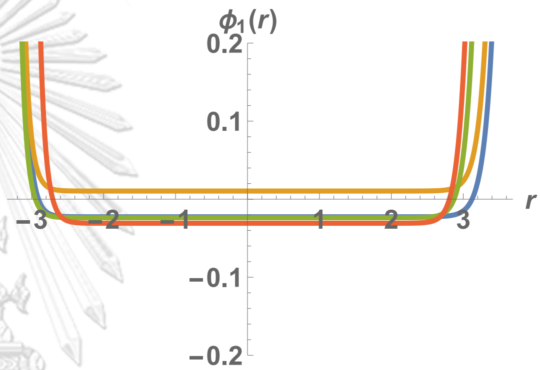
(a) U solution



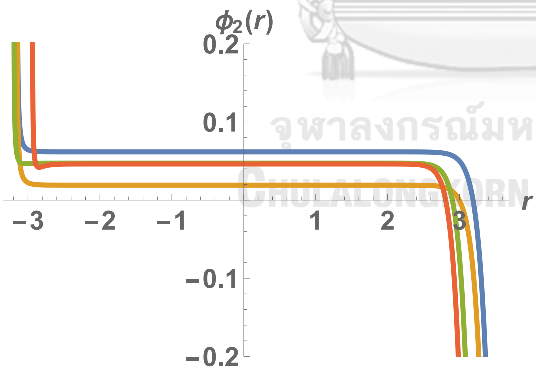
(b) V solution



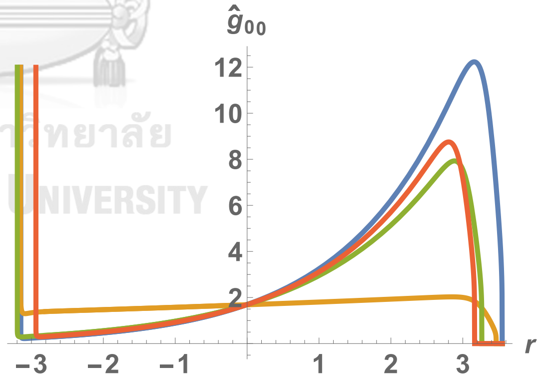
(c) W solution



(d) ϕ_1 solution



(e) ϕ_2 solution



(f) Profiles of $\hat{g}_{00}$

Figure 4.27: Numerical flows for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(3,2)$ gauge group. The flows start from $AdS_3 \times S^2 \times S^2$ fixed points at $r = 0$ to (un)physical singularities in the form of $Mkw_2 \times S^2 \times S^2$-sliced DWs in the region $r > 0$ ($r < 0$). The blue, orange, green, and red curves refer to $(z_1, z_2) = (-0.55, -0.55), (-0.55, -0.6), (-0.35, -0.87), (-1, -0.3)$.

To implement the twist, we turn on the following $SO(3)$ gauge fields

$$A^{IJ}_{(1)} = -\frac{p}{k}(f'_k(\psi) - 1)\varepsilon^{IJK}\tau_K, \qquad I, J, K = 1, 2, 3 \tag{4.468}$$

with the modified two-forms given by

$$\mathcal{F}^{12}_{(2)} = F^{12}_{(2)} = e^{-2V}p(e^{\hat{3}} \wedge e^{\hat{4}} - e^{\hat{5}} \wedge e^{\hat{6}}), \tag{4.469}$$

$$\mathcal{F}^{23}_{(2)} = F^{23}_{(2)} = e^{-2V}p(e^{\hat{4}} \wedge e^{\hat{5}} - e^{\hat{3}} \wedge e^{\hat{6}}), \tag{4.470}$$

$$\mathcal{F}^{31}_{(2)} = F^{31}_{(2)} = e^{-2V}p(e^{\hat{5}} \wedge e^{\hat{3}} - e^{\hat{4}} \wedge e^{\hat{6}}). \tag{4.471}$$

Unlike the previous case, we do not need to turn on the modified three-forms since $\epsilon_{MNPQR}\mathcal{F}^{NP}_{(2)} \wedge \mathcal{F}^{QR}_{(2)} = 0$ in this case.

We then impose the twist condition (4.409) together with the following three projection conditions

$$\gamma^{\hat{3}\hat{4}}\epsilon^a = -(\Gamma_{12})^a{}_b\epsilon^b, \qquad \gamma^{\hat{4}\hat{5}}\epsilon^a = -(\Gamma_{23})^a{}_b\epsilon^b, \qquad \gamma^{\hat{3}\hat{4}}\epsilon^a = -\gamma^{\hat{5}\hat{6}}\epsilon^a\,. \tag{4.472}$$

Using the scalar coset representative (4.29) and the projection (4.388), we find the following BPS equations

$$U' = \frac{g}{40}e^{6\phi_1}\left[3e^{-10\phi_1} + (\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3 + (\rho-\sigma)\sinh 2\phi_3\right] - \frac{12}{5}e^{-2(V-2\phi_1)}p, \tag{4.473}$$

$$V' = \frac{g}{40}e^{6\phi_1}\left[3e^{-10\phi_1} + (\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3 + (\rho-\sigma)\sinh 2\phi_3\right] + \frac{18}{5}e^{-2(V-2\phi_1)}p, \tag{4.474}$$

$$\phi_1' = \frac{g}{40}e^{6\phi_1}\left[2e^{-10\phi_1} + (\rho-\sigma)\sinh 2\phi_3 - (\rho+\sigma)\cosh 2\phi_2\cosh 2\phi_3\right] - \frac{8}{5}e^{-2(V-2\phi_1)}p, \tag{4.475}$$

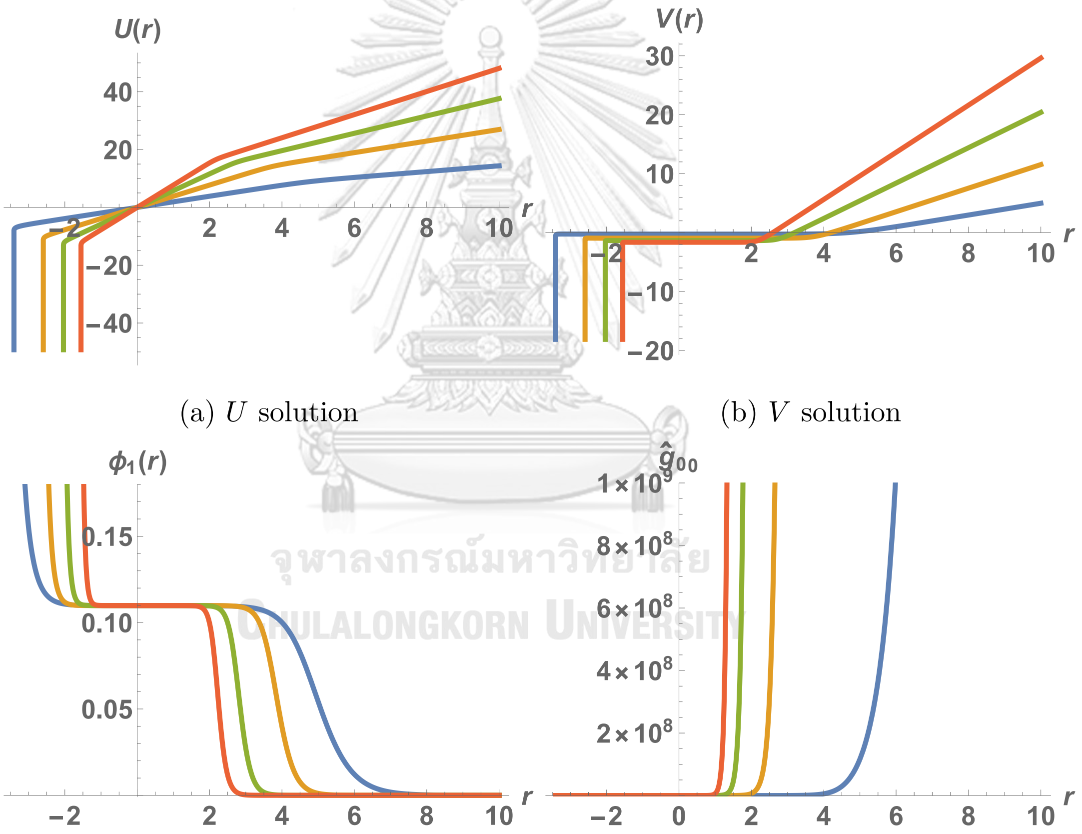
$$\phi_2' = -\frac{g}{8}e^{6\phi_1}(\rho+\sigma)\sinh 2\phi_2\,\text{sech}2\phi_3, \tag{4.476}$$

$$\phi_3' = -\frac{g}{8}e^{6\phi_1}\left((\rho+\sigma)\cosh 2\phi_2\sinh 2\phi_3 + (\rho-\sigma)\cosh 2\phi_3\right). \tag{4.477}$$

It turns out that only $SO(5)$ gauge group admits an $AdS_3 \times CH^2$ fixed point given by

$$V = \frac{1}{2}\ln\left[\frac{16\times 3^{4/5}}{g^2}\right], \qquad \phi_1 = \frac{1}{10}\ln 3,$$
$$\phi_2 = \phi_3 = 0, \qquad L_{\text{AdS}_3} = \frac{8}{3^{3/5}g}\,. \tag{4.478}$$

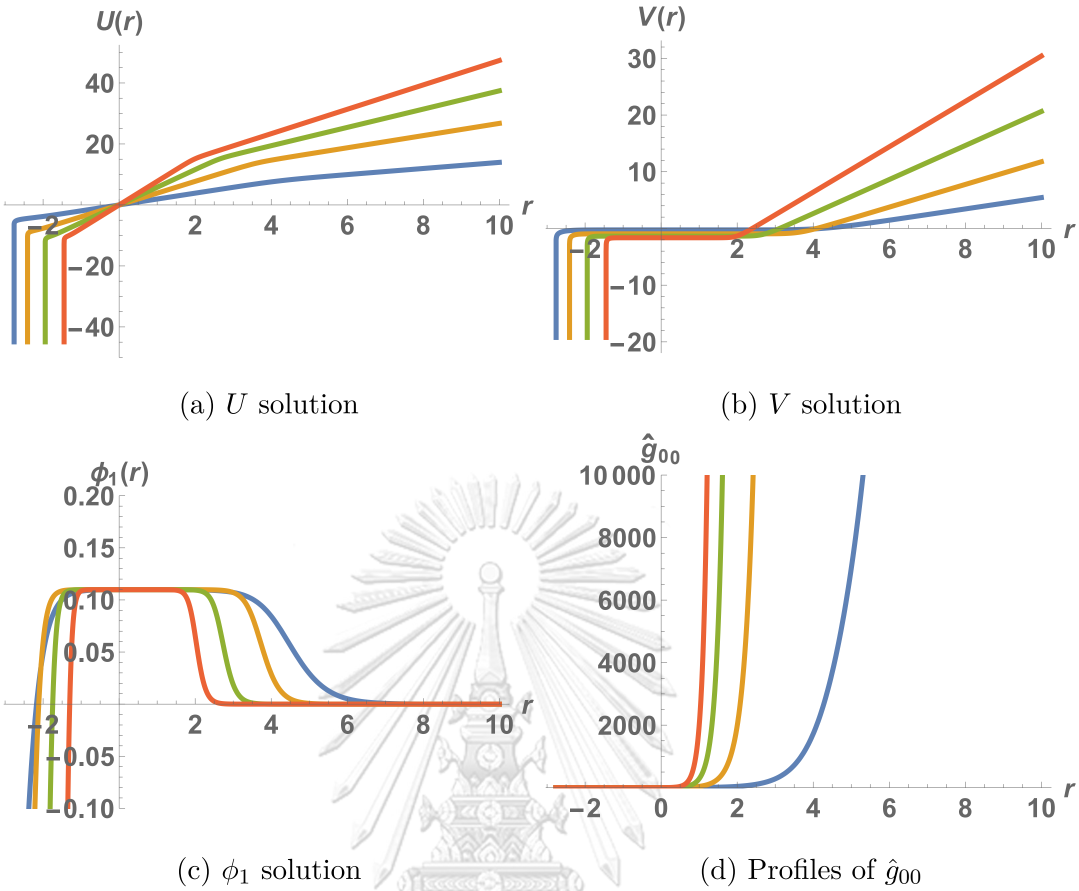
This is the $AdS_3 \times CH^2$ solution found in [53]. The solution preserves four supercharges and corresponds to a two-dimensional $N = (2, 0)$ SCFT with $SO(3) \times SO(2)$ symmetry. Note here that the scalar coset representative is invariant under $SO(3) \times SO(2) \subset SO(5)$ for $\phi_2 = \phi_3 = 0$. Various examples of general RG flows, from the supersymmetric AdS_7 critical point to this $AdS_3 \times CH^2$ fixed point and then to curved DWs, are shown in Figures 4.28, 4.29, and 4.30. From these figures, we find that both singularities for $\phi_1 \to \pm\infty$ in the flows with $\phi_2 = \phi_3 = 0$ are physical. In the flows with ϕ_1, ϕ_2, and ϕ_3 non-vanishing, the IR singularities are unphysical because of $\hat{g}_{00} \to \infty$ near the singularities.



(a) U solution (b) V solution

(c) ϕ_1 solution (d) Profiles of $\hat{g}_{00}$

Figure 4.28: Numerical solutions for $SO(3)$ twists with $\phi_2 = \phi_3 = 0$ along the flows in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to the $AdS_3 \times CH^2$ fixed point at $r = 0$ and then to physical singularities in the form of $Mkw_2 \times CH^2$-sliced DWs with $\phi_1 \to +\infty$ in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.



(a) U solution (b) V solution

(c) ϕ_1 solution (d) Profiles of $\hat{g}_{00}$

Figure 4.29: Numerical solutions for $SO(3)$ twists with $\phi_2 = \phi_3 = 0$ along the flows in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to the $AdS_3 \times CH^2$ fixed point at $r = 0$ and then to physical singularities in the form of $Mkw_2 \times CH^2$-sliced DWs with $\phi_1 \to -\infty$ in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.

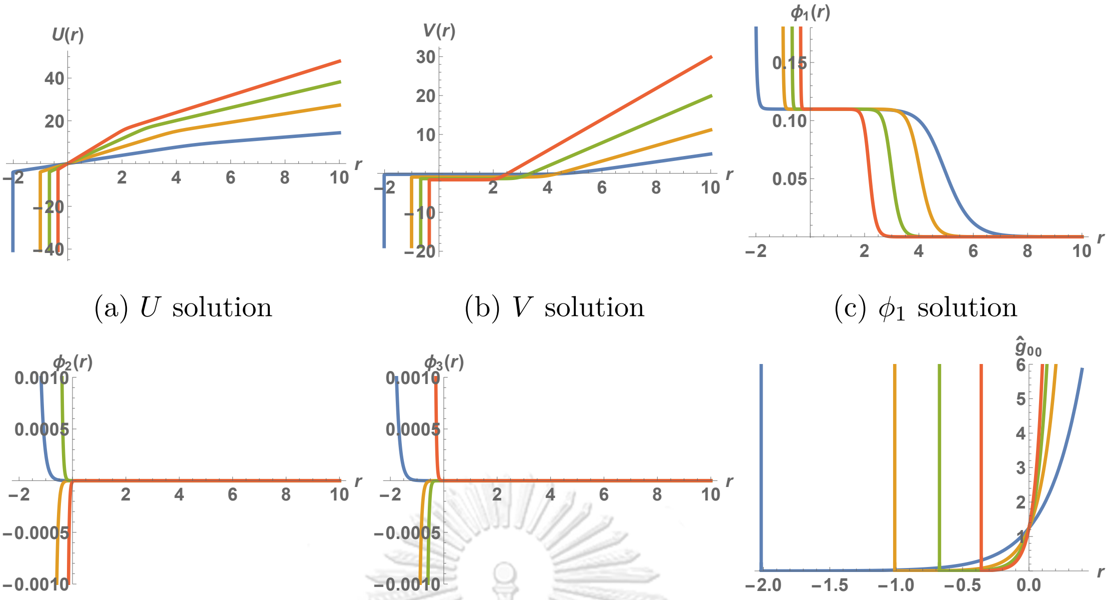
4.3.1.5.2 Solutions with $SO(3)_+$ Twists

We now move to $AdS_3 \times K^4$ solutions with the twist given by identifying the $SU(2)$ parts of the spin connection with the self-dual $SO(3)_+ \subset SO(3)_+ \times SO(3)_- \sim SO(4) \subset SO(5)_R$. Starting from $SO(3) \times SO(3)$ gauge fields of the form

$$A^{IJ}_{(1)} = -\frac{p}{2k}(f'_k(\psi) - 1)\epsilon^{IJK}\tau_K \qquad \text{and} \qquad A^{I4}_{(1)} = -\frac{p}{2k}(f'_k(\psi) - 1)\delta^{IJ}\tau_J\,, \quad (4.479)$$

the self-dual $SO(3)_+$ gauge fields can be defined as

$$A^{I}_{(1)} = \frac{1}{2}\epsilon^{IJK}A^{JK}_{(1)} + A^{I4}_{(1)} = -\frac{p}{k}(f'_k(\psi) - 1)\tau_I\,. \quad (4.480)$$



(a) U solution (b) V solution (c) ϕ_1 solution

(d) ϕ_2 solution (e) ϕ_3 solution (f) Profiles of $\hat{g}_{00}$

Figure 4.30: Numerical solutions for $SO(3)$ twists in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to the $AdS_3 \times CH^2$ fixed point at $r = 0$ and then to unphysical singularities in the form of $Mkw_2 \times CH^2$-sliced DWs with ϕ_1, ϕ_2, and ϕ_3 non-vanishing in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.

Using these gauge fields, we can perform the twist by imposing the twist condition (4.409) and the three projections given in (4.472) together with an additional projection condition for the self-duality of $SO(3)_+$

$$(\Gamma_{12})^a{}_b\epsilon^b = (\Gamma_{34})^a{}_b\epsilon^b\,. \qquad (4.481)$$

Furthermore, by turning on the above $SO(3)_+$ gauge fields, we also need to turn on the modified three-forms of the form

$$\mathcal{H}_{\hat{m}\hat{n}\hat{2}5} = -\frac{192}{g}\rho e^{-4(V+2\phi)}p^2\varepsilon_{\hat{m}\hat{n}}\,. \qquad (4.482)$$

We will consider only $SO(5)$ and $SO(4,1)$ gauge groups with $\rho \neq 0$ as in the case of $SO(4)$ symmetric solutions.

Using the embedding tensor (4.5) and the coset represenvative (4.7) for the

$SO(4)$ invariant scalar, we find the following BPS equations

$$U' = \frac{g}{40}(4e^{-2\phi} + \rho e^{8\phi}) - \frac{12}{5}e^{-2(V-\phi)}p + \frac{144}{5g}\rho e^{-4(V+\phi)}p^2, \quad (4.483)$$

$$V' = \frac{g}{40}(4e^{-2\phi} + \rho e^{8\phi}) + \frac{18}{5}e^{-2(V-\phi)}p - \frac{96}{5g}\rho e^{-4(V+\phi)}p^2, \quad (4.484)$$

$$\phi' = \frac{g}{20}(e^{-2\phi} - \rho e^{8\phi}) - \frac{6}{5}e^{-2(V-\phi)}p - \frac{48}{5g}\rho e^{-4(V+\phi)}p^2 \quad (4.485)$$

in which we have also imposed the γ^r projection (4.388). From these equations, an AdS_3 fixed point is obtained only in $SO(5)$ gauge group with $k = -1$ and $\rho = 1$. This $AdS_3 \times CH^2$ solution is given by

$$V = \frac{1}{2}\ln\left[\frac{4^{7/5} \times 3^{2/5} \times 7^{3/5}}{g^2}\right], \quad \phi = \frac{1}{10}\ln\left[\frac{12}{7}\right], \quad L_{\text{AdS}_3} = \frac{4^{6/5} \times 3^{1/5}}{g^2 7^{1/5}} \quad (4.486)$$

which is the $AdS_3 \times CH^2$ fixed point found in [53]. The solution preserves two supercharges and corresponds to a two-dimensional $N = (1, 0)$ SCFT with $SO(3)$ symmetry. Supersymmetric RG flows, from the AdS_7 vacuum to this $AdS_3 \times CH^2$ fixed point and curved DWs in the IR, are given in Figure 4.31. The IR singularities are physically acceptable, as indicated by the behavior $\hat{g}_{00} \to 0$.

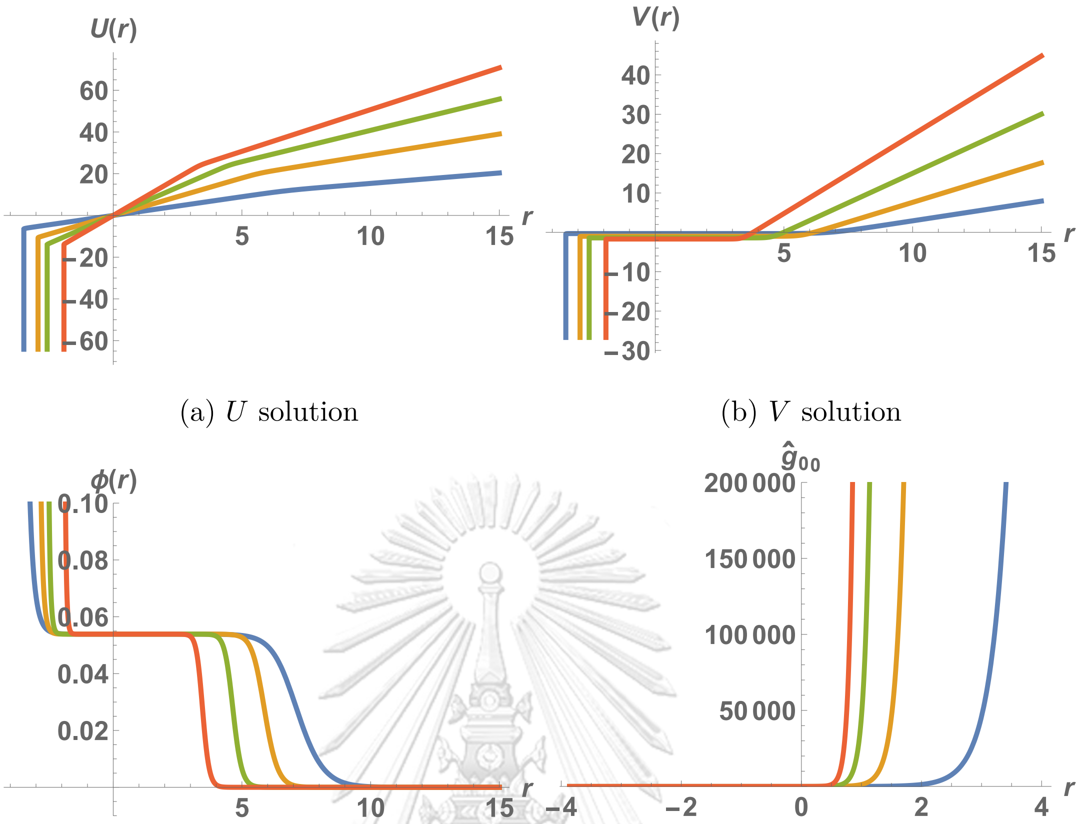
4.3.1.5.3 Solutions with $SO(2) \times SO(2)$ Twists

As a final case for $AdS_3 \times K^4$ solutions, we perform another twist by canceling the $U(1)$ part of the spin connection on the Kahler four-cycle. To make this $U(1)$ part manifest, we choose the metric on K^4_k in (3.137) together with the following choice of vielbein

$$e^{\hat{m}} = e^U dx^m, \qquad e^{\hat{2}} = dr, \qquad e^{\hat{3}} = \frac{e^V\psi}{\sqrt{k\psi^2+1}}\tau_1,$$
$$e^{\hat{4}} = \frac{e^V\psi}{\sqrt{k\psi^2+1}}\tau_2, \qquad e^{\hat{5}} = \frac{e^V\psi}{(k\psi^2+1)}\tau_3, \qquad e^{\hat{6}} = \frac{e^V d\psi}{(k\psi^2+1)}. \quad (4.487)$$

All non-vanishing components of the spin connection, in this case, are given by

$$\omega^{\hat{m}\hat{2}}_{(1)} = U' e^{\hat{m}}, \qquad \omega^{\hat{i}\hat{2}}_{(1)} = V' e^{\hat{i}}, \qquad \hat{i} = \hat{3}, ..., \hat{6},$$
$$\omega^{\hat{3}\hat{6}}_{(1)} = \omega^{\hat{4}\hat{5}}_{(1)} = \frac{\tau_1}{\sqrt{k\psi^2+1}}, \qquad \omega^{\hat{3}\hat{4}} = \frac{(2k\psi^2+1)}{(k\psi^2+1)}\tau_3,$$
$$\omega^{\hat{5}\hat{6}}_{(1)} = \frac{(1-k\psi^2)}{(k\psi^2+1)}\tau_3, \qquad \omega^{\hat{5}\hat{3}}_{(1)} = \omega^{\hat{4}\hat{6}}_{(1)} = \frac{\tau_2}{\sqrt{k\psi^2+1)}}. \quad (4.488)$$



(a) U solution (b) V solution

(c) ϕ solution (d) Profiles of $\hat{g}_{00}$

Figure 4.31: Numerical solutions for $SO(3)_+$ twists in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 15$ to the $AdS_3 \times CH^2$ fixed point at $r = 0$ and then to physical singularities in the form of $Mkw_2 \times CH^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.

We perform the twist by turning on the $SO(2) \times SO(2)$ gauge fields

$$A^{12}_{(1)} = p_1 \frac{3\psi^2}{\sqrt{k\psi^2 + 1}} \tau_3 \qquad \text{and} \qquad A^{34}_{(1)} = p_2 \frac{3\psi^2}{\sqrt{k\psi^2 + 1}} \tau_3 \tag{4.489}$$

and imposing the following projection conditions

$$\gamma^{\hat{3}\hat{4}} \epsilon^a = -\gamma^{\hat{5}\hat{6}} \epsilon^a = -(\Gamma_{12})^a{}_b \epsilon^b = -(\Gamma_{34})^a{}_b \epsilon^b \tag{4.490}$$

together with the twist condition (4.386). The associated modified two-forms are

$$\mathcal{F}^{12}_{(2)} = F^{12}_{(2)} = 3e^{-2V} p_1 (e^{\hat{3}} \wedge e^{\hat{4}} - e^{\hat{5}} \wedge e^{\hat{6}}), \tag{4.491}$$

$$\mathcal{F}^{34}_{(2)} = F^{34}_{(2)} = 3e^{-2V} p_2 (e^{\hat{3}} \wedge e^{\hat{4}} - e^{\hat{5}} \wedge e^{\hat{6}}). \tag{4.492}$$

With the above non-vanishing $SO(2) \times SO(2)$ gauge fields, we need to turn on the modified three-form

$$\mathcal{H}_{\hat{m}\hat{n}\hat{2}5} = \frac{576}{g}\rho e^{-4(V+\phi_1+\phi_2)}p_1 p_2 \varepsilon_{\hat{m}\hat{n}} \tag{4.493}$$

with ρ being the parameter in the embedding tensor (4.49) for gauge groups with an $SO(2) \times SO(2)$ subgroup. As in the previous cases, the appearance of ρ in (4.493) implies that the resulting BPS equations are not compatible with the field equations for the case of $\rho = 0$. We will accordingly consider only gauge groups with $\rho \neq 0$ in the following analysis.

With the γ^r projector (4.388) and the scalar coset representative (4.50), the corresponding BPS equations read

$$\begin{aligned} U' &= \frac{g}{40}(2e^{-2\phi_1} + \rho e^{4(\phi_1+\phi_2)} + 2\sigma e^{-2\phi_2}) - \frac{12}{5}e^{-2V}(e^{2\phi_1}p_1 + e^{2\phi_2}p_2) \\ &\quad + \frac{1728}{5g}\rho e^{-2(2V+\phi_1+\phi_2)}p_1 p_2, \end{aligned} \tag{4.494}$$

$$\begin{aligned} V' &= \frac{g}{40}(2e^{-2\phi_1} + \rho e^{4(\phi_1+\phi_2)} + 2\sigma e^{-2\phi_2}) + \frac{18}{5}e^{-2V}(e^{2\phi_1}p_1 + e^{2\phi_2}p_2) \\ &\quad - \frac{1152}{5g}\rho e^{-2(2V+\phi_1+\phi_2)}p_1 p_2, \end{aligned} \tag{4.495}$$

$$\begin{aligned} \phi_1' &= \frac{g}{20}(3e^{-2\phi_1} - \rho e^{4(\phi_1+\phi_2)} - 2\sigma e^{-2\phi_2}) - \frac{12}{5}e^{-2V}(3e^{2\phi_1}p_1 - 2e^{2\phi_2}p_2) \\ &\quad - \frac{576}{5g}\rho e^{-2(2V+\phi_1+\phi_2)}p_1 p_2, \end{aligned} \tag{4.496}$$

$$\begin{aligned} \phi_2' &= \frac{g}{20}(3\sigma e^{-2\phi_2} - \rho e^{4(\phi_1+\phi_2)} - 2e^{-2\phi_1}) + \frac{12}{5}e^{-2V}(2e^{2\phi_1}p_1 - 3e^{2\phi_2}p_2) \\ &\quad - \frac{576}{5g}\rho e^{-2(2V+\phi_1+\phi_2)}p_1 p_2\,. \end{aligned} \tag{4.497}$$

From these BPS equations, we find the following AdS_3 fixed points

$$e^{2V} = -\frac{48(p_1 e^{2\phi_1} + p_2 e^{2\phi_2})}{g\rho e^{4(\phi_1+\phi_2)}}, \tag{4.498}$$

$$e^{10\phi_1} = \frac{p_2^2\rho\left((p_1 + p_1\rho^2 - p_2\sigma)(p_1 + p_2\sigma)\right)^2}{p_1^3(2+\rho^2)(p_2(1+\rho^2)\sigma - p_1)^3}, \tag{4.499}$$

$$e^{10\phi_2} = \frac{p_1^2\rho\left(p_1 - p_2(1+\rho^2)\sigma)(p_1 + p_2\sigma)\right)^2}{p_2^3(2+\rho^2)(p_1 + p_1\rho^2 - p_2\sigma)^3}, \tag{4.500}$$

$$L_{\text{AdS}_3} = \frac{8e^{-4(\phi_1+\phi_2)}(p_1 e^{2\phi_1} + p_2 e^{2\phi_2})^2}{g\rho\left(p_1^2 e^{4\phi_1} + p_2^2 e^{4\phi_2} + 2p_1 p_2 e^{2(\phi_1+\phi_2)}(1+\rho^2)\right)}. \tag{4.501}$$

These solutions preserve four supercharges and are dual to $N = (2, 0)$ SCFTs in two dimensions with $SO(2) \times SO(2)$ symmetry.

For $SO(5)$ gauge group, there exist $AdS_3 \times CH^2$ fixed points in the range

$$-\frac{2}{3} < gp_2 < -\frac{1}{3} \tag{4.502}$$

in which we have taken $g > 0$ for convenience. Up to some differences in notations, these $AdS_3 \times CH^2$ fixed points are the same as the solutions studied in [58]. As in the previous cases, we also study RG flows from the supersymmetric AdS_7 critical point to the $AdS_3 \times CH^2$ fixed points and curved DWs in the IR. Some examples of these flows are given in Figure 4.32 for $g = 16$ and different values of p_2. In these examples, the behaviors of the eleven-dimensional metric component $\hat{g}_{00}$ indicate that the singularities are physical, as shown in Figure 4.33.

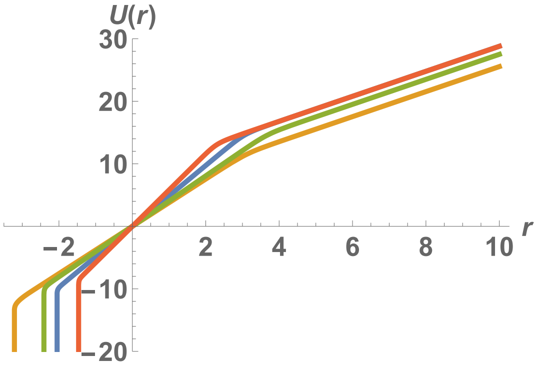
Apart from the $AdS_3 \times CH^2$ solutions, we find new $AdS_3 \times CP^2$ fixed points in non-compact $SO(4,1)$ and $SO(3,2)$ gauge groups respectively within the following ranges, with $g > 0$,

$$gp_2 < 0 \ \cup \ gp_2 > 1 \qquad \text{and} \qquad -\frac{(3-\sqrt{3})}{6} < gp_2 < -\frac{2}{3}\,. \tag{4.503}$$

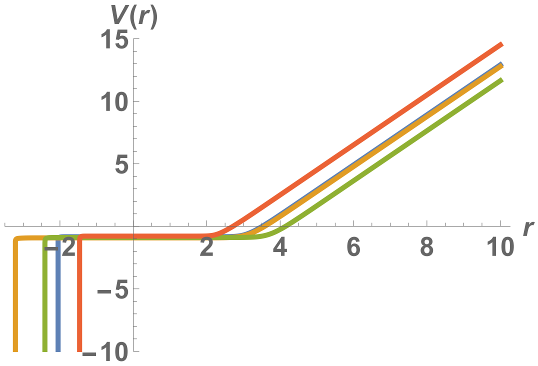
Recall that there is no supersymmetric AdS_7 critical point for $SO(4,1)$ and $SO(3,2)$ gauge groups. We will study supersymmetric RG flows between these $AdS_3 \times CP^2$ fixed points and curved DWs with $SO(2) \times SO(2)$ symmetry. With $g = 16$ and different values of p_2, examples of these RG flows in $SO(4,1)$ and $SO(3,2)$ gauge groups are shown respectively in Figures 4.34 and 4.35. From the behaviors of $\hat{g}_{00}$ in Figure 4.36, we find that the singularities on the left (right) with $\phi_1 \to \pm\infty$ and $\phi_2 \to \mp\infty$ ($\phi_1 \to +\infty$ and $\phi_2 \to -\infty$) of the flows in $SO(4,1)$ ($SO(3,2)$) gauge group are physically acceptable.

4.3.1.6 Supersymmetric $AdS_2 \times \Sigma^3 \times \Sigma^2$ Solutions with $SO(3) \times SO(2)$ Symmetry

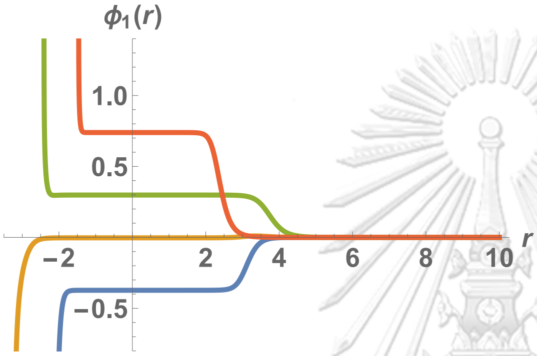
We end this section by considering solutions with AdS_2 vacua. For the five-manifold Σ^5 being S^5 or H^5, $AdS_2 \times \Sigma^5$ solutions have been given in [53] by performing the twist using $SO(5)$ gauge fields. These solutions are possible only for $SO(5)$ gauge group. Besides, there is no scalar in $SL(5)/SO(5)$ coset invariant



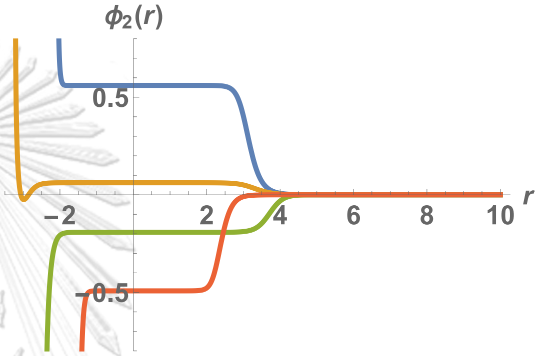
(a) U solution



(b) V solution



(c) ϕ_1 solution



(d) ϕ_2 solution

Figure 4.32: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to $AdS_3 \times CH^2$ fixed points at $r = 0$ and then to singularities in the form of $Mkw_2 \times CH^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $p_2 = -\frac{1}{25}, -\frac{1}{31}, -\frac{1}{40}, -\frac{1}{46}$.

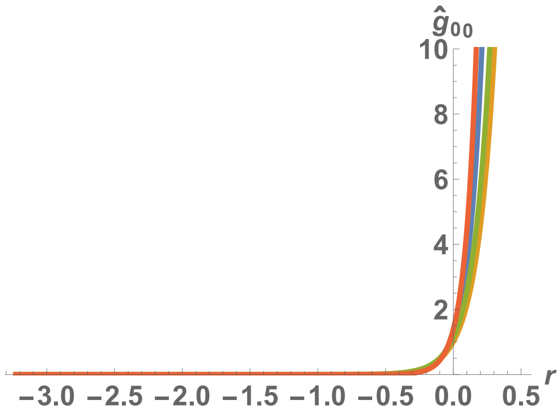
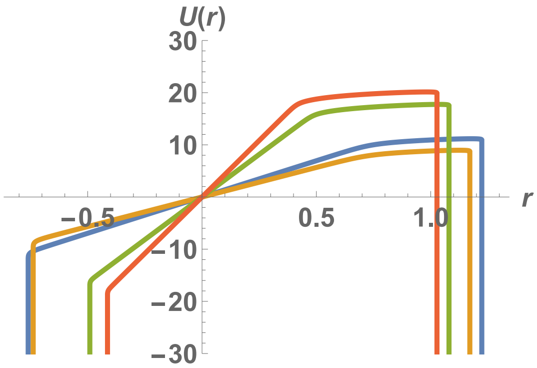
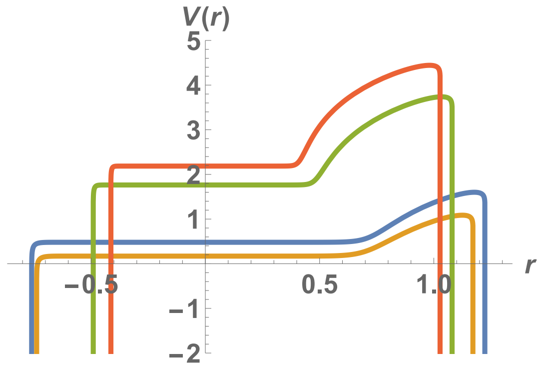


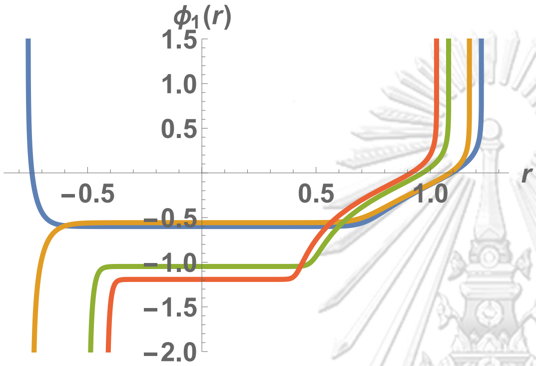
Figure 4.33: The behavior of $\hat{g}_{00}$ for RG flows given in Figure 4.32 where $\hat{g}_{00} \to 0$ in the region $r < 0$.



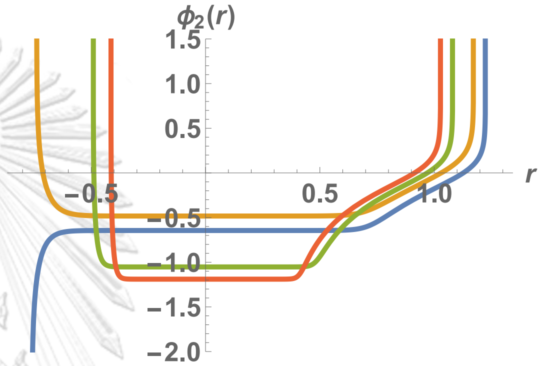
(a) U solution



(b) V solution



(c) ϕ_1 solution



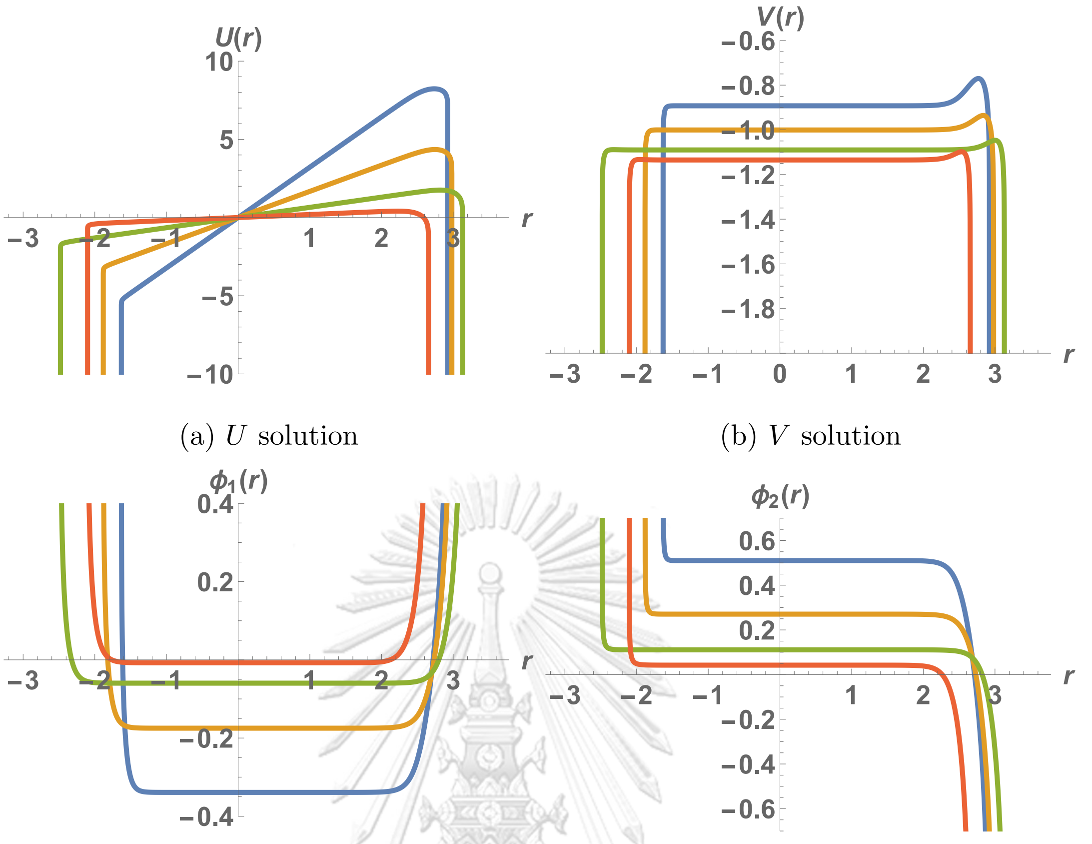
(d) ϕ_2 solution

Figure 4.34: Numerical solutions for $SO(2)\times SO(2)$ twists with $g = 16$ in $SO(4,1)$ gauge group. The flows start from $AdS_3\times CP^2$ fixed points at $r = 0$ to singularities in the form of $Mkw_2 \times CP^2$-sliced DWs on both $r \neq 0$ sides. The blue, orange, green, and red curves refer to $p_2 = \frac{1}{2}, -\frac{1}{4}, 4, -8, -\frac{1}{47}$.

under $SO(5)$ unbroken symmetry, so the solutions are purely given in terms of the seven-dimensional metric. The corresponding RG flows from the supersymmetric AdS_7 vacuum to the $AdS_2 \times H^5$ or $AdS_2 \times S^5$ fixed points have already been analytically given in [53]. We will not repeat the analysis for this case here.

However, if we consider Σ^5 as a product of Riemannian three- and two-manifolds $\Sigma^3 \times \Sigma^2$, it is possible to perform a twist by turning on $SO(3) \times SO(2)$ gauge fields along $\Sigma^3 \times \Sigma^2$. In this case, there are two gauge groups with an $SO(3) \times SO(2)$ subgroup, namely $SO(5)$ and $SO(3,2)$. The ansatz for the seven-dimensional metric takes the form of

$$ds_7^2 = -e^{2U(r)}dt^2 + dr^2 + e^{2V(r)}ds^2_{\Sigma^3_{k_1}} + e^{2W(r)}ds^2_{\Sigma^2_{k_2}}\,. \tag{4.504}$$



(a) U solution (b) V solution

(c) ϕ_1 solution (d) ϕ_2 solution

Figure 4.35: Numerical solutions for $SO(2)\times SO(2)$ twists with $g=16$ in $SO(3,2)$ gauge group. The flows start from $AdS_3\times CP^2$ fixed points at $r=0$ to singularities in the form of $Mkw_2\times CP^2$-sliced DWs on both $r\neq 0$ sides. The blue, orange, green, and red curves refer to $p_2=-\frac{1}{23},-\frac{1}{22},-\frac{1}{21},-\frac{2}{41}$.

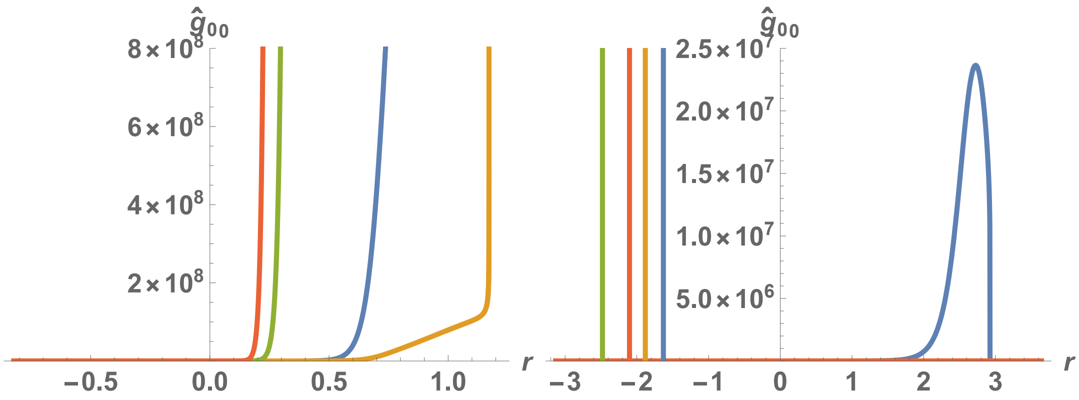
The explicit form of the metrics on the $\Sigma^3_{k_1}$ and $\Sigma^2_{k_2}$ are given in (4.405) and (3.79), respectively.

Using the vielbein

$$\begin{aligned} e^{\hat{0}} &= e^U dt, \quad e^{\hat{1}} = dr, \quad e^{\hat{2}} = e^V d\psi_1, \quad e^{\hat{3}} = e^V f_{k_1}(\psi_1)d\theta_1, \\ e^{\hat{4}} &= e^V f_{k_1}(\psi_1)\sin\theta_1 d\varphi_1, \quad e^{\hat{5}} = e^W d\theta_2, \quad e^{\hat{6}} = e^W f_{k_2}(\theta_2)d\varphi_2, \end{aligned} \qquad (4.505)$$

we find non-vanishing components of the spin connection as follow

$$\begin{aligned} \omega^{\hat{0}\hat{1}}_{(1)} &= U'e^{\hat{0}}, \quad \omega^{\hat{i}_1\hat{1}}_{(1)} = V'e^{\hat{i}_1}, \quad \omega^{\hat{i}_2\hat{1}}_{(1)} = W'e^{\hat{i}_2}, \quad \omega^{\hat{3}\hat{2}}_{(1)} = \frac{f'_{k_1}(\psi_1)}{f_{k_1}(\psi_1)}e^{-V}e^{\hat{3}}, \\ \omega^{\hat{4}\hat{2}}_{(1)} &= \frac{f'_{k_1}(\psi_1)}{f_{k_1}(\psi_1)}e^{-V}e^{\hat{4}}, \quad \omega^{\hat{4}\hat{3}}_{(1)} = \frac{\cot\theta_1}{f_{k_1}(\psi_1)}e^{-V}e^{\hat{4}}, \quad \omega^{\hat{6}\hat{5}}_{(1)} = \frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)}e^{-W}e^{\hat{6}} \end{aligned} \qquad (4.506)$$



(a) $SO(4,1)$ gauge group (b) $SO(3,2)$ gauge group

Figure 4.36: The behavior of $\hat{g}_{00}$ for RG flows given in Figures 4.34 and 4.35 where $\hat{g}_{00} \to 0$ in the region $r < 0$ $(r > 0)$ for $SO(4,1)$ $(SO(3,2))$ gauge group.

where $\hat{i}_1 = \hat{2}, \hat{3}, \hat{4}$ and $\hat{i}_2 = \hat{5}, \hat{6}$ are flat indices on $\Sigma^3_{k_1}$ and $\Sigma^2_{k_2}$, respectively.

We now turn on the $SO(3) \times SO(2)$ gauge fields of the form

$$\begin{aligned} A^{12}_{(1)} &= -\frac{p_1}{k_1}\frac{f'_{k_1}(\psi_1)}{f_{k_1}(\psi_1)}e^{-V}e^{\hat{3}}, \qquad A^{13}_{(1)} = -\frac{p_1}{k_1}\frac{f'_{k_1}(\psi_1)}{f_{k_1}(\psi_1)}e^{-V}e^{\hat{4}}, \\ A^{23}_{(1)} &= -\frac{p_1}{k_1}\frac{\cot(\theta_1)}{f_{k_1}(\psi_1)}e^{-V}e^{\hat{4}}, \qquad A^{45}_{(1)} = -\frac{p_2}{k_2}\frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)}e^{-W}e^{\hat{6}} \end{aligned} \tag{4.507}$$

with the modified two-forms given by

$$\begin{aligned} \mathcal{F}^{12}_{\hat{2}\hat{3}} &= F^{12}_{\hat{2}\hat{3}} = e^{-2V}p_1, \qquad \mathcal{F}^{23}_{\hat{3}\hat{4}} = F^{23}_{\hat{3}\hat{4}} = e^{-2V}p_1, \\ \mathcal{F}^{31}_{\hat{4}\hat{2}} &= F^{31}_{\hat{4}\hat{2}} = e^{-2V}p_1, \qquad \mathcal{F}^{45}_{\hat{5}\hat{6}} = F^{45}_{\hat{5}\hat{6}} = e^{-2W}p_2\,. \end{aligned} \tag{4.508}$$

With these $SO(3) \times SO(2)$ gauge fields non-vanishing, we need to turn on the modified three-forms

$$\mathcal{H}_{\hat{0}\hat{1}\hat{i}_1 M} = -\frac{32}{g}\delta_{\hat{i}_1, M+1}e^{4\phi-2V-2W}p_1p_2 \tag{4.509}$$

in which ϕ is the $SO(3) \times SO(2)$ invariant scalar field corresponding to the coset representative (4.20).

We then impose the twist conditions

$$gp_1 = k_1 \qquad \text{and} \qquad \sigma g p_2 = k_2 \tag{4.510}$$

and the following projection conditions on the Killing spinors

$$\gamma^{\hat{2}\hat{3}}\epsilon^a = -(\Gamma_{12})^a{}_b\epsilon^b, \quad \gamma^{\hat{3}\hat{4}}\epsilon^a = -(\Gamma_{23})^a{}_b\epsilon^b, \quad \gamma^{\hat{5}\hat{6}}\epsilon^a = -(\Gamma_{45})^a{}_b\epsilon^b\,. \tag{4.511}$$

Using the embedding tensor (4.19), we can derive the following BPS equations

$$\begin{aligned}
U' &= \frac{g}{40}(3e^{-4\phi}+2\sigma e^{6\phi})+\frac{288e^{2\phi}p_1p_2}{5ge^{2(V+W)}}-\frac{2}{5}(3e^{-2V+4\phi}p_1+e^{-2W-6\phi}p_2), &(4.512)\\
V' &= \frac{g}{40}(3e^{-4\phi}+2\sigma e^{6\phi})-\frac{32e^{2\phi}p_1p_2}{5ge^{2(V+W)}}+\frac{2}{5}(7e^{-2V+4\phi}p_1-e^{-2W-6\phi}p_2), &(4.513)\\
W' &= \frac{g}{40}(3e^{-4\phi}+2\sigma e^{6\phi})-\frac{192e^{2\phi}p_1p_2}{5ge^{2(V+W)}}-\frac{2}{5}(3e^{-2V+4\phi}p_1-2e^{-2W-6\phi}p_2), &(4.514)\\
\phi' &= \frac{g}{20}(e^{-4\phi}-\sigma e^{6\phi})+\frac{32e^{2\phi}p_1p_2}{5ge^{2(V+W)}}-\frac{2}{5}(2e^{-2V+4\phi}p_1-e^{-2W-6\phi}p_2) &(4.515)
\end{aligned}$$

in which we have also used the γ^r projector in (4.388).

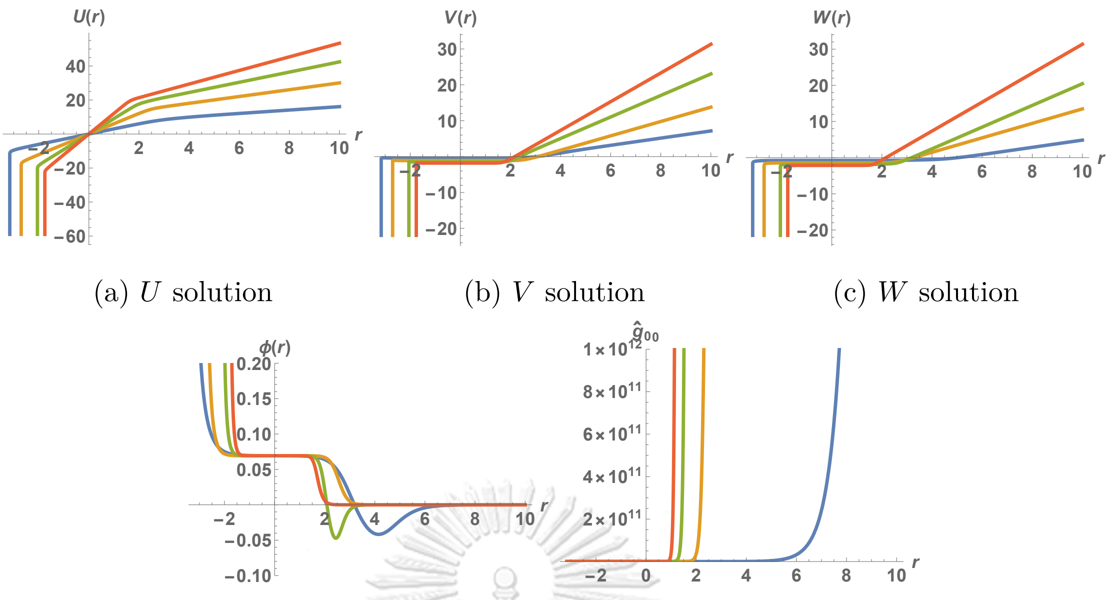
From these BPS equations, we find an AdS_2 fixed point only for $\sigma = 1$ and $k_1 = k_2 = -1$. The resulting $AdS_2 \times H^3 \times H^2$ fixed point is given by

$$\begin{aligned}
V &= \frac{1}{2}\ln\left[\frac{16\times 2^{4/5}}{g^2}\right], \qquad W = \frac{1}{2}\ln\left[\frac{16}{g^2 2^{1/5}}\right],\\
\phi &= \frac{1}{10}\ln 2, \qquad L_{\text{AdS}_2} = \frac{2\times 2^{2/5}}{g} &(4.516)
\end{aligned}$$

which is the solution found in [54]. The three projectors in (4.511) imply that this $AdS_2 \times H^3 \times H^2$ fixed point preserves four supercharges. The solution is dual to superconformal quantum mechanics. Examples of RG flows, from the supersymmetric AdS_7 critical point to this $AdS_2 \times H^3 \times H^2$ fixed point and curved DWs in the IR, are given in Figure 4.37. From the behaviors of the eleven-dimensional metric component $\hat{g}_{00}$, we see that the singularities are physically acceptable. Therefore, these singularities are expected to describe supersymmetric quantum mechanics obtained from twisted compactifications of the six-dimensional $N = (2, 0)$ SCFT on $H^3 \times H^2$.

4.3.2 Gaugings in $\overline{\mathbf{40}}$ Representation

We repeat the same analysis for gaugings from $\overline{\mathbf{40}}$ representation in this section. As in the case of gaugings in **15** representation, the modified three-forms need to be turned on when the compact manifold has dimension more than three in order to satisfy the corresponding Bianchi's identity. However, with $Y_{MN} = 0$, there are no massive three-form fields. In this case, the contribution to $\mathcal{H}_{(3)M}$ arises solely



(a) U solution (b) V solution (c) W solution

(d) ϕ solution (e) Profiles of $\hat{g}_{00}$

Figure 4.37: Numerical solutions for $SO(3) \times SO(2)$ twists in $SO(5)$ gauge group. The flows start from the AdS_7 critical point as $r \to 10$ to the $AdS_2 \times H^3 \times H^2$ fixed point at $r = 0$ and then to physical singularities in the form of $\mathbb{R} \times H^3 \times H^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 8, 16, 24, 32$.

from the two-form fields. For $s = \text{rank}\, Z$, there are respectively $5 - s$ massless and s massive two-form fields. The latter also appear in the modified two-forms. In particular, with the embedding tensor given in (2.90), we find

$$\mathcal{F}^{ij}_{(2)} = F^{ij}_{(2)} \qquad \text{and} \qquad \mathcal{F}^{5i}_{(2)} = \frac{g}{2} w^{ij} B_{(2)j} \tag{4.517}$$

in which $B_{(2)j}$ are massive two-form fields. However, we are unable to find a consistent set of BPS equations that are compatible with the field equations for non-vanishing massive two-form fields. Therefore, we will truncate out all the massive two-form fields in the following analysis. Finally, we point out here that the $CSO(p, q, 4 - p - q)$ gauge group is not large enough to accommodate $SO(5)$ or $SO(3) \times SO(2)$ subgroup so that it is not possible to have $AdS_2 \times \Sigma^5$ or $AdS_2 \times \Sigma^3 \times \Sigma^2$ solutions in this section.

4.3.2.1 Solutions with the Twists on Σ^2

We first look for $AdS_5\times\Sigma^2$ solutions with Σ^2 being a Riemann surface. The ansatz for the seven-dimensional metric is given in (4.382). We will consider solutions obtained from $SO(2)\times SO(2)$ and $SO(2)$ twists on Σ^2. The procedure is the same as in the gaugings in **15** representation, so we will not give all the details here to avoid repetition.

4.3.2.1.1 Solutions with $SO(2)\times SO(2)$ Twists

We now perform the twist by turning on the following $SO(2)\times SO(2)$ gauge fields

$$A^{12}_{(1)}=e^{-V}\frac{p_2}{4k}\frac{f'_k(\theta)}{f_k(\theta)}e^{\hat{6}}\qquad\text{and}\qquad A^{34}_{(1)}=e^{-V}\frac{p_1}{4k}\frac{f'_k(\theta)}{f_k(\theta)}e^{\hat{6}}\tag{4.518}$$

and imposing the projection conditions given in (4.387) and

$$\gamma^r\epsilon^a=-(\Gamma_5)^a{}_b\epsilon^b\tag{4.519}$$

together with the twist condition (4.386).

With the embedding tensor (4.128) and the coset representative (4.129), we find the following BPS equations

$$U'=\frac{g}{5}e^{-2(\phi_0+\phi)}(e^{4\phi}+\sigma)-\frac{1}{10}e^{-2(V-\phi_0)}(e^{-2\phi}p_1+e^{2\phi}p_2),\tag{4.520}$$

$$V'=\frac{g}{5}e^{-2(\phi_0+\phi)}(e^{4\phi}+\sigma)+\frac{2}{5}e^{-2(V-\phi_0)}(e^{-2\phi}p_1+e^{2\phi}p_2),\tag{4.521}$$

$$\phi_0'=\frac{g}{10}e^{-2(\phi_0+\phi)}(e^{4\phi}+\sigma)-\frac{1}{20}e^{-2(V-\phi_0)}(e^{-2\phi}p_1+e^{2\phi}p_2),\tag{4.522}$$

$$\phi'=-\frac{g}{2}e^{-2(\phi_0+\phi)}(e^{4\phi}-\sigma)+\frac{1}{4}e^{-2(V-\phi_0)}(e^{-2\phi}p_1-e^{2\phi}p_2).\tag{4.523}$$

From these BPS equations, there are no AdS_5 fixed point solutions satisfying the conditions $\phi'=\phi_0'=V'=0$ and $U'=\frac{1}{L_{\text{AdS}_5}}$. In the following analysis, we will consider RG flows interpolating between an asymptotically locally flat DW and curved DWs in $SO(4)$ gauge group. Note that similar solutions can also be found in $SO(2,2)$ gauge group.

When V is large, the contribution from the gauge fields is highly suppressed.

In this limit, we find

$$\phi \sim \frac{1}{r^5}, \qquad \phi_0 \sim -\frac{1}{10}\log\phi, \qquad U \sim V \sim 2\phi_0 \tag{4.524}$$

which implies $U \sim V \to \infty$ as $r \to \infty$. Examples of the flow solutions with this asymptotic behavior are given in Figures 4.38, 4.39, and 4.40 for $g = 16$ and $\Sigma^2 = S^2, \mathbb{R}^2, H^2$, respectively. We note here that the flows to the flat $Mkw_4 \times \mathbb{R}^2$-sliced DWs given in Figure 4.39 are possible by setting $p_2 = -p_1$ as required from the twist condition. It should be pointed out that the green curve in Figure 4.39 is simply the usual flat DW since $p_1 = p_2 = k = 0$. Due to the vanishing of the $SO(2) \times SO(2)$ singlet scalar ϕ, the solution preserves the full $SO(4)$ gauge symmetry in this case. This solution has already been given analytically in Section 4.1.2.3.

As shown in [65], the maximal gauged supergravity in seven dimensions with $CSO(p, q, 4-p-q)$ gauge group can be embedded in type IIB theory via a truncation on $H^{p,q} \circ T^{4-p-q}$. For the present analysis, we only need the ten-dimensional metric which is given by

$$\hat{g}_{\mu\nu} = \mathcal{K}^{\frac{3}{4}}\Delta^{\frac{1}{4}} g_{\mu\nu} \tag{4.525}$$

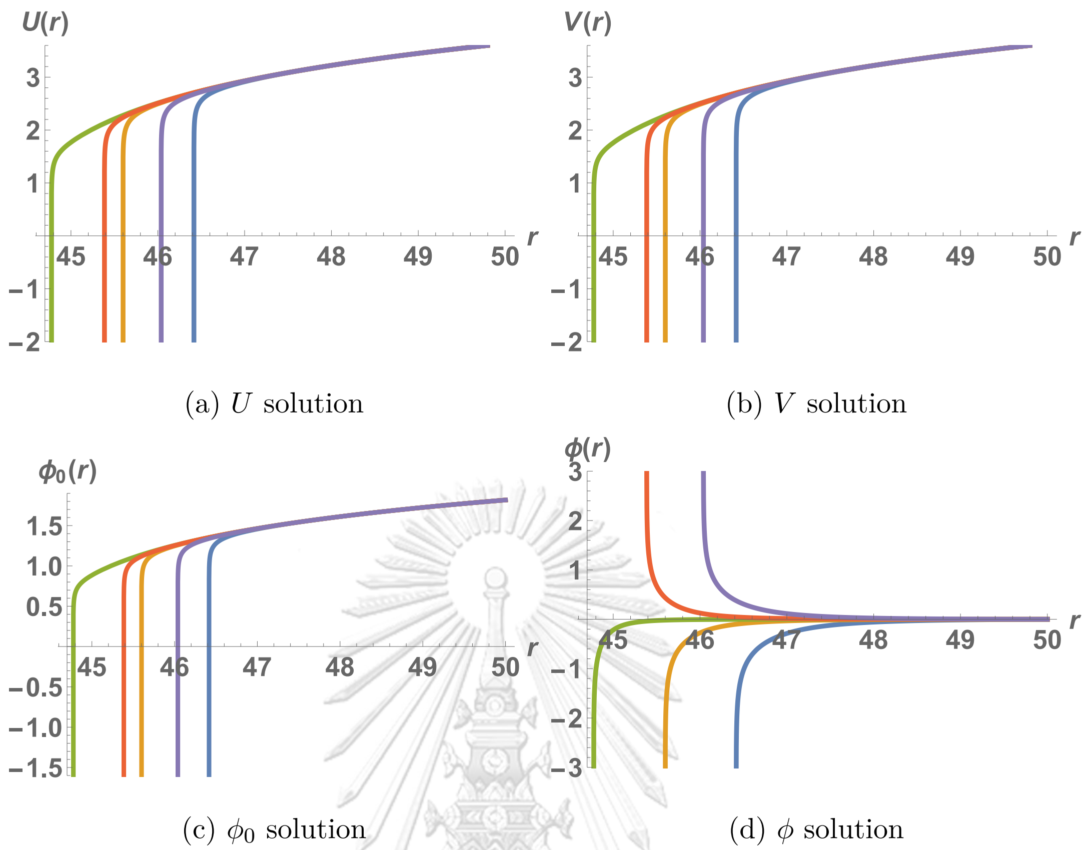
in which

$$\Delta = \mu_i\mu_j\eta^{ik}\eta^{jl}\widetilde{\mathcal{M}}_{kl}, \qquad i, j = 1, \ldots p+q. \tag{4.526}$$

η^{ij} is the $SO(p, q)$ invariant tensor, and μ_i are coordinates on $H^{p,q}$ satisfying $\mu_i\mu_j\eta^{ij} = 1$. In term of the parametrization (2.93), $\mathcal{K}$ is identified as follows

$$\mathcal{K} = e^{2\phi_0}\,. \tag{4.527}$$

For the present case with $SO(4)$ gauge group, we simply have $\eta^{ij} = \delta^{ij}$ for $i, j = 1, 2, 3, 4$. For the flow solutions in Figures 4.38, 4.39, and 4.40, the behaviors of the of the ten-dimensional metric component $\hat{g}_{00}$ are shown in Figure 4.41. We find that the IR singularities are physically acceptable since $\hat{g}_{00} \to 0$ near the singularities.



(a) U solution (b) V solution

(c) ϕ_0 solution (d) ϕ solution

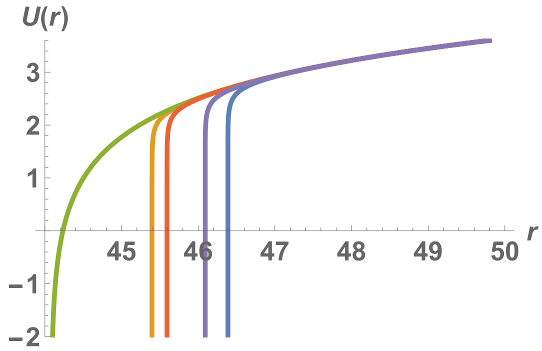
Figure 4.38: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_4 \times S^2$-sliced DWs in the region $r < 50$. The blue, orange, green, red, and purple curves refer to $p_2 = -0.5, -0.03, 0.03, 0.06, 0.25$.

4.3.2.1.2 Solutions with $SO(2)$ Twists

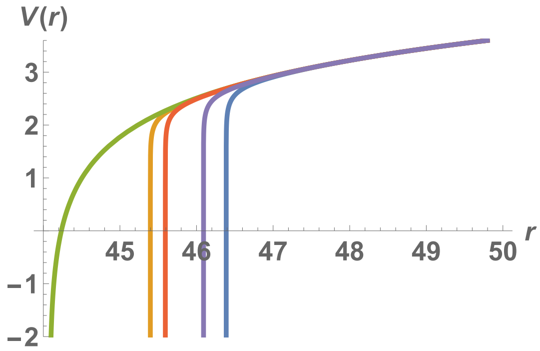
We then consider another twist on Σ^2 by turning on only an $SO(2)$ gauge field. From the $SO(2) \times SO(2)$ gauge fields given in (4.518), this can be achieved by setting $p_2 = 0$ and $p_1 = p$. In this case, the $SL(4)/SO(4)$ coset representative and the embedding tensor are the same as in Section 4.1.2.4.

Imposing the twist condition (4.409) and the projector (4.519) together with

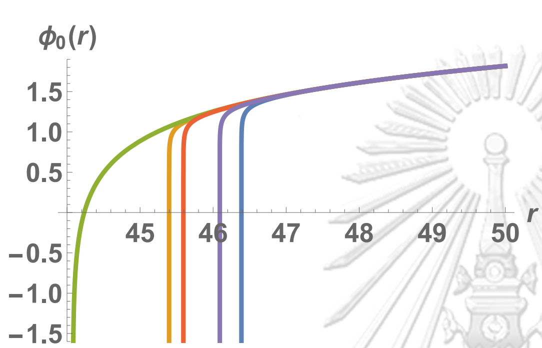
$$\gamma^{\hat{5}\hat{6}}\epsilon^a = -(\Gamma_{12})^a{}_b\epsilon^b, \tag{4.528}$$



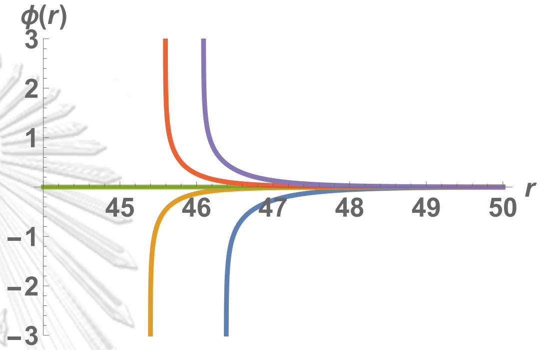
(a) U solution



(b) V solution



(c) ϕ_0 solution



(d) ϕ solution

Figure 4.39: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_4 \times \mathbb{R}^2$-sliced DWs in the region $r < 50$. The blue, orange, green, red, and purple curves refer to $p_2 = -0.5, -0.03, 0, 0.06, 0.25$.

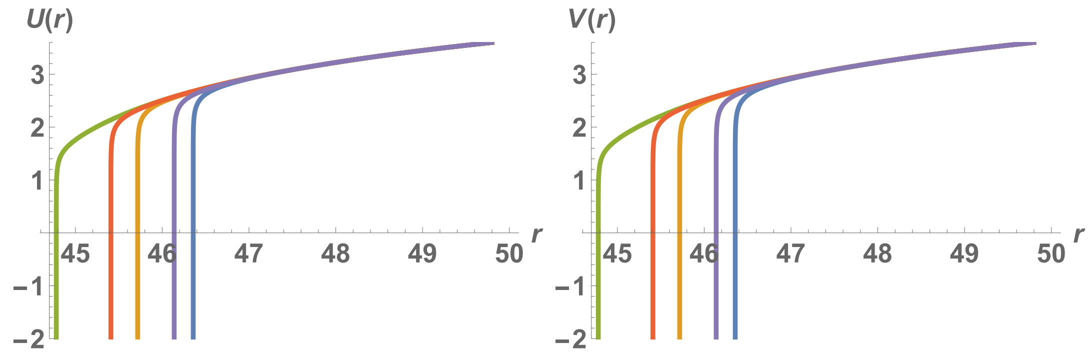
we find the BPS equations

$$\begin{aligned} U' &= \frac{g}{10}e^{-2(\phi_0+\phi_1)}\left[2e^{4\phi_1} - (\rho-\sigma)\sinh 2\phi_3 + (\rho+\sigma)\cosh 2\phi_3 \cosh 2\phi_2\right] \\ &\quad -\frac{1}{10}pe^{-2(V-\phi_0+\phi_1)}, \end{aligned} \tag{4.529}$$

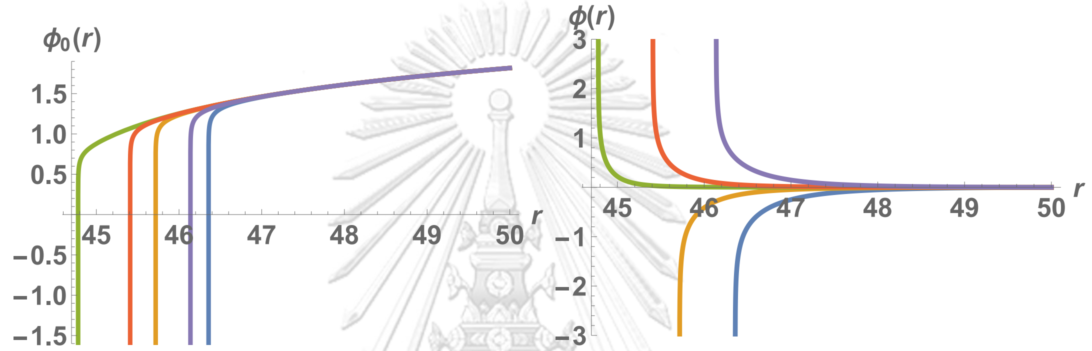
$$\begin{aligned} V' &= \frac{g}{10}e^{-2(\phi_0+\phi_1)}\left[2e^{4\phi_1} - (\rho-\sigma)\sinh 2\phi_3 + (\rho+\sigma)\cosh 2\phi_3 \cosh 2\phi_2\right] \\ &\quad +\frac{2}{5}pe^{-2(V-\phi_0+\phi_1)}, \end{aligned} \tag{4.530}$$

$$\begin{aligned} \phi_0' &= \frac{g}{20}e^{-2(\phi_0+\phi_1)}\left[2e^{4\phi_1} - (\rho-\sigma)\sinh 2\phi_3 + (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3\right] \\ &\quad -\frac{1}{20}pe^{-2(V-\phi_0+\phi_1)}, \end{aligned} \tag{4.531}$$

$$\begin{aligned} \phi_1' &= -\frac{g}{4}e^{-2(\phi_0+\phi_1)}\left[2e^{4\phi_1} + (\rho-\sigma)\sinh 2\phi_3 - (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3\right] \\ &\quad +\frac{1}{4}pe^{-2(V-\phi_0+\phi_1)}, \end{aligned} \tag{4.532}$$

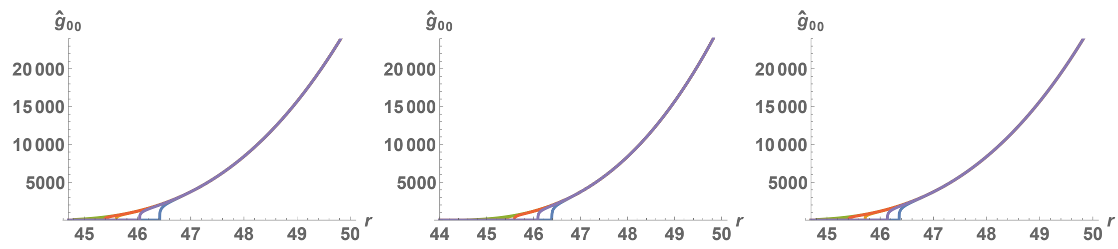


(a) U solution (b) V solution



(c) ϕ_0 solution (d) ϕ solution

Figure 4.40: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_4 \times H^2$-sliced DWs in the region $r < 50$. The blue, orange, green, red, and purple curves refer to $p_2 = -0.5, -0.12, -0.03, 0, 0.25$.



(a) $\Sigma^2 = S^2$ (b) $\Sigma^2 = \mathbb{R}^2$ (c) $\Sigma^2 = H^2$

Figure 4.41: The behavior of $\hat{g}_{00}$ for RG flows given in Figures 4.38, 4.39, and 4.40, respectively, where $\hat{g}_{00} \to 0$ in the region $r < 50$ for every case.

$$\phi_2' = -\frac{g}{2}e^{-2(\phi_0+\phi_1)}(\rho+\sigma)\sinh 2\phi_2\,\mathrm{sech}\,2\phi_3, \tag{4.533}$$

$$\phi_3' = \frac{g}{2}e^{-2(\phi_0+\phi_1)}\left[(\rho-\sigma)\cosh 2\phi_3 - (\rho+\sigma)\cosh 2\phi_2\sinh 2\phi_3\right]. \tag{4.534}$$

As in the previous case, there do not exist any $AdS_5 \times \Sigma^2$ fixed points from these BPS equations. Moreover, the numerical solutions interpolating between locally asymptotically flat DWs and $Mkw_4 \times \Sigma^2$-sliced curved DWs, in this case, can also be obtained in the same way.

4.3.2.2 Solutions with the Twists on Σ^3

In this section, we repeat the same analysis for $AdS_4 \times \Sigma^3$ solutions with the ansatz for the seven-dimensional metric given in (4.404). As in the cases with gaugings from **15** representation, we also consider two different twists by turning on $SO(3)$ and $SO(3)_+$ gauge fields.

4.3.2.2.1 Solutions with $SO(3)$ Twists

With the embedding tensor (4.118) and the $SL(4)/SO(4)$ coset (4.119), we turn on the following $SO(3)$ gauge fields in order to perform the twist,

$$A^{34}_{(1)} = e^{-V}\frac{p}{4k}\frac{f_k'(\psi)}{f_k(\psi)}e^{\hat{5}},\quad A^{42}_{(1)} = e^{-V}\frac{p}{4k}\frac{f_k'(\psi)}{f_k(\psi)}e^{\hat{6}},\quad A^{14}_{(1)} = e^{-V}\frac{p}{4k}\frac{\cot(\theta)}{f_k(\psi)}e^{\hat{6}} \tag{4.535}$$

and impose the projectors (4.410) on the Killing spinors together with the twist condition (4.409). With all these and the γ^r projector (4.519), the resulting BPS equations read

$$U' = \frac{g}{10}e^{-2(\phi_0+3\phi)}(3e^{8\phi}+\rho) - \frac{3}{10}e^{-2(V-\phi_0+\phi)}p, \tag{4.536}$$

$$V' = \frac{g}{10}e^{-2(\phi_0+3\phi)}(3e^{8\phi}+\rho) + \frac{7}{10}e^{-2(V-\phi_0+\phi)}p, \tag{4.537}$$

$$\phi_0' = \frac{g}{20}e^{-2(\phi_0+3\phi)}(3e^{8\phi}+\rho) - \frac{3}{20}e^{-2(V-\phi_0+\phi)}p, \tag{4.538}$$

$$\phi' = -\frac{g}{4}e^{-2(\phi_0+3\phi)}(e^{8\phi}-\rho) + \frac{1}{4}e^{-2(V-\phi_0+\phi)}p. \tag{4.539}$$

As in the previous case, there do not exist any AdS_4 fixed points from these BPS equations. We then look for flow solutions interpolating between asymptotically locally flat DWs and $Mkw_3 \times \Sigma^3$-sliced curved DWs.

For $CSO(3,0,1)$ gauge group with $\rho = 0$, these BPS equations can be solved analytically. First of all, the BPS equations (4.536) and (4.538) give

$$U = 2\phi_0\,. \tag{4.540}$$

We have set an additive integration constant for U to zero. This corresponds to rescaling the coordinates on Mkw_3. When $\rho = 0$, we find that $\phi_0' + \frac{3}{5}\phi' = 0$ which gives

$$\phi_0 = -\frac{3}{5}\phi + C_0 \tag{4.541}$$

with an integration constant C_0.

Taking a linear combination $V' + \frac{6}{5}\phi'$ and changing to a new radial coordinate $\tilde{r}$ defined by $\frac{d\tilde{r}}{dr} = e^{-\frac{4}{5}\phi}$, we find

$$V = \frac{1}{2}\ln(2p\tilde{r} + C_1) - \frac{6}{5}\phi\,. \tag{4.542}$$

The integration constant C_1 can also be neglected by shifting the coordinate $\tilde{r}$. With all these results, the equation for ϕ' gives

$$\phi = -\frac{1}{4}\ln\left[\frac{2}{3}g\tilde{r} - \frac{C_2}{p\sqrt{p\tilde{r}}}\right] \tag{4.543}$$

in which we have set $C_1 = 0$ for simplicity, and C_2 is another integration constant.

As $\tilde{r} \to 0$, we find that the above solution becomes a locally flat DW with $U \sim V \to \infty$. The asymptotic behavior is given by

$$\phi \sim \frac{1}{8}\ln\tilde{r}, \qquad \phi_0 \sim -\frac{3}{40}\ln\tilde{r}, \qquad U \sim V \sim -\frac{3}{20}\ln\tilde{r}\,. \tag{4.544}$$

For $\tilde{r} \to \infty$, we find

$$\phi \sim -\frac{1}{4}\ln\tilde{r}, \qquad \phi_0 \sim \frac{3}{20}\ln\tilde{r}, \qquad V \sim \frac{4}{5}\ln\tilde{r}, \qquad U \sim \frac{3}{10}\ln\tilde{r}\,. \tag{4.545}$$

Computing the (00)-component of type IIB metric, we obtain

$$\hat{g}_{00} \sim e^{2U + \frac{3}{2}\phi_0 + \frac{1}{2}\phi} \sim \tilde{r}^{\frac{7}{10}} \to \infty, \tag{4.546}$$

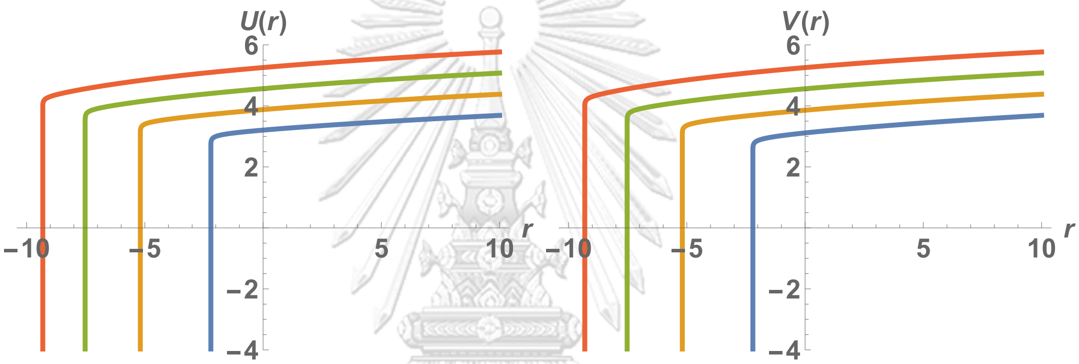
as $\tilde{r} \to \infty$, which indicates that the singularity is unphysical.

For $\rho \neq 0$, the solutions can be obtained numerically. When V is large as

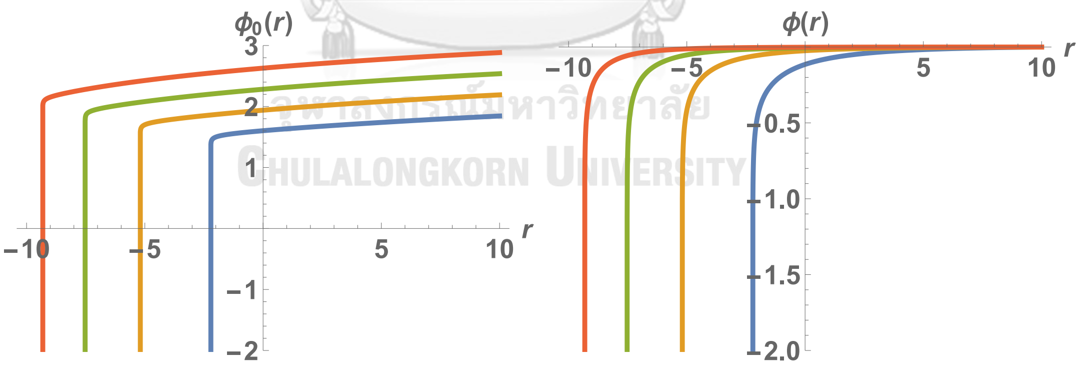
$r \to \infty$, we find

$$\phi \sim \frac{1}{r^5}, \qquad \phi_0 \sim -\frac{1}{10}\log\phi, \qquad U \sim V \sim 2\phi_0\,. \tag{4.547}$$

Examples of the flow solutions with this asymptotic behavior for $SO(4)$ gauge group are given in Figures 4.42 and 4.43 for $\Sigma^2 = S^2, H^2$, respectively. The behavior of the ten-dimensional metric $\hat{g}_{00}$ for these flow solutions is shown in Figure 4.44, from which only the IR singularities of $Mkw_3 \times H^3$-sliced DWs are physical. For $Mkw_3 \times \mathbb{R}^3$-sliced DWs with $k = 0$, the twist condition gives $p = 0$, resulting in the usual flat DWs.

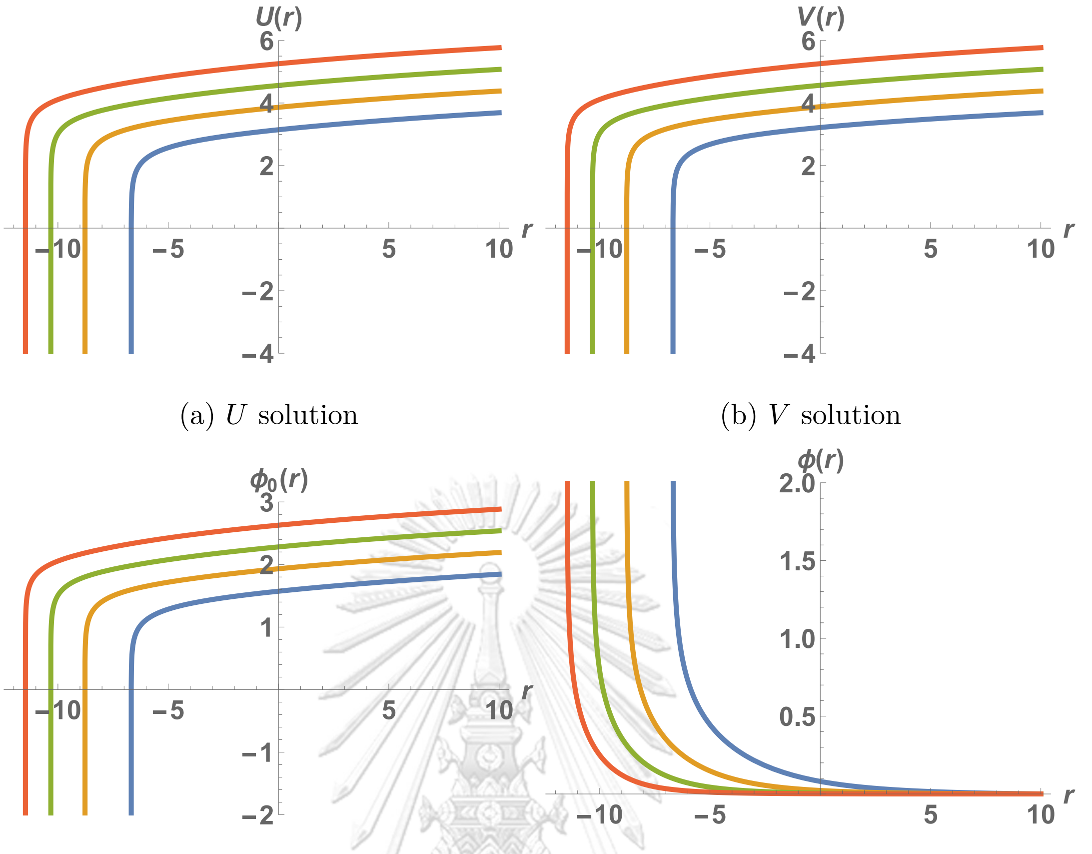


(a) U solution (b) V solution



(c) ϕ_0 solution (d) ϕ solution

Figure 4.42: Numerical solutions for $SO(3)$ twists in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 10$ to singularities in the form of $Mkw_3 \times S^3$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 4, 8, 16, 32$.



(a) U solution (b) V solution

(c) ϕ_0 solution (d) ϕ solution

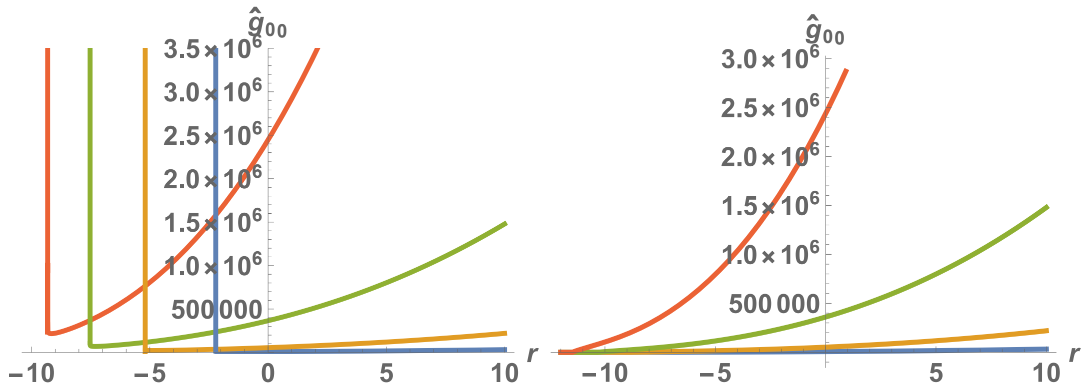
Figure 4.43: Numerical solutions for $SO(3)$ twists in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 10$ to singularities in the form of $Mkw_3 \times H^3$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 4, 8, 16, 32$.

4.3.2.2.2 Solutions with $SO(3)_+$ Twists

We now move on to another twist by turning on the self-dual $SO(3)_+$ gauge fields

$$\begin{aligned} A^{12}_{(1)} &= A^{34}_{(1)} = e^{-V}\frac{p}{8k}\frac{f_k'(\psi)}{f_k(\psi)}e^{\hat{5}}, \\ A^{13}_{(1)} &= A^{42}_{(1)} = e^{-V}\frac{p}{8k}\frac{f_k'(\psi)}{f_k(\psi)}e^{\hat{6}}, \\ A^{23}_{(1)} &= A^{14}_{(1)} = e^{-V}\frac{p}{8k}\frac{\cot(\theta)}{f_k(\psi)}e^{\hat{6}}. \end{aligned} \tag{4.548}$$

Only the dilaton scalar field ϕ_0 is singlet under $SO(3)_+$, so we have $\widetilde{\mathcal{M}}_{ij} = \delta_{ij}$. Moreover, we consider only $SO(4)$ gauge group in this case since this is the only



(a) $\Sigma^3 = S^3$ (b) $\Sigma^3 = H^3$

Figure 4.44: The behavior of $\hat{g}_{00}$ for RG flows given in Figures 4.42 and 4.43 where $\hat{g}_{00} \to 0$ in the region $r < 0$ only for the case with $\Sigma^3 = H^3$.

gauge group containing the $SO(3)_+$ subgroup.

With the projectors (4.410), (4.481), and (4.519) together with the twist condition (4.409), the resulting BPS equations are given by

$$U' = \frac{2g}{5}e^{-2\phi_0} - \frac{3}{10}e^{-2(V-\phi_0)}p, \tag{4.549}$$

$$V' = \frac{2g}{5}e^{-2\phi_0} + \frac{7}{10}e^{-2(V-\phi_0)}p, \tag{4.550}$$

$$\phi_0' = \frac{g}{5}e^{-2\phi_0} - \frac{3}{20}e^{-2(V-\phi_0)}p\,. \tag{4.551}$$

As in the previous case with $SO(3)$ twist, we do not find any AdS_4 fixed points.

However, these BPS equations can be analytically solved. Starting from (4.549) and (4.551), we again find that $U = 2\phi_0$. Defining a new radial coordinate $\tilde{r}$ by $\frac{d\tilde{r}}{dr} = e^{-V}$ and taking a linear combination $V' - 2\phi_0'$, we obtain

$$V = \ln(p\tilde{r} + C) + 2\phi_0\,. \tag{4.552}$$

Using ϕ_0 from (4.552) in equation (4.550) and changing to the new radial coordinate $\tilde{r}$, we find

$$V = \frac{g}{5p}(p\tilde{r} + C)^2 + \frac{7}{10}\ln(p\tilde{r} + C). \tag{4.553}$$

With C set to zero by shifting $\tilde{r}$, as $\tilde{r} \to \infty$, we find

$$U \sim V \sim \frac{1}{5}gp\tilde{r}^2 \qquad \text{and} \qquad \phi_0 \sim \frac{1}{10}gp\tilde{r}^2 \tag{4.554}$$

which is identified with the flat DW solution given in Section 4.1.2.1. On the other hand, as $\tilde{r} \to 0$, the solution becomes singular

$$U \sim -\frac{3}{5}\ln(p\tilde{r}), \qquad V \sim \frac{7}{10}\ln(p\tilde{r}), \qquad \phi_0 \sim -\frac{3}{10}\ln(p\tilde{r}). \tag{4.555}$$

This singularity is unphysical since the ten-dimensional metric gives

$$\hat{g}_{00} \sim e^{2U+\frac{3}{2}\phi_0} \to \infty\,. \tag{4.556}$$

Note here that this solution is the same as that given in Section 4.3.1.2.2 for $CSO(4,0,1)$ gauge group. The two gauged supergravities can be obtained from consistent truncations on S^3 of type IIB and type IIA theories, respectively. As pointed out in [65], there is a duality between these solutions.

4.3.2.3 Solutions with the Twists on Σ^4

We finally look for solutions obtained from the twists on a four-manifold Σ^4. In this section, we consider two types of Σ^4, a product of two Riemann surfaces $\Sigma^2 \times \Sigma^2$ and a Kahler four-cycle K^4. For the case with the internal space being a Riemannian four-manifold M^4, we do not find any consistent set of BPS equations that are compatible with the field equations, especially the deformed Bianchi's identity for the modified three-forms.

4.3.2.3.1 Solutions with $SO(2)\times SO(2)$ Twists on $\Sigma^2 \times \Sigma^2$

For the twists on $\Sigma^2 \times \Sigma^2$, we consider solutions with $SO(2)\times SO(2)$ unbroken symmetry in $SO(4)$ and $SO(2,2)$ gauge groups corresponding to the embedding tensor (4.128). The ansatz for the metric is given in (4.441). To cancel the spin connection on $\Sigma^2_{k_1} \times \Sigma^2_{k_2}$, we turn on the following $SO(2)\times SO(2)$ gauge fields

$$\begin{aligned} A^{12}_{(1)} &= \frac{p_{11}}{4k_1}\frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)}e^{-V}e^{\hat{4}} + \frac{p_{12}}{4k_2}\frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)}e^{-W}e^{\hat{6}}, \\ A^{34}_{(1)} &= \frac{p_{21}}{4k_1}\frac{f'_{k_1}(\theta_1)}{f_{k_1}(\theta_1)}e^{-V}e^{\hat{4}} + \frac{p_{22}}{4k_2}\frac{f'_{k_2}(\theta_2)}{f_{k_2}(\theta_2)}e^{-W}e^{\hat{6}} \end{aligned} \tag{4.557}$$

with the corresponding modified two-forms

$$
\begin{aligned}
\mathcal{F}^{12}_{(2)} = F^{12}_{(2)} &= -e^{-2V}\frac{p_{11}}{4}e^{\hat{3}}\wedge e^{\hat{4}} - e^{-2W}\frac{p_{12}}{4}e^{\hat{5}}\wedge e^{\hat{6}},\\
\mathcal{F}^{34}_{(2)} = F^{34}_{(2)} &= -e^{-2V}\frac{p_{21}}{4}e^{\hat{3}}\wedge e^{\hat{4}} - e^{-2W}\frac{p_{22}}{4}e^{\hat{5}}\wedge e^{\hat{6}}\,. \qquad (4.558)
\end{aligned}
$$

Following a similar analysis for gaugings in **15** representation, we also turn on the modified three-forms using the ansatz

$$\mathcal{H}_{\hat{m}\hat{n}\hat{2}5} = \beta e^{-2(V+W+4\phi_0)}\varepsilon_{\hat{m}\hat{n}} \qquad (4.559)$$

in which β is a constant. We now impose the twist conditions

$$g(\sigma p_{11}+p_{21}) = k_1 \qquad \text{and} \qquad g(\sigma p_{12}+p_{22}) = k_2 \qquad (4.560)$$

together with the projection conditions (4.450) and (4.519).

With all these and the coset representative given in (4.129), the resulting BPS equations are given by

$$
\begin{aligned}
U' &= -\frac{e^{2\phi_0}}{10}\left[e^{-2(V+\phi)}(e^{4\phi}p_{11}+p_{21}) + e^{-2(W+\phi)}(e^{4\phi}p_{12}+p_{22})\right]\\
&\quad +\frac{g}{5}e^{-2(\phi_0+\phi)}(e^{4\phi}+\sigma) + \frac{3}{5}e^{-2(V+W+2\phi_0)}\beta, \qquad (4.561)\\
V' &= -\frac{e^{2\phi_0}}{10}\left[4e^{-2(V+\phi)}(e^{4\phi}p_{11}+p_{21}) - e^{-2(W+\phi)}(e^{4\phi}p_{12}+p_{22})\right]\\
&\quad +\frac{g}{5}e^{-2(\phi_0+\phi)}(e^{4\phi}+\sigma) - \frac{2}{5}e^{-2(V+W+2\phi_0)}\beta, \qquad (4.562)\\
W' &= -\frac{e^{2\phi_0}}{10}\left[e^{-2(V+\phi)}(e^{4\phi}p_{11}+p_{21}) - 4e^{-2(W+\phi)}(e^{4\phi}p_{12}+p_{22})\right]\\
&\quad +\frac{g}{5}e^{-2(\phi_0+\phi)}(e^{4\phi}+\sigma) - \frac{2}{5}e^{-2(V+W+2\phi_0)}\beta, \qquad (4.563)\\
\phi_0' &= -\frac{e^{2\phi_0}}{20}\left[e^{-2(V+\phi)}(e^{4\phi}p_{11}+p_{21}) + e^{-2(W+\phi)}(e^{4\phi}p_{12}+p_{22})\right]\\
&\quad +\frac{g}{10}e^{-2(\phi_0+\phi)}(e^{4\phi}+\sigma) - \frac{1}{5}e^{-2(V+W+2\phi_0)}\beta, \qquad (4.564)\\
\phi' &= -\frac{e^{2\phi_0}}{4}\left[e^{-2(V+\phi)}(e^{4\phi}p_{11}-p_{21}) + e^{-2(W+\phi)}(e^{4\phi}p_{12}-p_{22})\right]\\
&\quad -\frac{g}{2}e^{-2(\phi_0+\phi)}(e^{4\phi}-\sigma). \qquad (4.565)
\end{aligned}
$$

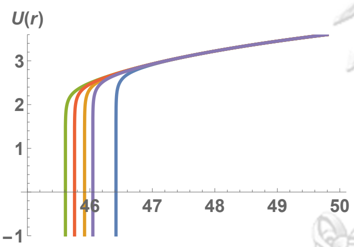
Unlike the similar case in Section 4.3.1.4, it turns out that compatibility between these BPS equations and the field equations requires

$$p_{12}p_{21} + p_{11}p_{22} = 0 \qquad (4.566)$$

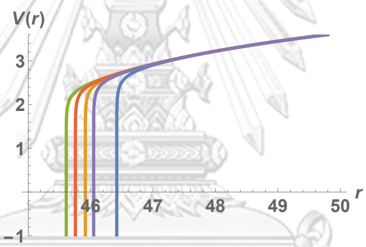
for any values of β. This implies that the constant β is a free parameter in this case. However, we do not find any AdS_3 fixed points from the BPS equations.

For $SO(4)$ gauge group, examples of flow solutions between asymptotically locally flat and curved DWs for various forms of $\Sigma^2 \times \Sigma^2$ are shown in Figures 4.45 to 4.50. In these numerical solutions, we have set $g = 16$ and $\beta = 2$. Note also that the green curve in Figure 4.48 is the flat DW solution given in Section 4.1.2.3. All of the IR singularities are physical, as can be seen from the behavior of the ten-dimensional metric given in Figure 4.51.

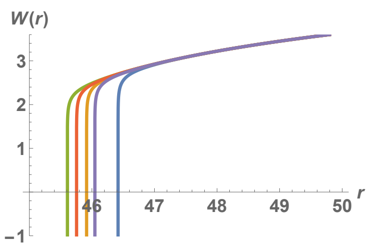
We have also examined $SO(2)$ twists on $\Sigma^2 \times \Sigma^2$ by setting $p_{11} = p_{12} = 0$ and obtain more complicated BPS equations. However, we will not give further detail on this analysis since there do not exist any AdS_3 fixed points.



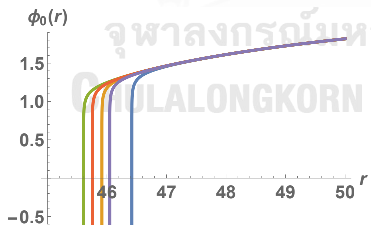
(a) U solution



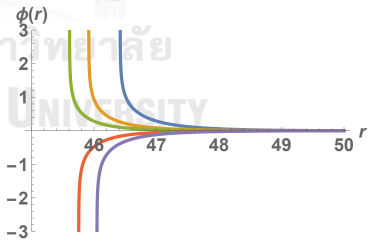
(b) V solution



(c) W solution

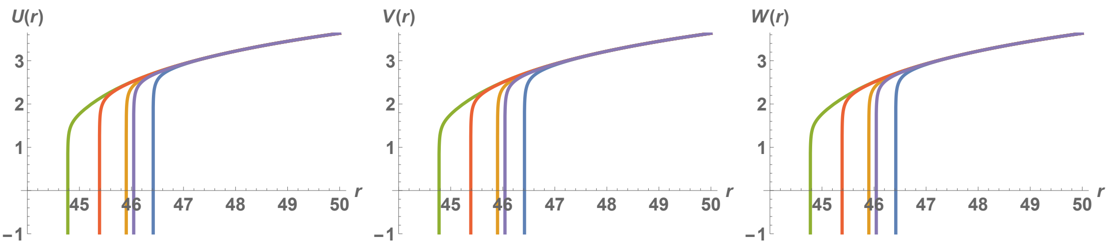


(d) ϕ_0 solution

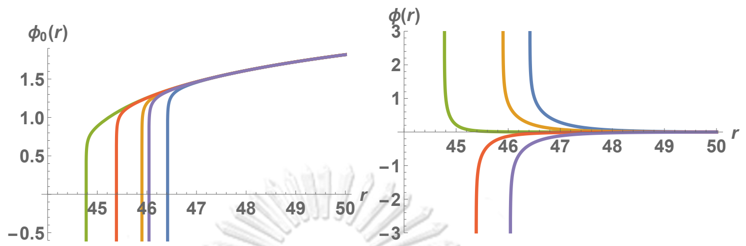


(e) ϕ solution

Figure 4.45: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ and $\beta = 2$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_2 \times S^2 \times S^2$-sliced DWs when $r < 50$. The blue, orange, green, red, and purple curves refer to $p_{21} = -0.5, -0.12, 0, 0.12, 0.25$.



(a) U solution (b) V solution (c) W solution



(d) ϕ_0 solution (e) ϕ solution

Figure 4.46: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ and $\beta = 2$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_2 \times S^2 \times \mathbb{R}^2$-sliced DWs when $r < 50$. The blue, orange, green, red, and purple curves refer to $p_{21} = -0.5, -0.12, 0.03, 0.06, 0.25$.

4.3.2.3.2 Solutions with $SO(3)$ Twists on K^4

For Σ^4 being a Kahler four-cycle K^4, we perform an $SO(3)$ twist to cancel the $SU(2)$ part of the spin connection, given in (4.467), by turning on the $SO(3)$ gauge fields

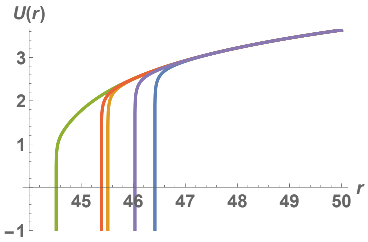
$$A^{i4}_{(1)} = \frac{p}{4k}(f_k'(\psi) - 1)\delta^{ij}\tau_j, \qquad i,j = 1,2,3 \tag{4.567}$$

with the modified two-forms given by

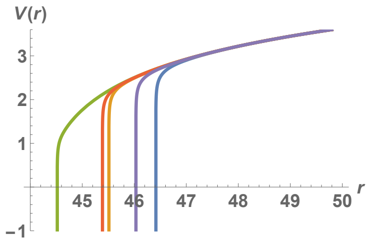
$$\begin{aligned}
\mathcal{F}^{14}_{(2)} = F^{14}_{(2)} &= -\frac{p}{4}e^{-2V}(e^{\hat{4}} \wedge e^{\hat{5}} - e^{\hat{3}} \wedge e^{\hat{6}}), \\
\mathcal{F}^{24}_{(2)} = F^{24}_{(2)} &= -\frac{p}{4}e^{-2V}(e^{\hat{5}} \wedge e^{\hat{3}} - e^{\hat{4}} \wedge e^{\hat{6}}), \\
\mathcal{F}^{34}_{(2)} = F^{34}_{(2)} &= -\frac{p}{4}e^{-2V}(e^{\hat{3}} \wedge e^{\hat{4}} - e^{\hat{5}} \wedge e^{\hat{6}})\,.
\end{aligned} \tag{4.568}$$

These modified two-forms do not lead to any problematic terms in the deformed Bianchi's identity for the modified three-forms. However, we can have a non-vanishing modified three-form by using the following ansatz

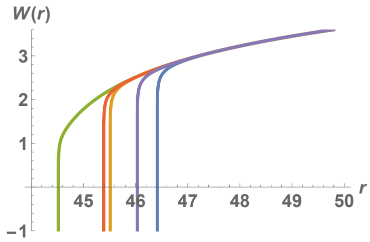
$$\mathcal{H}_{\hat{m}\hat{n}\hat{2}5} = \beta e^{-4(V+\phi_0)}\varepsilon_{\hat{m}\hat{n}}. \tag{4.569}$$



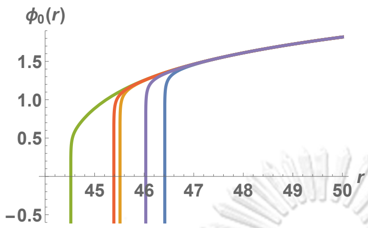
(a) U solution



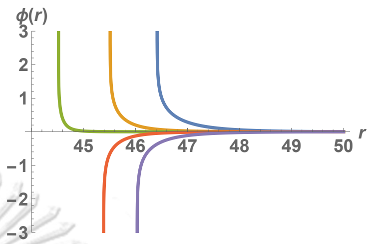
(b) V solution



(c) W solution



(d) ϕ_0 solution



(e) ϕ solution

Figure 4.47: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ and $\beta = 2$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_2 \times S^2 \times H^2$-sliced DWs when $r < 50$. The blue, orange, green, red, and purple curves refer to $p_{21} = -0.5, -0.03, 0, 0.08, 0.25$.

which is a manifestly closed three-form for a constant β.

With the $SL(4)/SO(4)$ coset representative and the embedding tensor given in (4.119) and (4.118) together with the projections (4.472) and (4.519), we find the following BPS equations

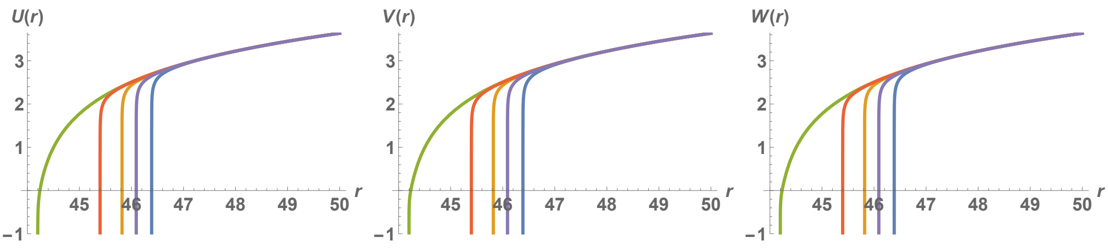
$$U' = \frac{g}{10}e^{-2(\phi_0+3\phi)}(3e^{8\phi}+\rho) - \frac{3}{5}e^{-2(V-\phi_0+\phi)}p - \frac{3}{5}e^{-4(V+\phi_0)}\beta, \qquad (4.570)$$

$$V' = \frac{g}{10}e^{-2(\phi_0+3\phi)}(3e^{8\phi}+\rho) + \frac{9}{10}e^{-2(V-\phi_0+\phi)}p + \frac{2}{5}e^{-4(V+\phi_0)}\beta, \qquad (4.571)$$

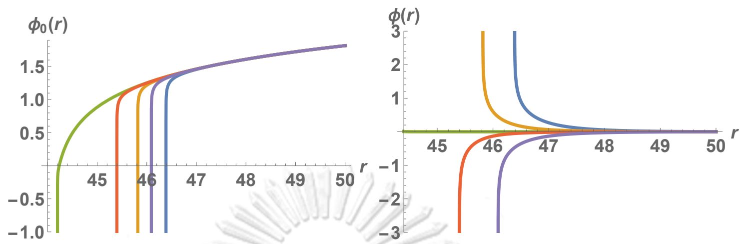
$$\phi_0' = \frac{g}{20}e^{-2(\phi_0+3\phi)}(3e^{8\phi}+\rho) - \frac{3}{10}e^{-2(V-\phi_0+\phi)}p + \frac{1}{5}e^{-4(V+\phi_0)}\beta, \qquad (4.572)$$

$$\phi' = -\frac{g}{4}e^{-2(\phi_0+3\phi)}(e^{8\phi}-\rho) + \frac{1}{2}e^{-2(V-\phi_0+\phi)}p \qquad (4.573)$$

in which we have used the twist condition (4.409). We again do not find any AdS_3 fixed points from these BPS equations. Examples of supersymmetric flows with $\beta = -2$ are given in Figures 4.52 and 4.53 for $k = 1$ and $k = -1$, respectively. From the behavior of the ten-dimensional metric component $\hat{g}_{00}$ given in Figure 4.54, we find that the IR singularities are physical for $k = -1$. For $k = 0$, we have



(a) U solution (b) V solution (c) W solution



(d) ϕ_0 solution (e) ϕ solution

Figure 4.48: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ and $\beta = 2$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_2 \times \mathbb{R}^2 \times \mathbb{R}^2$-sliced DWs when $r < 50$. The blue, orange, green, red, and purple curves refer to $p_{21} = -0.5, -0.12, 0, 0.03, 0.25$.

$p = 0$ by the twist condition resulting in the standard flat DW solutions.

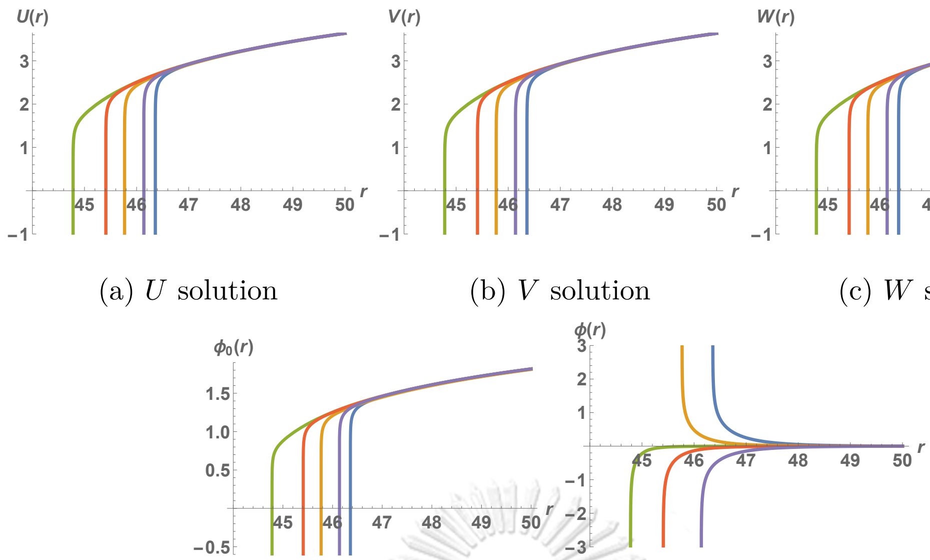
4.3.2.3.3 Solutions with $SO(2)$ Twists on K^4

As the finale case, we briefly consider the $SO(2)$ twist canceling the $U(1)$ part of the spin connection in (4.488). This procedure can be achieved by turning on an $SO(2)$ gauge field of the form

$$A^{34}_{(1)} = -p\frac{3k\psi^2}{4\sqrt{f_k(\psi)}}\tau_3\,. \tag{4.574}$$

The embedding tensor for gauge groups containing an $SO(2)$ subgroup is given in (4.139). Moreover, we can also turn on the modified three-form (4.569) in this case. With the coset representative (4.137), the twist condition (4.409), and the projections (4.519) together with

$$\gamma^{\hat{3}\hat{4}}\epsilon^a = -\gamma^{\hat{5}\hat{6}}\epsilon^a = -(\Gamma_{12})^a{}_b\epsilon^b, \tag{4.575}$$



(a) U solution (b) V solution (c) W solution

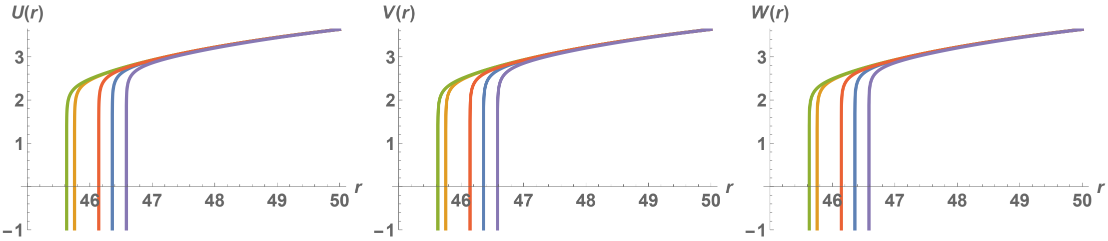
(d) ϕ_0 solution (e) ϕ solution

Figure 4.49: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ and $\beta = 2$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_2 \times H^2 \times \mathbb{R}^2$-sliced DWs when $r < 50$. The blue, orange, green, red, and purple curves refer to $-0.5, -0.12, -0.03, 0, 0.25$.

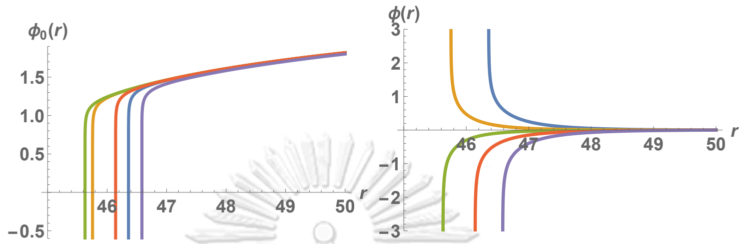
the corresponding BPS equations are given by

$$
\begin{aligned}
U' &= \frac{g}{10}e^{-2(\phi_0+\phi_1)}\left[2e^{4\phi_1} - (\rho-\sigma)\sinh 2\phi_3 + (\rho+\sigma)\cosh 2\phi_3 \cosh 2\phi_2\right] \\
&\quad -\frac{3}{5}e^{-2(V-\phi_0+\phi_1)}p - \frac{3}{5}e^{-4(V+\phi_0)}\beta, \qquad (4.576)\\
V' &= \frac{g}{10}e^{-2(\phi_0+\phi_1)}\left[2e^{4\phi_1} - (\rho-\sigma)\sinh 2\phi_3 + (\rho+\sigma)\cosh 2\phi_3 \cosh 2\phi_2\right] \\
&\quad +\frac{9}{10}e^{-2(V-\phi_0+\phi_1)}p + \frac{2}{5}e^{-4(V+\phi_0)}\beta, \qquad (4.577)\\
\phi_0' &= \frac{g}{20}e^{-2(\phi_0+\phi_1)}\left[2e^{4\phi_1} - (\rho-\sigma)\sinh 2\phi_3 + (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3\right] \\
&\quad -\frac{3}{10}e^{-2(V-\phi_0+\phi_1)}p + \frac{1}{5}e^{-4(V+\phi_0)}\beta, \qquad (4.578)\\
\phi_1' &= -\frac{g}{4}e^{-2(\phi_0+\phi_1)}\left[2e^{4\phi_1} + (\rho-\sigma)\sinh 2\phi_3 - (\rho+\sigma)\cosh 2\phi_2 \cosh 2\phi_3\right] \\
&\quad +\frac{3}{2}e^{-2(V-\phi_0+\phi_1)}p, \qquad (4.579)\\
\phi_2' &= -\frac{g}{2}e^{-2(\phi_0+\phi_1)}(\rho+\sigma)\sinh 2\phi_2 \,\mathrm{sech}\, 2\phi_3, \qquad (4.580)\\
\phi_3' &= \frac{g}{2}e^{-2(\phi_0+\phi_1)}\left[(\rho-\sigma)\cosh 2\phi_3 - (\rho+\sigma)\cosh 2\phi_2 \sinh 2\phi_3\right]. \qquad (4.581)
\end{aligned}
$$

As in other previous cases, there are no AdS_3 fixed points from these equations.

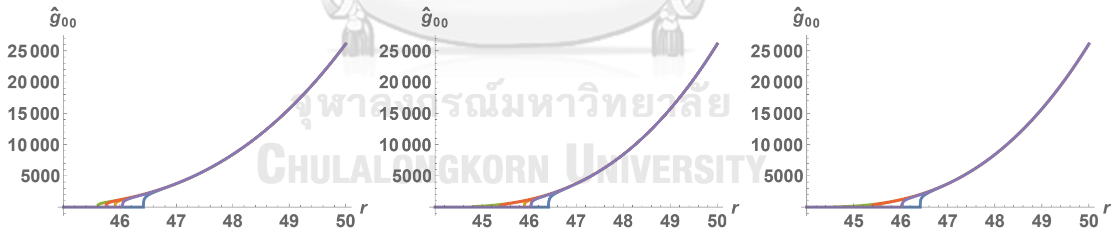


(a) U solution (b) V solution (c) W solution

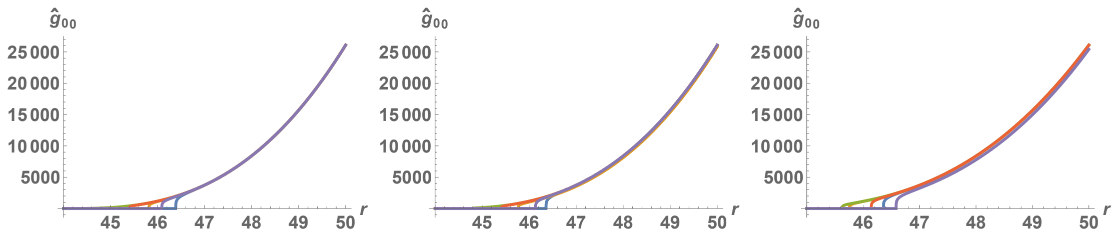


(d) ϕ_0 solution (e) ϕ solution

Figure 4.50: Numerical solutions for $SO(2) \times SO(2)$ twists with $g = 16$ and $\beta = 2$ in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 50$ to singularities in the form of $Mkw_2 \times H^2 \times H^2$-sliced DWs when $r < 50$. The blue, orange, green, red, and purple curves refer to $p_{21} = -0.5, -0.12, -0.01, 0.25, 0.6$.

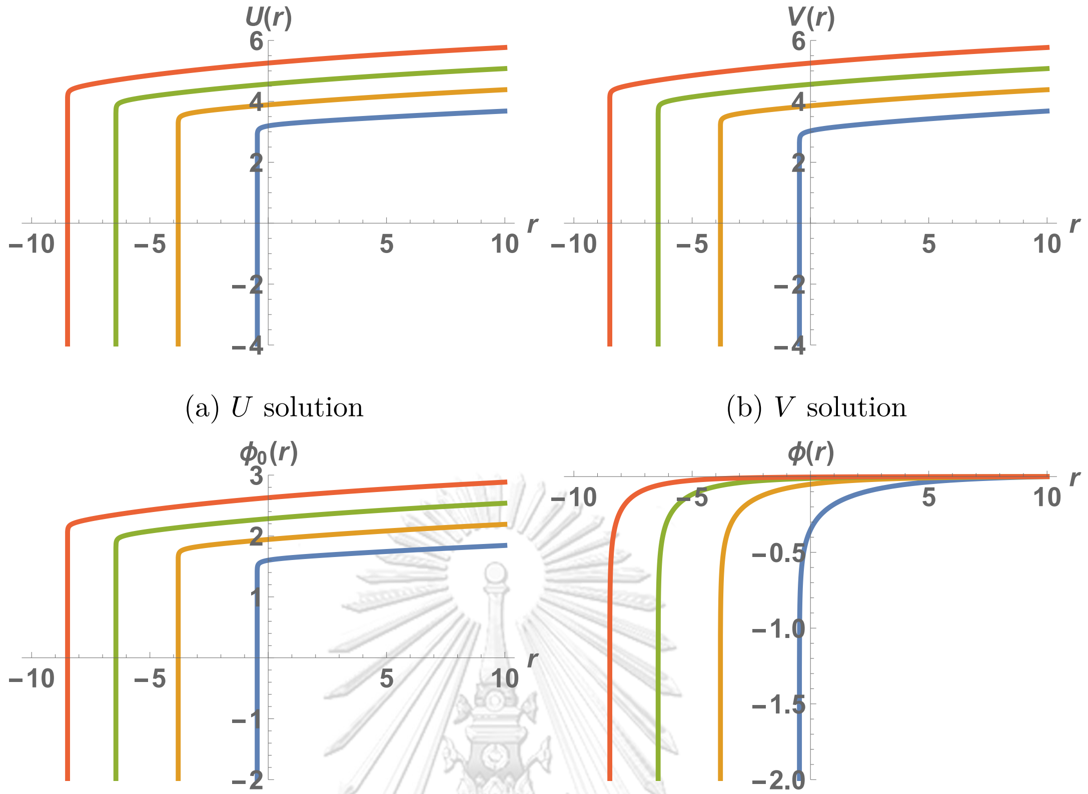


(a) $k_1 = k_2 = 1$ (b) $k_1 = 1, k_2 = 0$ (c) $k_1 = -k_2 = 1$



(d) $k_1 = k_2 = 0$ (e) $k_1 = -1, k_2 = 0$ (f) $k_1 = k_2 = -1$

Figure 4.51: The behavior of $\hat{g}_{00}$ for RG flows given in Figures 4.45 to 4.50 where $\hat{g}_{00} \to 0$ in the region $r < 50$ for every case.

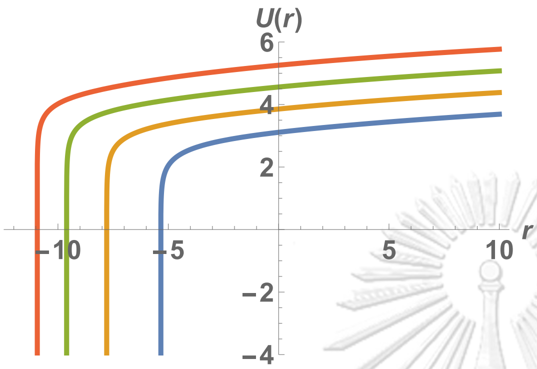


(a) U solution (b) V solution

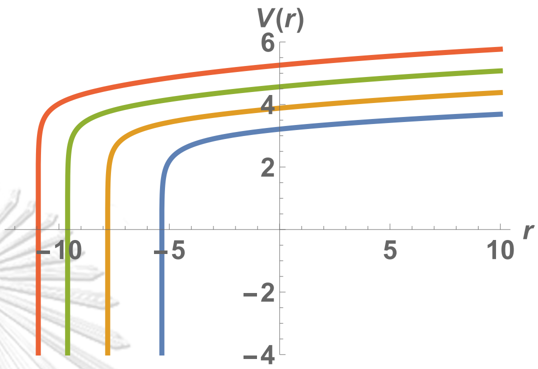
(c) ϕ_0 solution (d) ϕ solution

Figure 4.52: Numerical solutions for $SO(3)$ twists in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 10$ to singularities in the form of $Mkw_2 \times CP^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 4, 8, 16, 32$, respectively.

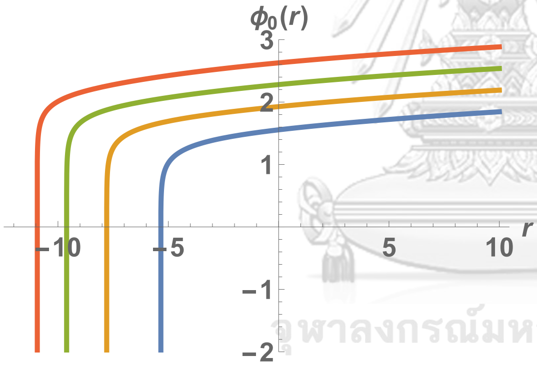
We end this chapter by commenting on the cases for gaugings in both **15** and $\overline{\mathbf{40}}$ representations with $SO(2,1) \ltimes \mathbf{R}^4$ and $SO(2) \ltimes \mathbf{R}^4$ gauge groups. In these gauge groups, the $SO(2)$ twists are also possible on Σ^2, $\Sigma^2 \times \Sigma^2$, and K^4 internal spaces. However, we do not find any fixed points from the resulting BPS equations. Therefore, we will not give further detail on these analyses.



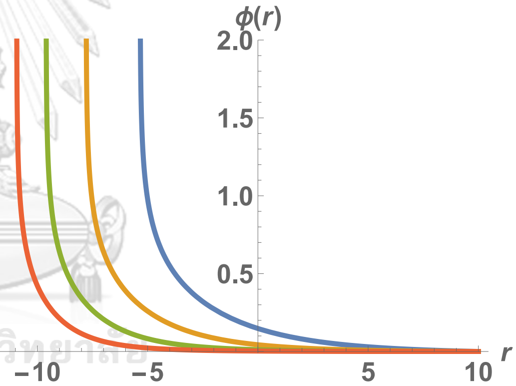
(a) U solution



(b) V solution

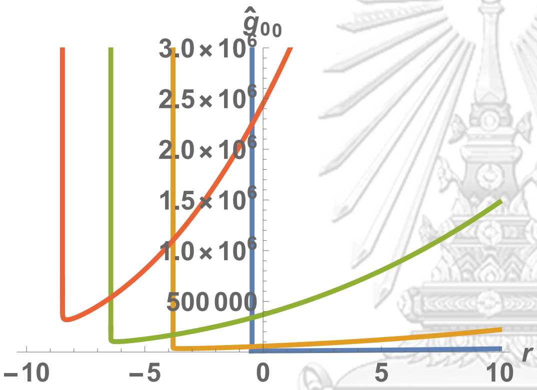


(c) ϕ_0 solution

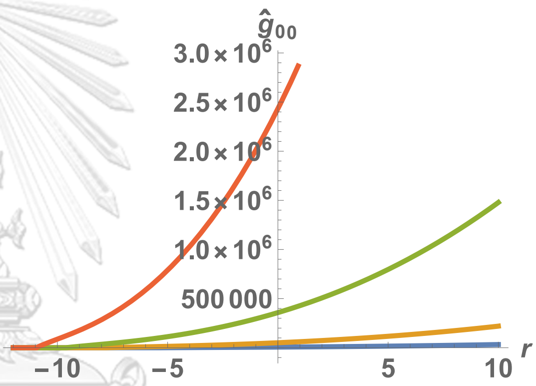


(d) ϕ solution

Figure 4.53: Numerical solutions for $SO(3)$ twists in $SO(4)$ gauge group. The flows start from locally flat DW as $r \to 10$ to singularities in the form of $Mkw_2 \times CH^2$-sliced DWs in the region $r < 0$. The blue, orange, green, and red curves refer to $g = 4, 8, 16, 32$, respectively.



(a) $k = 1$



(b) $k = -1$

Figure 4.54: The behavior of $\hat{g}_{00}$ for RG flows given in Figures 4.52 and 4.53 where $\hat{g}_{00} \to 0$ in the region $r < 0$ only the case with $k = -1$.

CHAPTER V

CONCLUSIONS AND DISCUSSIONS

Throughout this dissertation, we have studied several supersymmetric solutions of seven-dimensional gauged supergravities, namely the matter-coupled $N = 2$, $SO(4)$ gauged theory and the maximal $N = 4$ gauged theory with various gauge groups. In many cases, the resulting solutions have higher dimensional origins and could be interpreted as different brane configurations in string/M-theory. This feature makes applications of these solutions in the holographic context more intriguing. We now end the dissertation with some comments on the results together with the remaining problems, which will give us some directions for future works.

In Section 3.1, we have found charged DW solution preserving $SO(3)_{\text{diag}}$ residual symmetry from the matter-coupled $SO(4)$ gauged supergravity. Unlike the solutions of the minimal $SO(3)$ gauged theory in [62], this solution is more restrictive. For vanishing gauge fields, only analytic solution in the form of an $AdS_3 \times S^3$-sliced DW is possible. The solution interpolates between the two $N = 2$ supersymmetric AdS_7 critical points dual to $N = (1, 0)$ SCFTs with $SO(4)$ and $SO(3)$ symmetries in six dimensions. We expect this solution to describe a supersymmetric conformal surface defect within the six-dimensional $N = (1, 0)$ SCFTs with $SO(3)$ flavor symmetry in the same way as in [84].

We have also coupled the solution to non-vanishing $SO(3)_{\text{diag}}$ gauge fields and obtained a consistent set of BPS equations together with an algebraic constraint. In this case, we have performed the investigation and showed that supersymmetric solutions do not exist, at least within the truncation considered here. This is because BPS solutions of the flow equations, in general, violate the constraint arising from the SUSY transformations of the gaugini.

Supersymmetric $AdS_3 \times M^4$ solutions of the matter-coupled $SO(4)$ gauged supergravity have been studied in Section 3.2. For M^4 being a product of two Riemann surfaces, there exist a large class of $AdS_3 \times H^2 \times \Sigma^2$ solutions with $SO(2) \times SO(2)$ symmetry for $\Sigma^2 = S^2, \mathbb{R}^2, H^2$ similar to the solutions of maximal $SO(5)$ gauged supergravity. Besides, we have found a number of $AdS_3 \times H^2 \times H^2$ solutions with $SO(2)_{\text{diag}}$ and $SO(2)_R$ symmetries. We have also given various numerical solutions from both $N = 2$ supersymmetric AdS_7 vacua to these AdS_3 fixed points. The solutions describe holographic RG flows across dimensions from the $N = (1, 0)$ SCFTs in six dimensions to $N = (2, 0)$ SCFTs in two dimensions.

For M^4 being a Kahler four-cycle with $U(2) \sim SU(2) \times U(1)$ connection, we have found $AdS_3 \times CH^2$ solutions with $SO(2) \times SO(2)$, $SO(2)_{\text{diag}}$, and $SO(2)_R$ symmetries via performing the twist by $U(1) \sim SO(2)_R \subset SO(3)_R$. These fixed points preserve four supercharges and correspond to $N = (2, 0)$ SCFTs in two dimensions. Moreover, we have performed the twist along the $SU(2) \sim SO(3)$ part by turning on the $SO(3)_{\text{diag}}$ gauge fields. Unlike the previous cases, the AdS_3 solutions, in this case, preserve only two supercharges and dual to $N = (1, 0)$ SCFTs in two dimensions. We have studied RG flows from both $N = 2$ supersymmetric AdS_7 vacua to these geometries as well. These flow solutions can be interpreted as supersymmetric black strings in asymptotically AdS_7 space. Our solutions should be useful in the study of black string entropy using twisted indices of $N = (1, 0)$ SCFTs along the line of [90].

Supersymmetric solutions obtained in Chapter 3 can be embedded in eleven-dimensional supergravity using truncation ansatze constructed in [74] for a particular case with equal $SO(3)$ coupling constants. We have performed these uplifts and given the explicit forms of the eleven-dimensional metric and, in some simple cases, the four-form field strength. These solutions with clear M-theory origins are of particular interest in the study of M5-branes. For solutions with different $SO(3)$ coupling constants, there is no known embedding in string and M-theories. Therefore, in this case, the holographic interpretation in the dual $N = (1, 0)$ SCFTs should be done with some caveats.

In Chapter 4, supersymmetric solutions of the maximal $N = 4$ gauged supergravity with various gauge groups have been studied. We have started from classifying flat DW solutions in Section 4.1. There are both half-supersymmetric and $\frac{1}{4}$-supersymmetric flat DWs depending on which components of the embedding tensor in **15** and $\overline{\mathbf{40}}$ representations of the $SL(5)$ global symmetry lead to the gauging. Only in $SO(5)$ gauge group, there exist flat DWs that are asymptotic to the $N = 4$ supersymmetric AdS_7 vacuum and can be described as holographic RG flows from the dual $N = (2, 0)$ SCFT to SQFTs in six dimensions.

Supersymmetric charged DWs with $M_3 \times S^3$ slices, for $M_3 = Mkw_3, AdS_3$, and non-vanishing three-form fluxes are considered in Section 4.2. All of these solutions can be obtained analytically. Moreover, the charged DWs preserving $SO(4)$ residual symmetry can couple to $SO(3) \subset SO(4)$ gauge fields, but the resulting solutions can only be obtained numerically. For $SO(3)$ symmetric solutions, coupling to $SO(3)$ gauge fields does not lead to a new BPS solution. Only solutions with vanishing three-form fluxes or gauge fields are possible in this case. Apart from these solutions, we have also given a number of $SO(2) \times SO(2)$ and $SO(2)$ symmetric charged DWs that cannot couple to SO(3) gauge fields due to the absence of any unbroken SO(3) gauge symmetry.

For $SO(5)$ gauge group, charged DW solutions with an $AdS_3 \times S^3$ slice can be interpreted as surface defects within the $N = (2, 0)$ SCFT. For other gauge groups, their supersymmetric vacua take the form of flat DWs DW/QFT dual to $N = (2, 0)$ SQFTs in six dimensions. We then expect these $AdS_3 \times S^3$-sliced DWs to describe $\frac{1}{4}$-BPS surface defects in $N = (2, 0)$ SQFTs. We have found that a number of charged DW solutions are given in terms of the flat DWs in Section 4.1 attached with three-form fluxes. However, the charged DWs preserve only 1/4 of the original SUSY as opposed to the flat ones, which are $\frac{1}{2}$-supersymmetric, except for the DWs from gaugings in both **15** and $\overline{\mathbf{40}}$ representations in which both charged and flat DWs are $\frac{1}{4}$-supersymmetric.

We have performed the uplift for flat and charged DW solutions for $SO(5)$ and $CSO(4,0,1)$ gauge groups. In these cases, the complete truncation ansatze of eleven-dimensional supergravity on S^4 and type IIA theory on S^3 are given in [26, 27] and [77], respectively. These uplifted solutions would be useful in the study of the AdS/CFT correspondence and several dynamical aspects of M5-branes and NS5-branes in different transverse spaces. Furthermore, the uplift of charged DW solutions in these two gauge groups should respectively describe bound states of M2- and M5-branes and of fundamental strings and NS5-branes similar to the solutions in [62].

Supersymmetric $AdS_n \times \Sigma^{7-n}$ solutions of the maximal gauged supergravity have been extensively studied in Section 4.3. For $SO(5)$ gauge group, all the previous results on $AdS_n \times \Sigma^{7-n}$ fixed points with $n = 2, 3, 4, 5$ have been recovered. We have provided numerical RG flows from the $N = 4$ supersymmetric AdS_7 vacuum in the UV to all these fixed points and then to singular geometries in the IR. These IR singularities take the form of curved DWs with $Mkw_{n-1} \times \Sigma^{7-n}$ slices and can be interpreted as $(n-1)$-dimensional SQFTs. The extended flows suggest that they describe non-conformal phases of the SCFTs in $n-1$ dimensions, dual to the $AdS_n \times \Sigma^{7-n}$ fixed points, obtained from twisted compactifications of the six-dimensional $N = (2,0)$ SCFT.

In addition to the previously known results from $SO(5)$ gauge group, we have discovered novel classes of $AdS_5 \times S^2$, $AdS_3 \times S^2 \times \Sigma^2$, and $AdS_3 \times CP^2$ solutions in $SO(3,2)$ gauge group. There are no supersymmetric AdS_7 critical points in this gauge group, so we have studied RG flow solutions interpolating between these new fixed points and curved DWs. A number of the singularities are physically acceptable and can be interpreted as SQFTs obtained from twisted compactifications of the $N = (2,0)$ SQFTs in six dimensions. We have further carried out a similar analysis for $SO(4,1)$ gauge group and found a new class of $AdS_3 \times CP^2$ solutions. For convenience, we summarize all the $AdS_n \times \Sigma^{7-n}$ fixed points in table 5.1.

AdS_n	Unbroken symmetry	Σ^{7-n}	Gauge group $SO(5)$	$SO(4,1)$	$SO(3,2)$
AdS_5	$SO(2) \times SO(2)$	S^2	8		8
		$\mathbb{R}^2$	16		
		H^2	8		
AdS_4	$SO(3)$	H^3	8		
	$SO(3)_+$	H^3	4		
AdS_3	$SO(4)$	H^4	4		
	$SO(2) \times SO(2)$	$S^2 \times \Sigma^2$			4
		$H^2 \times \Sigma^2$	4		
	$SO(3)$	CH^2	4		
	$SO(3)_+$	CH^2	2		
	$SO(2) \times SO(2)$	CP^2		4	4
		CH^2	4		
AdS_2	$SO(3) \times SO(2)$	$H^3 \times H^2$	4		
	$SO(5)$	S^5	2		
		H^5	2		

Table 5.1: $AdS_n \times \Sigma^{7-n}$ fixed points from maximal gauged supergravity in seven dimensions together with the corresponding symmetries and numbers of unbroken supercharges. Σ^2 can be S^2, $\mathbb{R}^2$ or H^2.

Similar to four-dimensional black hole solutions with curved DW asymptotics studied in [91], the novel AdS_5 and AdS_3 fixed points in $SO(3,2)$ and $SO(4,1)$ gauge groups can be respectively interpreted as black three-branes and black strings in asymptotically curved DW spacetime. It has also been pointed out in [92] that, from a higher-dimensional perspective, the four-dimensional black holes should be seen as black string solutions in AdS_5 spacetime studied in [58]. However, our solutions cannot be related to any supersymmetric black objects in eight dimensions with asymptotically AdS_8 spacetime. This is because of the absence of supersymmetric AdS_d vacua for $d > 7$.

For other gauge groups, we have performed a similar analysis but have not found any AdS_n fixed points. Instead, in $CSO(p, q, 4-p-q)$ gauge group, we have studied supersymmetric flow solutions interpolating between asymptotically locally flat DWs, in which the effect of magnetic charges is highly suppressed, and curved DWs with $Mkw_{n-1} \times \Sigma^{7-n}$ world-volume. By the DW/QFT duality, these solutions should be interpreted as RG flows across dimensions between SQFTs in six and $n-1$ dimensions. Our results suggest that these six-dimensional $N = (2, 0)$ field theories have no conformal fixed points in lower dimensions. It could be interesting to study these field theories on the world-volume of five-branes in type IIB theory and find a definite conclusion whether this is true in general.

Apart from intriguing investigations in field theories, many works remain to be done on the supergravity side. It would be interesting to find the embedding of solutions, from the matter-coupled $SO(4)$ gauged theory in cases with different $SO(3)$ coupling constants, in ten or eleven dimensions. This could provide the full holographic duals of the effective theories on five-branes. Besides, supersymmetric solutions in seven-dimensional matter-coupled $N = 2$ gauged supergravity with other gauge groups are worth considering.

It is also interesting to look for flat DWs from the maximal gauged supergravity with $CSO(1, 0, 4)$ and $CSO(1, 0, 3)$ gauge groups. These solutions, called elementary DWs in [52], would probably involve many non-vanishing scalars. Since we have not found any AdS_n fixed points in other gauge groups, it would also be interesting to extend our analysis by using more general ansatz including non-vanishing massive two-form fields and find new classes of $AdS_n \times \Sigma^{7-n}$ solutions of seven-dimensional gauged supergravity. Moreover, it is natural to extend our study by constructing the complete truncation ansatze of eleven-dimensional supergravity on $H^{p,q} \circ T^{5-p-q}$ and type IIB theory on $H^{p,q} \circ T^{4-p-q}$. These ansatze can be used to uplift the solutions in $CSO(p, q, 5-p-q)$ and $CSO(p, q, 4-p-q)$ gauge groups for any values of p and q leading to the full holographic interpretation.

Finally, finding supersymmetric solutions of six-dimensional maximal $N = (2,2)$ gauged supergravity is very interesting. It was shown in [93] that $SO(5)$ gauged supergravity in six dimensions is inherited from circle reduction of the seven-dimensional $SO(5)$ gauged theory. Besides, by using the embedding tensor formalism, the most general gauging of the maximal gauged supergravity in six dimensions has been classified in [94]. Some consistent gaugings are related to circle reductions of seven-dimensional $CSO(p,q,r)$ gauged theory. Nevertheless, there is no $N = (2,2)$ supersymmetric AdS_6 vacuum admitted in this theory, as pointed out in [30, 95]. Therefore, supersymmetric solutions of six-dimensional $N = (2,2)$ gauged supergravity will be useful to study a non-conformal extension of the AdS/CFT correspondence.

REFERENCES

[1] Marc H. Goroff and Augusto Sagnotti, "The ultraviolet behavior of Einstein gravity," **Nuclear Physics B** 266 (1986): 709-736.

[2] Steven Carlip, "Quantum gravity: a progress report," **Reports on Progress in Physics** 64 (2001): 885-957.

[3] Michael B. Green, John H. Schwarz, and Edward Witten, **Superstring theory, Vol. 1: Introduction**, (Cambridge: Cambridge University Press, 1987).

[4] Michael B. Green, John H. Schwarz, and Edward Witten, **Superstring theory, Vol. 2: Loop amplitudes, anomalies and phenomenology**, (Cambridge: Cambridge University Press, 1987).

[5] Joseph G. Polchinski, **String theory. Vol. 1: An introduction to the bosonic string**, (Cambridge: Cambridge University Press, 2007).

[6] Joseph G. Polchinski, **String theory. Vol. 2: Superstring theory and beyond**, (Cambridge: Cambridge University Press, 2007).

[7] Katrin Becker, Melanie Becker, and John H. Schwarz, **String theory and M-theory: A modern introduction**, (New York: Cambridge University Press, 2007).

[8] Elias Kiritsis, **String theory in a nutshell**, (New Jersey: Princeton University Press, 2007).

[9] Juan M. Maldacena, "The large N limit of superconformal field theories and supergravity," **Advances in Theoretical and Mathematical Physics** 2 (1998): 231-252.

[10] Steven S. Gubser, Igor R. Klebanov, and Alexander M. Polyakov, "Gauge theory correlators from noncritical string theory," **Physics Letters B** 428 (1998): 105-114.

[11] Edward Witten, "Anti-de Sitter space and holography," **Advances in Theoretical and Mathematical Physics** 2 (1998): 253-291.

[12] Gerard 't Hooft, "A planar diagram theory for strong interactions," **Nuclear Physics B** 72 (1974): 461-473.

[13] Motohiko Ezawa, "Supersymmetry and Unconventional Quantum Hall Effect in Graphene," **Physics Letters A** 372 (2008): 924-929.

[14] Sean A. Hartnoll, Christopher P. Herzog, and Gary T. Horowitz, "Building an AdS/CFT superconductor," **Physical Review Letters** 101 (2008): 031601.

[15] Sean A. Hartnoll, "Lectures on holographic methods for condensed matter physics," **Classical and Quantum Gravity** 26 (2009): 224002.

[16] Subir Sachdev, "Condensed matter and AdS/CFT," **Lecture Notes in Physics** 828 (2011): 273-311.

[17] Subir Sachdev, "What can gauge-gravity duality teach us about condensed matter physics?," **Annual Review of Condensed Matter Physics** 3 (2012): 9-33.

[18] Andrew G. Green, "An introduction to gauge gravity duality and its application in condensed matter," **Contemporary Physics** 54 (2013): 33-48.

[19] César A. Dartora and Guillermo G. Cabrera, "Wess-Zumino supersymmetric phase and superconductivity in graphene," **Physics Letters A** 377 (2013): 907-909.

[20] Mahya Mohammadi and Ahmad Sheykhi, "Conductivity of the one-dimensional holographic p-wave superconductors in the presence of nonlinear electrodynamics," **Physical Review D** 100 (2019): 086012.

[21] Laura Andrianopoli, Bianca L. Cerchiai, Riccardo D'Auria, Antonio Gallerati et al., "$\mathcal{N}$-Extended $D = 4$ Supergravity, Unconventional SUSY and Graphene," **Journal of High Energy Physics** 01 (2020): 084.

[22] Eugène Cremmer, Bernard L. Julia, and Joël Scherk, "Supergravity theory in 11 dimensions," **Physics Letters B** 76 (1978): 409-412.

[23] Bernard De Witt and Hermann Nicolai, "The consistency of the S^7 truncation in $D = 11$ supergravity," **Nuclear Physics B** 281 (1987): 211-240.

[24] Oscar Varela, "Complete $D = 11$ embedding of $SO(8)$ supergravity," **Physical Review D** 97 (2008): 045010.

[25] Krzysztof A. Pilch, Pieter van Nieuwenhuizen, and Paul K. Townsend, "Compactification of $d = 11$ supergravity on $S(4)$ (or $11 = 7 + 4$, too)," **Nuclear Physics B** 242 (1984): 377–392.

[26] Horatiu Nastase, Diana Vaman, and Pieter van Nieuwenhuizen, "Consistent nonlinear KK reduction of $11-d$ supergravity on $AdS(7)\times S(4)$ and selfduality in odd dimensions," **Physics Letters B** 469 (1999): 96–102.

[27] Horatiu Nastase, Diana Vaman, Pieter van Nieuwenhuizen, "Consistency of the $AdS_7 \times S^4$ reduction and the origin of self-duality in odd dimensions," **Nuclear Physics B** 581 (2000): 179–239.

[28] Micha Berkooz, "A supergravity dual of a $(1,0)$ field theory in six dimensions," **Physics Letters B** 437 (1998): 315-317.

[29] Sergio Ferrara, Alex A. Kehagias, Hervé Partouche, and Alberto Zaffaroni, "Membranes and fivebranes with lower supersymmetry and their AdS supergravity duals," **Physics Letters B** 431 (1998): 42-48.

[30] Werner Nahm, "Supersymmetries and their representations," **Nuclear Physics B** 135 (1978): 149-166.

[31] Edward Witten, **Conformal field theory in four and six dimensions** [Online]. Available from: https://arxiv.org/pdf/0712.0157.pdf [2019, June 8]

[32] Mario Pernici, Krzysztof A. Pilch, and Pieter van Nieuwenhuizen, "Gauged maximally extended supergravity in seven-dimensions," **Physics Letters B** 143 (1984): 103-107.

[33] Mario Pernici, Krzysztof A. Pilch, Pieter van Nieuwenhuizen, and Nicholas P. Warner, "Noncompact gaugings and critical points of maximal supergravity in seven-dimensions," **Nuclear Physics B** 249 (1985): 381-395.

[34] Eric Bergshoeff, I. G. Koh, and Ergin Sezgin, "Yang-Mills-Einstein supergravity in seven dimensions," **Physical Review D** 32 (1985): 1353-1357.

[35] Youngjai Park, "Gauged Yang-Mills-Einstein supergravity with three index field in seven dimensions," **Physical Review D** 38 (1988): 1087.

[36] Paul K. Townsend and Pieter van Nieuwenhuizen, "Gauged seven-dimensional supergravity," **Physics Letters B** 125 (1983): 41-46.

[37] Luca Mezincescu, Paul K. Townsend, and Pieter van Nieuwenhuizen, "Stability of gauged $d = 7$ supergravity and the definition of masslessness in AdS_7," **Physics Letters B** 143 (1984): 384-388.

[38] Eric Bergshoeff, Der-Chyn Jong, and Ergin Sezgin, "Noncompact gaugings, chiral reduction and dual sigma model in supergravity," **Classical and Quantum Gravity** 23 (2006): 2803-2832.

[39] Parinya Karndumri, "RG flows in 6D $N = (1, 0)$ SCFT from $SO(4)$ half-maximal gauged supergravity," **Journal of High Energy Physics** 06 (2014): 101.

[40] Parinya Karndumri, "Noncompact gauging of $N = 2$ 7D supergravity and AdS/CFT holography," **Journal of High Energy Physics** 02 (2015): 034.

[41] Abdus Salam and Ergin Sezgin, "$SO(4)$ gauging of $N = 2$ supergravity in seven dimensions," **Physics Letters B** 126 (1983): 295-300.

[42] Michela Petrini and Alberto Zaffaroni, **The holographic RG flow to conformal and non-conformal theory** [Online]. Available from: https://arxiv.org/pdf/hep-th/0002172.pdf [2019, June 12]

[43] Steven S. Gubser, "Non-conformal examples of AdS/CFT," **Classical and Quantum Gravity** 17 (2000): 1081–1092.

[44] Eric D'Hoker and Daniel Z. Freedman, **Supersymmetric gauge theories and the AdS/CFT correspondence** [Online]. Available from: https://arxiv.org/pdf/hep-th/0201253.pdf [2019, June 18]

[45] Harm J. Boonstra, Konstantinos Skenderis, and Paul K. Townsend, "The domain-wall/QFT correspondence," **Journal of High Energy Physics** 01 (1999): 003.

[46] Tony Gherghetta and Yaron Oz, "Supergravity, non-conformal field theories and brane-worlds," **Physical Review D** 65 (2002): 046001.

[47] Ingmar Kanitscheider, Kostas Skenderis, and Marika Taylor, "Precision holography for non-conformal branes," **Journal of High Energy Physics** 09 (2008): 094.

[48] P. M. Cowdall, Hong Lu, Christopher N. Pope, Kellogg S. Stelle, and Paul K. Townsend, "Domain walls in massive supergravities," **Nuclear Physics B** 486 (1997): 49-76.

[49] Ioannis Bakas, Andreas Brandhuber, and Konstantinos Sfetsos, "Domain walls of gauged supergravity, M-branes, and algebraic curves," **Advances in Theoretical and Mathematical Physics** 3 (1999): 1657-1719.

[50] Mirjam Cvetic, Steven S. Gubser, Hong Lu, and Christopher N. Pope, "Symmetric potentials of gauged supergravities in diverse dimensions and coulomb branch of gauge theories," **Physical Review D** 62 (2000): 086003.

[51] Eric Bergshoeff, Mikkel Nielsen, and Diederik Roest, "The domain walls of gauged maximal supergravities and their M-theory origin," **Journal of High Energy Physics** 07 (2004): 006.

[52] Eric Bergshoeff, Axel Kleinschmidt, and Fabio Riccioni, "Supersymmetric domain walls," **Physical Review D** 86 (2012): 085043.

[53] Jerome P. Gauntlett, Nakwoo Kim, and Daniel Waldram, "M five-branes wrapped on supersymmetric cycles," **Physical Review D** 63 (2001): 126001.

[54] Jerome P. Gauntlett and Nakwoo Kim, "M five-branes wrapped on supersymmetric cycles 2," **Physical Review D** 65 (2002): 086003.

[55] Jerome P. Gauntlett, Nakwoo Kim, Stathis Pakis, and Daniel Waldram, "M-theory solutions with AdS factors," **Classical and Quantum Gravity** 19 (2002): 3927-3946.

[56] Sorin Cucu, Hong Lu, and Justin F. Vazquez-Poritz, "Interpolating from $AdS_{(D-2)} \times S^2$ to AdS_D," **Nuclear Physics B** 677 (2004): 181-222.

[57] Ibrahima Bah, Christopher Beem, Nikolay Bobev, and Brian Wecht, "Four-dimensional SCFTs from M5-branes," **Journal of High Energy Physics** 06 (2012): 005.

[58] Francesco Benini and Nikolay Bobev, "Two-dimensional SCFTs from wrapped branes and c-extremization," **Journal of High Energy Physics** 06 (2013): 005.

[59] Parinya Karndumri and Eoin O. Colgain, "3D supergravity from wrapped M5-branes," **Journal of High Energy Physics** 03 (2016): 188.

[60] Parinya Karndumri, "RG flows from $(1, 0)$ 6D SCFTs to $N = 1$ SCFTs in four and three dimensions," **Journal of High Energy Physics** 06 (2015): 027.

[61] Bobby S. Acharya, Jerome P. Gauntlett, and Nakwoo Kim, "Five-branes wrapped on associative three cycles," **Physical Review D** 63 (2001): 106003.

[62] Giuseppe Dibitetto and Nicolò Petri, "BPS objects in $D = 7$ supergravity and their M-theory origin," **Journal of High Energy Physics** 12 (2017): 041.

[63] Henning Samtleben and Martin Weidner, "The maximal D=7 supergravities," **Nuclear Physics B** 725 (2005): 383-419.

[64] Olaf Hohm and Henning Samtleben, "Consistent Kaluza-Klein truncations via exceptional field theory," **Journal of High Energy Physics** 01 (2015): 131.

[65] Emanuel Malek and Henning Samtleben, "Dualising consistent IIA/IIB truncations," **Journal of High Energy Physics** 12 (2015): 029.

[66] Yuri Gol'fand and Evgeny Likhtman, "Extension of the algebra of Poincaré group generators and violation of P invariance," **Journal of Experimental and Theoretical Physics Letters** 13 (1971): 323–326.

[67] Dimitrii Volkov and Vladimir Akulov, "Is the neutrino a Goldstone particle?," **Physics Letters B** 46 (1973): 109–110.

[68] Julius Wess and Bruno Zumino, "Supergauge transformations in four-dimensions," **Nuclear Physics B** 70 (1974): 39–50.

[69] Daniel Z. Freedman, Pieter van Nieuwenhuizen, and Sergio Ferrara, "Progress toward a theory of supergravity," **Physical Review D** 13 (1976): 3214.

[70] Stanley Deser and Bruno Zumino, "Consistent supergravity," **Physics Letters B** 62 (1976): 335-337.

[71] Eugène Cremmer and Bernard L. Julia, "The SO(8) supergravity," **Nuclear Physics B** 159 (1979): 141-212.

[72] Bernard De Wit and Daniel Z.Freedman, "On SO(8) extended supergravity," **Nuclear Physics B** 130 (1977): 105-113.

[73] Arnaud Baguet, Olaf Hohm, and Henning Samtleben "Consistent type IIB reductions to maximal 5D supergravity," **Physical Review D** 92 (2015): 065004.

[74] Parinya Karndumri, "$N = 2$ $SO(4)$ 7D gauged supergravity with topological mass term from 11 dimensions," **Journal of High Energy Physics** 11 (2014): 063.

[75] Giuseppe Dibitetto, Jose J. Fernndez-Melgarejo, and Diego Marqus, "All gaugings and stable de Sitter in $D = 7$ half-maximal supergravity," **Journal of High Energy Physics** 11 (2015): 037.

[76] Nicholas P. Warner, "Some new extrema of the scalar potential of gauged $N = 8$ supergravity," **Physics Letters B** 128 (1983): 169-173.

[77] Mirjam Cvetic, Hong Lu, Christopher N. Pope, A. Sadrzadeh, and Tuan A. Tran, "S^3 and S^4 reductions of type IIA supergravity," **Nuclear Physics B** 590 (2000): 233-251.

[78] Joël Scherk and John H. Schwarz, "How to get masses from extra dimensions," **Nuclear Physics B** 153 (1979): 61–88.

[79] Abdus Salam and Ergin Sezgin, "$D = 8$ supergravity," **Nuclear Physics B** 258 (1985): 284-304.

[80] Nathan Seiberg, "Non-trivial fixed points of the renormalization group in six-dimensions," **Physics Letters B** 390 (1997): 169-171.

[81] Nathan Seiberg and Edward Witten, "Comments on string dynamics in six dimensions," **Nuclear Physics B** 471 (1996): 121-134.

[82] Nathan Seiberg, "Note on theories with 16 supercharges," **Nuclear Physics B - Proceedings Supplements** 67 (1998): 158-171.

[83] Hong Lu and Christopher N. Pope, "Exact embedding of $N = 1$, $D = 7$ gauged supergravity in $D = 11$," **Physics Letters B** 467 (1999): 67–72.

[84] Giuseppe Dibitetto and Nicolò Petri, "6d surface defects from massive type IIA," **Journal of High Energy Physics** 01 (2018): 039.

[85] Steven S. Gubser, "Curvature singularities: the good, the bad and the naked," **Advances in Theoretical and Mathematical Physics** 4 (2000): 679-745.

[86] Juan M. Maldacena and Carlos Nunez, "Supergravity description of field theories on curved manifolds and a no go theorem," **International Journal of Modern Physics A** 16 (2001): 822-855.

[87] Achilleas Passias, Andrea Rota, and Alessandro Tomasiello, "Universal consistent truncation for 6d/7d gauge/gravity duals," **Journal of High Energy Physics** 10 (2015): 187.

[88] Madoka Nishimura and Yoshiaki Tanii, "Local symmetries in the $\mathrm{AdS}_7/\mathrm{CFT}_6$ correspondence," **Modern Physics Letters A** 14 (1999): 2709-2719.

[89] Nikolay Bobev, Giuseppe Dibitetto, Fridrik F. Gautason, and Brecht Truijen, "Holography, brane intersections and six-dimensional SCFTs," **Journal of High Energy Physics** 02 (2017): 116.

[90] Seyed M. Hosseini, Kiril Hristov, and Alberto Zaffaroni, "A note on the entropy of rotating BPS $AdS_7 \times S^4$ black holes," **Journal of High Energy Physics** 05 (2018): 121.

[91] Sergio L. Cacciatori and Dietmar Klemm, "Supersymmetric AdS_4 black holes and attractors," **Journal of High Energy Physics** 01 (2010): 085.

[92] Kiril Hristov, "Dimensional reduction of BPS attractors in AdS gauged supergravities," **Journal of High Energy Physics** 12 (2014): 066.

[93] P.M. Cowdall, "On gauged maximal supergravity in six-dimensions," **Journal of High Energy Physics** 06 (1999): 018.

[94] Eric Bergshoeff, Henning Samtleben, and Ergin Sezgin, "The gaugings of maximal $D = 6$ supergravity," **Journal of High Energy Physics** 03 (2008): 068.

[95] Severin Lust, Philipp Ruter, and Jan Louis, "Maximally supersymmetric AdS solutions and their moduli spaces," **Journal of High Energy Physics** 03 (2018): 019.

[96] Mijan F. Huq and Mirza A. Namazie, "Kaluza-Klein supergravity in ten dimensions," **Classical and Quantum Gravity** 2 (1985): 293-308.

APPENDICES

APPENDIX A

EINSTEIN'S GENERAL RELATIVITY

Einstein's general relativity (GR) is one of the cornerstones of classical physics describing gravity as curvature of spacetime encoded in symmetric metric tensor $g_{\mu\nu} = g_{\nu\mu}$. In this appendix, $\mu, \nu = 0, 1, ..., D-1$ refer to D-dimensional curved spacetime indices lowered and raised by $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$, respectively. For examples,

$$U_\mu = g_{\mu\nu} U^\nu \quad \text{and} \quad U^{\mu\nu} = g^{\mu\rho} g^{\nu\sigma} U_{\rho\sigma} \tag{A.1}$$

for any tensor U. Throughout this dissertation, we regularly apply Einstein's summation convention in which a pair of upper and lower repeated indices are summed

$$g_{\mu\nu} U^\nu = \Sigma_{\nu=0}^{D-1} g_{\mu\nu} U^\nu. \tag{A.2}$$

To decode the information on curvature, we use vielbein formalism in the language of differential forms. In this appendix, we start with a brief introduction of differential forms. After that, the vielbein formalism used to compute spacetime curvature will be reviewed.

A.1 Differential Forms

For an integer p such that $0 \leq p \leq D$, a differential p-form is a mathematical object in D dimensional spacetime defined by

$$\Omega_{(p)} \equiv \frac{1}{p!} \, \Omega_{\mu_1 ... \mu_p} dx^{\mu_1} \wedge ... \wedge dx^{\mu_p} \tag{A.3}$$

where the component $\Omega_{\mu_1 ... \mu_p}$ is totally antisymmetric

$$\Omega_{\mu_1 ... \mu_i ... \mu_j ... \mu_p} = -\Omega_{\mu_1 ... \mu_j ... \mu_i ... \mu_p} \tag{A.4}$$

in any pair of indices μ_i and μ_j. The basis of a p-form is also totally antisymmetric described by the wedge product

$$dx^{\mu_1} \wedge ... \wedge dx^{\mu_p} = p!\ dx^{[\mu_1} \otimes ... \otimes dx^{\mu_p]} \tag{A.5}$$

in which x^μ are some coordinates parameterizing the spacetime and $\otimes$ is the usual tensor product. Besides, antisymmetrization and symmetrization in spacetime indices $\mu_1...\mu_p$ of any tensor T are denoted by

$$T^{[\mu_1...\mu_p]} = \frac{1}{p!}(T^{\mu_1...\mu_p} + (-1)^P \text{ all permutations of } \mu_1...\mu_p), \tag{A.6}$$

$$T^{(\mu_1...\mu_p)} = \frac{1}{p!}(T^{\mu_1...\mu_p} + \text{all permutations of } \mu_1...\mu_p) \tag{A.7}$$

with $P = 0, 1$ for even or odd permutation, respectively.

In particular, zero-forms are scalars

$$\Omega_{(0)} = \Omega, \tag{A.8}$$

and one-forms are vectors

$$\Omega_{(1)} = \Omega_\mu dx^\mu = \Omega_{\mu'} dx^{\mu'} \tag{A.9}$$

where $x^{\mu'}$ refer to other coordinate systems.

Some useful operations of differential forms are reviewed as follows:

- Wedge product

  If $p + q \leq D$, a p-form and a q-form can be multiplied to give a $(p+q)$-form through a wedge product

$$\begin{aligned}\Omega_{(p)} \wedge \Pi_{(q)} &= \left(\frac{1}{p!}\ \Omega_{\mu_1...\mu_p} dx^{\mu_1} \wedge ... \wedge dx^{\mu_p}\right) \wedge \left(\frac{1}{q!}\ \Pi_{\nu_1...\nu_q} dx^{\nu_1} \wedge ... \wedge dx^{\nu_q}\right) \\ &= \frac{1}{p!q!}\ \Omega_{\mu_1...\mu_p} \Pi_{\nu_1...\nu_q} dx^{\mu_1} \wedge ... \wedge dx^{\mu_p} \wedge dx^{\nu_1} \wedge ... \wedge dx^{\nu_q}\end{aligned} \tag{A.10}$$

  satisfying

$$\Omega_{(p)} \wedge \Pi_{(q)} = (-1)^{pq} \Pi_{(q)} \wedge \Omega_{(p)}. \tag{A.11}$$

- <u>Exterior derivative</u>

  Denoted by d, exterior derivative is a linear operation mapping a p-form to be a $(p+1)$-form

$$d\Omega_{(p)} = \frac{1}{p!}\,\partial_\mu \Omega_{\mu_1\ldots\mu_p} dx^\mu \wedge dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p}, \tag{A.12}$$

  and satisfying the following conditions

$$d(\Omega_{(p)} \wedge \Omega_{(q)}) = d\Omega_{(p)} \wedge \Omega_{(q)} + (-1)^p \Omega_{(p)} \wedge d\Omega_{(q)}, \tag{A.13}$$

$$d^2\Omega_{(p)} = dd\Omega_{(p)} = 0 \tag{A.14}$$

  for any p- and q-froms, $\Omega_{(p)}$ and $\Omega_{(q)}$.

- <u>Hodge duality</u>

  From the definition of the Hodge duality of a p-form's basis

$$*(dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p}) \equiv \frac{1}{(D-p)!}\,\epsilon_{\mu_{p+1}\ldots\mu_D}{}^{\mu_1\ldots\mu_p} dx^{\mu_{p+1}} \wedge \ldots \wedge dx^{\mu_D}, \tag{A.15}$$

  the Hodge duality of a p-form in D-dimensional spacetime is the following $(D-p)$-form

$$*\Omega_{(p)} = \frac{1}{p!(D-p)!}\,\Omega_{\mu_1\ldots\mu_p} \epsilon_{\mu_{p+1}\ldots\mu_D}{}^{\mu_1\ldots\mu_p} dx^{\mu_{p+1}} \wedge \ldots \wedge dx^{\mu_D}. \tag{A.16}$$

Thorough this work, $\epsilon_{\mu_1\ldots\mu_D}$ is a totally antisymmetric Levi-Civita tensor in D dimensions given by

$$\epsilon_{\mu_1\ldots\mu_D} = \sqrt{|g|}\varepsilon_{\mu_1\ldots\mu_D} \tag{A.17}$$

in which $g = \det g_{\mu\nu}$ and $\varepsilon_{\mu_1\ldots\mu_D}$ is a totally antisymmetric Levi-Civita symbol defined as

$$\varepsilon_{\mu_1\ldots\mu_D} = \begin{cases} +1 & \text{if } \mu_1\ldots\mu_D \text{ is an even permutation of } 0,1,\ldots,D-1, \\ -1 & \text{if } \mu_1\ldots\mu_D \text{ is an odd permutation of } 0,1,\ldots,D-1, \\ 0 & \text{otherwise.} \end{cases} \tag{A.18}$$

Note also that the Levi-Civita tensor becomes the Levi-Civita symbol in flat spacetime where $g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-,+,\ldots,+)$ with $\det \eta_{\mu\nu} = -1$.

We can see that Hodge duality of a pure number 1 (zero-form) is a D-form whose component is the Levi-Civita tensor,

$$\begin{aligned} *1 = \epsilon_{(D)} &= \frac{1}{D!}\ \epsilon_{\mu_1 \ldots \mu_D} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D} \\ &= \sqrt{|g|} dx^0 \wedge \ldots \wedge dx^{D-1}. \end{aligned} \tag{A.19}$$

In the second line, the Levi-Civita tensor is changed to be the Levi-Civita symbol thus $*1$ is the generally coordinate invariant volume element on D-dimensional spacetime $\mathcal{M}$ called volume form,

$$Vol_{\mathcal{M}} = \sqrt{|g|} d^D x = \sqrt{|g|} dx^0 \wedge \ldots \wedge dx^{D-1}. \tag{A.20}$$

This volume form always appears in Lagrangian densities in the language of differential forms. To obtain the corresponding action, one should integrate a Lagrangian density over the spacetime. However, in the differential-form point of view, a scalar (zero-form) cannot be integrated over a D-dimensional spacetime but a D-form can. Consequently, in the language of differential forms, the Lagrangian density in D-dimensional spacetime is a D-form. For example, the Einstein-Hilbert Lagrangian density is written as

$$\mathcal{L}_{EH} = \frac{1}{2}\ R * 1 \tag{A.21}$$

where R is called Ricci scalar representing the curvature of spacetime that will be introduced in the following section.

Moreover, an inner product between any two p-forms, A and B, can be given by using the wedge product and Hodge duality as

$$*A \wedge B = *B \wedge A = \frac{1}{p!}\ |A \cdot B| * 1 \tag{A.22}$$

in which

$$|A \cdot B| = A_{\mu_1 \ldots \mu_p} B^{\mu_1 \ldots \mu_p}. \tag{A.23}$$

This is an inner product between two tensors that always appears as kinetic terms in Lagrangian densities.

A.2 Spacetime Curvature from Vielbein Formalism

To deal with curved D-dimensional spacetime, general relativity considers curved spacetime as a D-dimensional differentiable real manifold $\mathcal{M}$, a smooth and continuous topological space that locally looks like Minkowski flat spacetime Mkw_D. Parameterized by some coordinate system, any distance on D-dimensional curved spacetime called metric or line element is written as

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu \tag{A.24}$$

where $g_{\mu\nu} = g_{\mu\nu}(x)$ is the metric tensor depending on coordinates x^μ. Since a differentiable real manifold locally looks like flat spacetime, there exists a Minkowski flat space called tangent space or Lorentz frame described by vielbein basis $e^{\hat{\mu}}(x)$ at each point $p \in \mathcal{M}$. Here, $\hat{\mu}, \hat{\nu} = \hat{0}, \hat{1}, ..., \widehat{D-1}$ are D-dimensional flat spacetime indices raised and lowered by $\eta^{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}} = \text{diag}\,(-, +, ..., +)$. The metric in this tangent space is given by

$$ds^2 = \eta_{\hat{\mu}\hat{\nu}} e^{\hat{\mu}}(x) e^{\hat{\nu}}(x) = \eta_{\hat{\mu}\hat{\nu}} e^{\hat{\mu}}_{\mu}(x) e^{\hat{\nu}}_{\nu}(x) dx^\mu dx^\nu. \tag{A.25}$$

In the last step, we have used

$$e^{\hat{\mu}}(x) = e^{\hat{\mu}}_{\mu}(x) dx^\mu \tag{A.26}$$

to describe the vielbein basis as a one-form in D-dimensional curved spacetime with components called vielbein $e^{\hat{\mu}}_{\mu}(x)$. Since the metric ds^2 is the same in every coordinate system (general coordinate transformation invariant), the following relation between the metric tensor and the vielbein can be derived

$$g_{\mu\nu} = \eta_{\hat{\mu}\hat{\nu}} e^{\hat{\mu}}_{\mu} e^{\hat{\nu}}_{\nu} \tag{A.27}$$

in which and also in the following we have suppressed an argument (x) for simplicity. As seen from (A.27), the vielbein can be interpreted as a "square-root" of the metric tensor so that $\sqrt{|g|} = e = \det\, e^{\hat{\mu}}_{\mu}$. It should be noted that curved and flat spacetime indices are related to each other via the vielbein component $e^{\hat{\mu}}_{\mu}$ and its inverse $e^{\mu}_{\hat{\mu}}$. Moreover, we practically use relation (A.27) to find vielbein components of a given tensor.

Unlike the usual formulation with the metric tensor $g_{\mu\nu}$, vielbein formalism extract the curvature of spacetime from the vielbein one-from. We start from the vielbein postulate

$$de^{\hat{\mu}} = -\omega^{\hat{\mu}}{}_{\hat{\nu}} \wedge e^{\hat{\nu}} \tag{A.28}$$

in which $\omega^{\hat{\mu}\hat{\nu}} = -\omega^{\hat{\nu}\hat{\mu}} = \omega_{\rho}{}^{\hat{\mu}\hat{\nu}} dx^{\rho}$ is also a one-form called spin connection. This postulate describes an exterior derivative of the vielbein one-from as a change of tangent space with respect to positions in curved spacetime represented by the spin connection.

To find the curvature of spacetime, we need the following curvature two-form calculated from the spin connection by

$$R^{\hat{\mu}\hat{\nu}}_{(2)} = d\omega^{\hat{\mu}\hat{\nu}} + \omega^{\hat{\mu}}{}_{\hat{\rho}} \wedge \omega^{\hat{\rho}\hat{\nu}}. \tag{A.29}$$

This two-form is defined as

$$R^{\hat{\mu}\hat{\nu}}_{(2)} = \frac{1}{2}\, R_{\rho\sigma}{}^{\hat{\mu}\hat{\nu}} dx^{\rho} \wedge dx^{\sigma} = \frac{1}{2}\, R_{\hat{\rho}\hat{\sigma}}{}^{\hat{\mu}\hat{\nu}} e^{\hat{\rho}} \wedge e^{\hat{\sigma}} \tag{A.30}$$

where the component $R_{\hat{\rho}\hat{\sigma}}{}^{\hat{\mu}\hat{\nu}} = e^{\rho}_{\hat{\rho}} e^{\sigma}_{\hat{\sigma}} R_{\rho\sigma}{}^{\hat{\mu}\hat{\nu}}$ is called Riemann curvature tensor measuring the curvature of spacetime as a deviation from flat spacetime.

By taking traces, the two quantities playing essential roles in the descriptions of curved spacetime can be derived. The first one is Ricci tensor

$$R_{\hat{\mu}\hat{\nu}} = R_{\hat{\mu}\hat{\rho}\hat{\nu}}{}^{\hat{\rho}}, \tag{A.31}$$

and the second one is called Ricci scalar

$$R = \eta^{\hat{\mu}\hat{\nu}} R_{\hat{\mu}\hat{\nu}}. \tag{A.32}$$

These two quantities appear in Einstein's field equations describing the relation between the curvature of spacetime and the distribution of energy and mass

$$R_{\hat{\mu}\hat{\nu}} - \frac{1}{2}\eta_{\hat{\mu}\hat{\nu}} R = T_{\hat{\mu}\hat{\nu}} \tag{A.33}$$

in which $T_{\hat{\mu}\hat{\nu}}$ is the energy-momentum tensor representing all energies, momentums, and also stresses in spacetime that can be sources of gravity.

APPENDIX B

SYMPLECTIC-MAJORANA SPINORS

Spinors in seven-dimensional spacetime are generally Dirac spinors carrying eigth complex components for each. These spinors are the corresponding representations of the Clifford algebra. The Clifford algebra is generated by Dirac gamma (8×8) matrices and can be written as

$$\{\gamma^{\hat{\mu}}, \gamma^{\hat{\nu}}\} = \gamma^{\hat{\mu}}\gamma^{\hat{\nu}} + \gamma^{\hat{\nu}}\gamma^{\hat{\mu}} = 2\eta^{\hat{\mu}\hat{\nu}}\mathbf{1}_8 \tag{B.1}$$

where $\eta^{\hat{\mu}\hat{\nu}} = \text{diag}(- + + + + + +)$. As in [62], we use the following explicit representation for the Dirac gamma matrices

$$\begin{aligned}
&\gamma^{\hat{0}} = i\sigma_2 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2, \quad \gamma^{\hat{1}} = \sigma_1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_2, \\
&\gamma^{\hat{2}} = \sigma_3 \otimes \sigma_1 \otimes \mathbf{1}_2, \quad \gamma^{\hat{3}} = \sigma_3 \otimes \sigma_3 \otimes \mathbf{1}_2, \\
&\gamma^{\hat{4}} = \sigma_3 \otimes \sigma_2 \otimes \sigma_1, \quad \gamma^{\hat{5}} = \sigma_3 \otimes \sigma_2 \otimes \sigma_2, \\
&\gamma^{\hat{6}} = \sigma_3 \otimes \sigma_2 \otimes \sigma_3
\end{aligned} \tag{B.2}$$

in which $\{\sigma_1, \sigma_2, \sigma_3\}$ are the usual Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{B.3}$$

The higher-rank gamma matrices are defined as an antisymmetric product

$$\gamma^{\hat{\mu}_1 \ldots \hat{\mu}_n} = \gamma^{[\hat{\mu}_1} \ldots \gamma^{\hat{\mu}_n]}. \tag{B.4}$$

For $\hat{\mu}_1 \neq \hat{\mu}_2 \neq \ldots \neq \hat{\mu}_n$, it can be written as a product

$$\gamma^{\hat{\mu}_1\hat{\mu}_2 \ldots \hat{\mu}_n} = \gamma^{\hat{\mu}_1}\gamma^{\hat{\mu}_2} \ldots \gamma^{\hat{\mu}_n} \tag{B.5}$$

due to the Clifford algebra (B.1). Moreover, one can check that the representation given in (B.2) satisfies the following identity

$$\gamma_* = \gamma^{\hat{0}\hat{1}\hat{2}\hat{3}\hat{4}\hat{5}\hat{6}} = \gamma^{\hat{0}}\gamma^{\hat{1}}\gamma^{\hat{2}}\gamma^{\hat{3}}\gamma^{\hat{4}}\gamma^{\hat{5}}\gamma^{\hat{6}} = \mathbf{1}_8. \tag{B.6}$$

With this explicit representation, Dirac, complex, and charge conjugation matrices can be respectively given by

$$A = \gamma^{\hat{0}}, \quad B = -i\gamma^{\hat{4}\hat{6}}, \quad C = i\gamma^{\hat{0}\hat{4}\hat{6}} \tag{B.7}$$

satisfying the following definitions

$$(\gamma^{\hat{\mu}})^\dagger = -A\gamma^{\hat{\mu}}A^{-1}, \quad (\gamma^{\hat{\mu}})^* = B\gamma^{\hat{\mu}}B^{-1}, \quad (\gamma^{\hat{\mu}})^T = -C\gamma^{\hat{\mu}}C^{-1} \tag{B.8}$$

as well as the properties

$$B^T = CA^{-1}, \quad B^*B = -\mathbf{1}_8, \quad C^T = -C^{-1} = -C^\dagger = C. \tag{B.9}$$

For seven-dimensional $N = 2$ gauged supergravity, fermionic fields and SUSY parameters are described by symplectic-Majorana (SM) spinors labeled by an $SU(2)_R$ doublet index $\alpha = 1, 2$. These SM spinors satisfy the following pseudo-reality condition in order to make sure that the amount of on-shell real degrees of freedom is sixteen

$$\zeta_\alpha = (\zeta^\alpha)^* = \varepsilon_{\alpha\beta}B\zeta^\beta \tag{B.10}$$

in which ζ^α is any SM spinor, $\varepsilon_{\alpha\beta}$ denotes the $SU(2)$-invariant Levi-Civita symbol, and B is the matrix involved in the complex conjugation of the Dirac gamma matrices given in (B.7).

For the maximal $N = 4$ theory, fermionic fields and SUSY parameters transform in representations of the local $SO(5)_R \sim USp(4)_R$ R-symmetry under which the total amount of on-shell real degrees of freedom is thirty-two. In this case, SM spinors carry an $USp(4)$ index $a = 1, 2, 3, 4$ and are subject to the condition

$$\overline{\zeta}_a^T = \Omega_{ab}C\zeta^b \tag{B.11}$$

where Ω_{ab} is the USp(4) symplectic form, C is the charge conjugation matrix given in (B.7), and the Dirac conjugate for an instant $USp(4)$ SM spinor ζ^a is defined by $\overline{\zeta} \equiv \zeta^\dagger\gamma^0$.

APPENDIX C

TRUNCATION ANSATZE

Essential formulae for truncations of eleven-dimensional supergravity on S^4 and type IIA theory on S^3 are assembled in this appendix. These truncations give rise to maximal $SO(5)$ and $CSO(4,0,1)$ gauged supergravities in seven dimensions. The complete S^4 and S^3 truncation ansatze have been given in [26, 27], and [77], respectively. By describing higher-dimensional fields in terms of lower-dimensional ones, truncation ansatze map field equations from the eleven- or ten-dimensional theory to the theories in seven dimensions. On the other hand, solutions to the seven-dimensional theories are also solutions to the higher-dimensional ones and vice versa. For the minimal $N = 2$ matter-coupled $SO(4)$ gauged theory, we can also embed the solutions into eleven-dimensional supergravity using the formula given in [74] where the S^4 truncation is further truncated.

C.1 Eleven-Dimensional Supergravity on S^4

In eleven-dimensional spacetime, there exists a unique supergravity theory [22] consists of a graviton $\hat{g}_{\mathcal{MN}}$, a gravitino $\hat{\Psi}_{\mathcal{M}}$, and a three-form potential $\hat{A}_{(3)}$. In this appendix, $\mathcal{M}$, $\mathcal{N}$, ... refer to higher-dimensional spacetime indices. The Lagrangian for the bosonic sector is given in the following differential form

$$\mathcal{L}_{11} = R\hat{*}1 - \frac{1}{2}\hat{*}\hat{F}_{(4)} \wedge \hat{F}_{(4)} - \frac{1}{6}\hat{F}_{(4)} \wedge \hat{F}_{(4)} \wedge \hat{A}_{(3)} \tag{C.1}$$

where the four-form field strength is $\hat{F}_{(4)} = d\hat{A}_{(3)}$. The associated field equations derived from this Lagrangian are

$$0 = R_{\mathcal{MN}} - \frac{1}{12}\left(\hat{F}_{\mathcal{MPQR}}\hat{F}_{\mathcal{N}}{}^{\mathcal{PQR}} - \frac{1}{12}g_{\mathcal{MN}}\hat{F}_{\mathcal{PQRS}}\hat{F}^{\mathcal{PQRS}}\right), \tag{C.2}$$

$$0 = d\hat{*}\hat{F}_{(4)} - \frac{1}{2}\hat{F}_{(4)} \wedge \hat{F}_{(4)}. \tag{C.3}$$

Here, we are using $\hat{*}$ to denote a Hodge duality in eleven-dimensional spacetime in contrast to $*$, which refers to seven-dimensional Hodge duality.

To truncate the eleven-dimensional supergravity on S^4 leading to the maximal $SO(5)$ gauged supergravity in seven dimensions, the following ansatz for the eleven-dimensional metric is needed

$$d\hat{s}_{11}^2 = \Delta^{\frac{1}{3}} ds_7^2 + \frac{1}{\hat{g}^2}\Delta^{-\frac{2}{3}} T^{-1}_{MN} D\mu^M D\mu^N \tag{C.4}$$

where μ^M, $M = 1, 2, 3, 4, 5$, are coordinates on S^4 satisfying $\mu^M \mu^M = 1$. The warped factor is defined by

$$\Delta = T_{MN}\mu^M \mu^N \tag{C.5}$$

in which T_{MN} is a unimodular 5×5 symmetric matrix describing scalar fields in $SL(5)/SO(5)$ coset. The ansatz for the four-form field strength reads

$$\begin{aligned}
\hat{F}_{(4)} \;=\;& \frac{1}{\hat{g}^3}\Delta^{-2}\left[\frac{1}{3!}\varepsilon_{M_1\ldots M_5}\mu^M \mu^N T^{M_1 M} DT^{M_2 N} \wedge D\mu^{M_3} \wedge D\mu^{M_4} \wedge D\mu^{M_5}\right] \\
& -\frac{1}{\hat{g}^3}\Delta^{-2}\mathcal{U}\epsilon_{(4)} + \frac{1}{4\hat{g}^2}\Delta^{-1}\varepsilon_{M_1\ldots M_5}\tilde{F}^{M_1 M_2}_{(2)} \wedge D\mu^{M_3} \wedge D\mu^{M_4} T^{M_5 N}\mu^N \\
& +\frac{1}{\hat{g}}\tilde{S}^M_{(3)} \wedge D\mu^M - T_{MN} * \tilde{S}^M_{(3)}\mu^N \,.
\end{aligned} \tag{C.6}$$

In these equations, we have used the following definitions

$$\mathcal{U} = 2T_{MN}T_{NP}\mu^M \mu^P - \Delta T_{MM}, \tag{C.7}$$

$$\epsilon_{(4)} = \frac{1}{4!}\varepsilon_{M_1\ldots M_5}\mu^{M_1} D\mu^{M_2} \wedge D\mu^{M_3} \wedge D\mu^{M_4} \wedge D\mu^{M_5}, \tag{C.8}$$

$$D\mu^M = d\mu^M + \hat{g}\tilde{A}^{MN}_{(1)}\mu^N, \quad \tilde{F}^{MN}_{(2)} = d\tilde{A}^{MN}_{(1)} + \hat{g}\tilde{A}^{MP}_{(1)} \wedge \tilde{A}^{PN}_{(1)}, \tag{C.9}$$

$$DT_{MN} = dT_{MN} + \hat{g}\tilde{A}^{MP}_{(1)} T_{PN} + \hat{g}\tilde{A}^{NP}_{(1)} T_{MP}\,. \tag{C.10}$$

In this appendix, the vector, two-form, and massive three-form fields in seven-dimensional truncated theories are denoted by $\tilde{A}^{MN}_{(1)}$, $\tilde{F}^{MN}_{(2)}$, and $\tilde{S}^M_{(3)}$ to avoid confusion with those appearing in (2.65).

Imposing these ansatze into the field equations, (C.2) - (C.3), we can find the seven-dimensional field equations given in [77] that can be derived from the following Lagrangian

$$\begin{aligned}\mathcal{L}_{S^4} &= \frac{1}{2}R*1+\frac{1}{8}*DT^{-1}_{MN}\wedge DT_{MN}-\frac{1}{4}\hat{g}^2\left[2T_{MN}T_{MN}-(T_{MM})^2\right]*1\\ &\quad +\frac{1}{4\hat{g}}\tilde{S}^M_{(3)}\wedge\tilde{H}^M_{(4)}-\frac{1}{16\hat{g}}\varepsilon_{MN_1\ldots N_4}\tilde{S}^M_{(3)}\wedge\tilde{F}^{N_1N_2}_{(2)}\wedge\tilde{F}^{N_4N_4}_{(2)}+\frac{1}{2\hat{g}}\Omega_{(7)}\\ &\quad -\frac{1}{8}T^{-1}_{MP}T^{-1}_{NQ}*F^{MN}_{(2)}\wedge F^{PQ}_{(2)}-\frac{1}{4}T_{MN}*\tilde{S}^M_{(3)}\wedge\tilde{S}^N_{(3)}\end{aligned}\tag{C.11}$$

in which $\Omega_{(7)}$ is the Chern-Simons term whose explicit form can be found in [32] and

$$\tilde{H}^M_{(4)}=d\tilde{S}^M_{(3)}+\hat{g}\tilde{A}^{MN}_{(1)}\wedge\tilde{S}^N_{(3)}.\tag{C.12}$$

Comparing this Lagrangian with (2.65) together with $Y_{MN}=\delta_{MN}$ and $Z^{MN,P}=0$ for $SO(5)$ gauge group, we can find the following relations between the seven-dimensional fields and parameters obtained from the S^4 truncation and those in seven-dimensional gauged supergravity of [63]

$$T_{MN}=\mathcal{M}^{MN},\quad \tilde{S}^M_{(3)}=2\mathcal{H}_{(3)M},\quad \tilde{F}^{MN}_{(2)}=4\mathcal{F}^{MN}_{(2)},\quad \hat{g}=\frac{1}{4}g\,.\tag{C.13}$$

Moreover, we will consider a further truncation giving rise to the matter-coupled $SO(4)$ gauged supergravity in seven dimensions [74]. By breaking the gauge group $SO(5)$ to $SO(4)$, $SO(5)$ gamma matrices, $\Gamma_{\hat{i}}$ with $\hat{i}=1,2,3,4,5$, are decomposed as $\Gamma_{\hat{i}}=(\Gamma_R,\Gamma_5)$ in which $R=1,2,3,4$ is an $SO(4)$ index. On the other hand, $\Gamma_5=\Gamma_1\Gamma_2\Gamma_3\Gamma_4$ acts as the chilarity matrix of $SO(4)$, $\Gamma_5\Psi^\pm=\pm\Psi^\pm$ for an instant $SO(4)$ spinor $\Psi=\Psi^++\Psi^-$. The following truncation is made on fermionic fields and SUSY parameters in the maximal $SO(5)$ gauged theory to reduce $N=4$ SUSY to $N=2$

$$\epsilon^-=\psi^-_\mu=\lambda^-_5=\lambda^+_R=0\tag{C.14}$$

in which $\psi^\pm_\mu$ are the gravitini and $\lambda^\pm_{\hat{i}}$ are the spin-$\frac{1}{2}$ fields that are decompsed into $\lambda^\pm_{\hat{i}}=(\lambda^\pm_R,\lambda^\pm_5)$. In the following, all $\pm$ superscript will be suppressed.

For bosonic fields, we set $T_{5\alpha}$, $\tilde{S}^\alpha_{(3)}$, and $\tilde{F}^{5\alpha}_{(2)}$ to zero while the index M is also split as $(\alpha,5)$ with $\alpha=1,2,3,4$. The corresponding scalar truncation is given by $T_{MN}=(T_{\alpha\beta},T_{55})=(X\tilde{T}_{\alpha\beta},X^{-4})$ in which X will be related to the $N=2$ dilaton scalar field and $\tilde{T}_{\alpha\beta}$ is a symmetric scalar matrix with unit determinant describing nine scalars in $SL(4,\mathbb{R})/SO(4)$ coset.

With these truncations, the bosonic Lagrangian for the resulting $N=2$, $SO(4)$ gauged supergravity reads

$$\begin{aligned}\mathcal{L}_{N=2} &= \frac{1}{2}R*1-\frac{1}{8}X^{-2}\tilde{T}^{-1}_{\alpha\gamma}\tilde{T}^{-1}_{\beta\delta}*\tilde{F}^{\alpha\beta}_{(2)}\wedge\tilde{F}^{\gamma\delta}_{(2)}-\frac{1}{8}\tilde{T}^{-1}_{\alpha\beta}*D\tilde{T}_{\beta\gamma}\wedge\tilde{T}^{-1}_{\gamma\delta}D\tilde{T}_{\delta\alpha}\\ &\quad -\frac{1}{4}X^{4}*F_{(4)}\wedge F_{(4)}+\frac{1}{16}\varepsilon_{\alpha\beta\gamma\delta}A_{(3)}\wedge\tilde{F}^{\alpha\beta}_{(2)}\wedge\tilde{F}^{\gamma\delta}_{(2)}-\frac{5}{2}X^{-2}*dX\wedge dX\\ &\quad -\frac{1}{4}\hat{g}F_{(4)}\wedge A_{(3)}-\mathbf{V}*1 \end{aligned}\qquad\text{(C.15)}$$

in which we have imposed

$$\tilde{S}^{5}_{(3)}=-\hat{g}A_{(3)}+\omega_{(3)}\qquad\text{(C.16)}$$

where $F_{(4)}=dA_{(3)}$ and $\omega_{(3)}$ is the Chern-Simons term, whose explicit form is given in [74]. The scalar potential is given by

$$\mathbf{V}=\frac{1}{4}\hat{g}^{2}\left[X^{-8}-2X^{-3}\tilde{T}_{\alpha\alpha}+2X^{2}\left(\tilde{T}_{\alpha\beta}\tilde{T}_{\alpha\beta}-\frac{1}{2}\tilde{T}^{2}_{\alpha\alpha}\right)\right].\qquad\text{(C.17)}$$

From the eleven-dimensional supergravity, this $SO(4)$ gauged theory can be obtained through the following truncation ansatze

$$\begin{aligned}d\hat{s}^{2}_{11} &= \Delta^{\frac{1}{3}}ds^{2}_{7}+\frac{2}{\hat{g}^{2}}\Delta^{-\frac{2}{3}}X^{3}\left[X\cos^{2}\xi+X^{-4}\sin^{2}\xi\tilde{T}^{-1}_{\alpha\beta}\mu^{\alpha}\mu^{\beta}\right]d\xi^{2}\\ &\quad -\frac{1}{\hat{g}^{2}}\Delta^{-\frac{2}{3}}X^{-1}\tilde{T}^{-1}_{\alpha\beta}\sin\xi\mu^{\alpha}d\xi D\mu^{\beta}+\frac{1}{2\hat{g}^{2}}\Delta^{-\frac{2}{3}}X^{-1}\tilde{T}^{-1}_{\alpha\beta}\cos^{2}\xi D\mu^{\alpha}D\mu^{\beta}\end{aligned}\qquad\text{(C.18)}$$

$$\begin{aligned}\hat{F}_{(4)} &= F_{(4)}\sin\xi+\frac{1}{\hat{g}}X^{4}\cos\xi*F_{(4)}\wedge\xi+\frac{1}{\hat{g}^{3}}\Delta^{-2}\mathcal{U}\cos^{5}\xi d\xi\wedge\epsilon_{(3)}\\ &\quad +\frac{\Delta^{-2}}{6\hat{g}^{3}X^{4}}\varepsilon_{\alpha\beta\gamma\delta}\sin\xi\cos^{4}\xi\mu^{\kappa}\left[5\tilde{T}^{\alpha\kappa}dX+XD\tilde{T}^{\alpha\kappa}\right]\wedge D\mu^{\beta}\wedge D\mu^{\gamma}\wedge D\mu^{\delta}\\ &\quad +\frac{\Delta^{-2}}{2\hat{g}^{3}}\varepsilon_{\alpha\beta\gamma\delta}\cos^{3}\xi\mu^{\kappa}\mu^{\lambda}\Big[\cos^{2}\xi X^{2}\tilde{T}^{\alpha\kappa}D\tilde{T}^{\beta\lambda}-\sin^{2}\xi X^{-3}\delta^{\beta\lambda}D\tilde{T}^{\alpha\kappa}\\ &\quad -5\sin^{2}\xi\tilde{T}^{\alpha\kappa}X^{-4}\delta^{\beta\lambda}dX\Big]\wedge D\mu^{\gamma}\wedge D\mu^{\delta}\wedge d\xi+\frac{X^{-4}}{2\hat{g}^{2}}\cos\xi\varepsilon_{\alpha\beta\gamma\delta}\times\\ &\quad \left[\frac{1}{2}\cos\xi\sin\xi D\mu^{\gamma}-\left(\sin^{2}\xi\mu^{\gamma}+X^{6}\cos^{2}\xi\tilde{T}^{\gamma\kappa}\mu^{\kappa}\right)d\xi\right]\wedge\tilde{F}^{\alpha\beta}_{(2)}\wedge D\mu^{\delta}\end{aligned}\qquad\text{(C.19)}$$

in which the S^4 coordinates μ^M are split to be $\mu^M = (\cos\xi\mu^\alpha, \sin\xi)$ with μ^α being coordinates on S^3 satisfying $\mu^\alpha\mu^\alpha = 1$. The following definitions are also used in the above ansatze

$$\epsilon_{(3)} = \frac{1}{3!}\varepsilon_{\alpha\beta\gamma\delta}\mu^\alpha D\mu^\beta \wedge D\mu^\gamma \wedge D\mu^\delta, \tag{C.20}$$

$$D\mu^\alpha = d\mu^\alpha + \hat{g}\tilde{A}^{\alpha\beta}_{(1)}\mu^\beta, \quad \Delta = \cos^2\xi X\tilde{T}_{\alpha\beta}\mu^\alpha\mu^\beta + X^{-4}\sin^2\xi, \tag{C.21}$$

$$\begin{aligned}\mathcal{U} &= \cos^2\xi X^2\mu^\alpha\mu^\beta(2\tilde{T}_{\alpha\gamma}\tilde{T}_{\gamma\beta} - \tilde{T}_{\alpha\beta}\tilde{T}_{\gamma\gamma} - X^{-5}\tilde{T}_{\alpha\beta}) \\ &\quad + \sin^2\xi(X^{-8} - X^{-3}\tilde{T}_{\alpha\alpha}).\end{aligned} \tag{C.22}$$

To identify $\tilde{T}^{-1}_{\alpha\beta}$ to the $SO(3,3)/SO(3)\times SO(3)$ coset representative $L_I{}^A = (L_I{}^i, L_I{}^r)$ used in the main text, we write $\tilde{T}^{-1}_{\alpha\beta}$ in the form of the $SL(4,\mathbb{R})/SO(4)$ coset representative $\mathcal{V}_\alpha{}^R$

$$\tilde{T}^{-1}_{\alpha\beta} = \mathcal{V}_\alpha{}^R\mathcal{V}_\beta{}^S\delta_{RS}\,. \tag{C.23}$$

Due to the isomorphisms $SO(3,3)\sim SL(4,\mathbb{R})$ and $SO(4)\sim SO(3)\times SO(3)$, the $SL(4,\mathbb{R})/SO(4)$ coset representative $\mathcal{V}_\alpha{}^R$ is related to $SO(3,3)/SO(3)\times SO(3)$ coset by the relation

$$L_I{}^A = \frac{1}{4}\zeta_I^{\alpha\beta}\eta^A_{RS}\mathcal{V}_\alpha{}^R\mathcal{V}_\beta{}^S \tag{C.24}$$

in which ζ^I and η^A are chirally projected gamma matrices of $SO(3,3)$ satisfying

$$(\zeta^I)_{\alpha\beta}(\zeta^J)^{\alpha\beta} = -4\eta^{IJ} \quad \text{and} \quad (\zeta^I)_{\alpha\beta}(\zeta_I)_{\gamma\delta} = -2\varepsilon_{\alpha\beta\gamma\delta} \tag{C.25}$$

where $\zeta^{I\alpha\beta} = (\zeta^i_{\alpha\beta}, -\zeta^r_{\alpha\beta})$ see more detail in [38]. Note that η^A_{RS} also satisfy similar relations which we will not repeat them here. We use the following choice of $\zeta^I_{\alpha\beta}$

$$\begin{aligned}\zeta^1 &= -i\sigma_2\otimes\sigma_1, \quad \zeta^2 = -i\sigma_2\otimes\sigma_3, \quad \zeta^3 = i\mathbf{I}_2\otimes\sigma_2, \\ \zeta^4 &= i\sigma_1\otimes\sigma_2, \quad\quad \zeta^5 = -i\sigma_2\otimes\mathbf{I}_2, \quad \zeta^6 = i\sigma_3\otimes\sigma_2\,.\end{aligned} \tag{C.26}$$

All these ingredients lead to the following identification of the fields and parameters in seven and eleven dimensions

$$\begin{aligned}g_2 &= g_1 = 16h = 2\hat{g}, \quad X = e^{-\frac{\sigma}{2}}, \\ H_{(3)} &= \frac{1}{\sqrt{2}}A_{(3)}, \quad \tilde{A}^{\alpha\beta}_{(1)} = \zeta^{\alpha\beta}_I A^I_{(1)}\,.\end{aligned} \tag{C.27}$$

C.2 Type IIA Supergravity on S^3

The low-energy effective theory of type IIA string theory is ten-dimensional type IIA supergravity obtained from a dimensional reduction of eleven-dimensional supergravity on a circle [96]. The field content of type IIA supergravity comprises a graviton $\hat{g}_{\mathcal{MN}}$, a scalar $\hat{\varphi}$, a Ramond-Ramond (R-R) one-form potential $\hat{A}_{(1)}$, an Neveu Schwarz-Neveu Schwarz (NS-NS) two-form potential $\hat{B}_{(2)}$, an R-R three-form potential $\hat{A}_{(3)}$ together with two gavitini $\hat{\Psi}^i_{\mathcal{M}}$ and two dilatini $\hat{\lambda}_i$ with $i = 1, 2$. The bosonic Lagrangian of this theory is given by

$$\begin{aligned}\mathcal{L}_{\text{IIA}} &= R\bar{*}1 - \frac{1}{2}\bar{*}d\hat{\varphi} \wedge d\hat{\varphi} - \frac{1}{2}e^{\frac{3}{2}\hat{\varphi}}\bar{*}\hat{F}_{(2)} \wedge \hat{F}_{(2)} - \frac{1}{2}e^{\frac{1}{2}\hat{\varphi}}\bar{*}\hat{F}_{(4)} \wedge \hat{F}_{(4)} \\ &\quad -\frac{1}{2}e^{-\hat{\varphi}}\bar{*}\hat{H}_{(3)} \wedge \hat{H}_{(3)} + \frac{1}{2}d\hat{A}_{(3)} \wedge d\hat{A}_{(3)} \wedge \hat{B}_{(2)} \end{aligned} \tag{C.28}$$

where $\bar{*}$ denotes ten-dimensional Hodge duality. The corresponding field strengths of the differential form potentials are given by

$$\hat{F}_{(4)} = d\hat{A}_{(3)} - d\hat{B}_{(2)} \wedge \hat{A}_{(1)}, \quad \hat{H}_{(3)} = d\hat{B}_{(2)}, \text{ and } \hat{F}_{(2)} = d\hat{A}_{(1)}. \tag{C.29}$$

The consistent truncation of type IIA supergravity on S^3 has been obtained in [77] by taking a degenerate limit of the S^4 truncation of eleven-dimensional supergravity. To write down this truncation ansatz, we first split the index M as $M = (i, 5)$, $i = 1, 2, 3, 4$. The scalar matrix of $SL(5)/SO(5)$ coset is then given by

$$T^{-1}_{MN} = \begin{pmatrix} \Phi^{-\frac{1}{4}}M^{-1}_{ij} + \Phi\chi_i\chi_j & \Phi\chi_i \\ \Phi\chi_j & \Phi \end{pmatrix} \tag{C.30}$$

where M_{ij} is a unimodular 4×4 symmetric matrix describing the $SL(4)/SO(4)$ coset. The fields χ_i and Φ are axion and dilaton scalars, respectively.

The truncation ansatze for the ten-dimensional metric, dilaton, and field strength tensors of various form fields are given by

$$d\hat{s}^2_{10} = \Phi^{\frac{3}{16}}\Delta^{\frac{1}{4}}ds^2_7 + \frac{1}{\hat{g}^2}\Phi^{-\frac{5}{16}}\Delta^{-\frac{3}{4}}M^{-1}_{ij}D\mu^i D\mu^j, \tag{C.31}$$

$$e^{2\hat{\varphi}} = \Delta^{-1}\Phi^{\frac{5}{4}}, \qquad \hat{F}_{(2)} = G^i_{(1)} \wedge D\mu^i + \hat{g}\mu^i G^i_{(2)}, \tag{C.32}$$

$$\hat{H}_{(3)} = \frac{1}{\hat{g}^3}\Delta^{-2}\left[-\mathcal{U}\epsilon_{(3)} + \frac{1}{2}\varepsilon_{i_1 i_2 i_3 i_4} M_{i_1 j}\mu^j\mu^k DM_{i_2 k}\wedge D\mu^{i_3}\wedge D\mu^{i_4}\right]$$
$$+\frac{1}{2\hat{g}^2}\Delta^{-1}\varepsilon_{ijkl}M_{im}\mu^m \tilde{F}^{jk}_{(2)}\wedge D\mu^l + \frac{1}{\hat{g}}\tilde{S}_{(3)}, \quad \text{(C.33)}$$
$$\hat{F}_{(4)} = \frac{1}{\hat{g}^3}\Delta^{-1}M_{ij}\mu^j G^i_{(1)}\wedge\epsilon_{(3)} + \frac{1}{2\hat{g}^2}\Delta^{-1}\varepsilon_{i_1 i_2 i_3 i_4}M_{i_4 j}\mu^j G^{i_1}_{(2)}\wedge D\mu^{i_2}\wedge D\mu^{i_3}$$
$$+M_{ij}\Phi^{\frac{1}{4}}\mu^j * G^i_{(3)} + \frac{1}{\hat{g}}G^i_{(3)}\wedge D\mu^i \quad \text{(C.34)}$$

with

$$\epsilon_{(3)} = \frac{1}{3!}\varepsilon_{ijkl}\mu^i D\mu^j\wedge D\mu^k\wedge D\mu^l, \quad D\mu^i = d\mu^i + \hat{g}\tilde{A}^{ij}_{(1)}\mu^j, \quad \text{(C.35)}$$
$$\mathcal{U} = 2M_{ij}M_{jk}\mu^i\mu^k - \Delta M_{ii}, \quad \Delta = M_{ij}\mu^i\mu^j, \quad \text{(C.36)}$$
$$G^i_{(1)} = D\chi_i + \hat{g}\tilde{A}^{i5}_{(1)}, \quad G^i_{(2)} = D\tilde{A}^{5i}_{(1)} + \chi_j F^{ji}_{(2)}, \quad \text{(C.37)}$$
$$G^i_{(3)} = \tilde{S}^i_{(3)} - \chi_i\tilde{S}_{(3)}, \quad \tilde{F}^{ij}_{(2)} = d\tilde{A}^{ij}_{(1)} + \hat{g}\tilde{A}^{ik}_{(1)}\wedge\tilde{A}^{kj}_{(1)}, \quad \text{(C.38)}$$
$$\tilde{S}_{(3)} = dB_{(2)} + \frac{1}{8}\varepsilon_{ijkl}\left(\tilde{F}^{ij}_{(2)}\wedge\tilde{A}^{kl}_{(1)} - \frac{1}{3}\hat{g}\tilde{A}^{ij}_{(1)}\wedge\tilde{A}^{km}_{(1)}\wedge\tilde{A}^{ml}_{(1)}\right) \quad \text{(C.39)}$$

where μ^i are coordinates on S^3 satisfying $\mu^i\mu^i = 1$ in this case.

Substituting these ansatze into the field equations derived from the Lagrangian (C.28), we obtain the resulting seven-dimensional field equations that can be derived from the following bosonic Lagrangian

$$\mathcal{L}_{S^3} = \frac{1}{2}R*1 - \frac{1}{8}\Phi^{-2}*d\Phi\wedge d\Phi - \frac{1}{8}M^{-1}_{ij}*DM_{jk}\wedge M^{-1}_{kl}DM_{li}$$
$$-\frac{1}{4}\Phi^{-1}*\tilde{S}_{(3)}\wedge\tilde{S}_{(3)} - \frac{1}{8}M^{-1}_{ik}M^{-1}_{jl}*\tilde{F}^{ij}_{(2)}\wedge\tilde{F}^{kl}_{(2)} - \frac{1}{4}\Phi M^{-1}_{ij}*G^i_{(2)}\wedge G^j_{(2)}$$
$$-\frac{1}{4}\Phi M_{ij}*G^i_{(1)}\wedge G^j_{(1)} - \frac{1}{4}\Phi M_{ij}*G^i_{(3)}\wedge G^j_{(3)} - \bar{\mathbf{V}}*1 + \frac{1}{2\hat{g}}\widetilde{\Omega}_{(7)} \quad \text{(C.40)}$$

where the explicit form for the scalar potential $\bar{\mathbf{V}}$ and topological terms of various form fields $\widetilde{\Omega}_{(7)}$ are given in [77]. By comparing this truncated Lagragian and the seven-dimensional gauged Lagrangian given in (2.65) with $Y_{ij} = \delta_{ij}$, $Y_{55} = 0$, and $Z^{MN,P} = 0$, we find the following relations

$$\hat{g} = \frac{1}{4}g, \quad \Phi = e^{8\phi_0}, \quad \chi_i = b_i, \quad M^{-1}_{ij} = \widetilde{\mathcal{M}}_{ij},$$
$$\tilde{S}^i_{(3)} = 2\mathcal{H}_{(3)i} \quad \tilde{F}^{ij}_{(2)} = 4\mathcal{F}^{ij}_{(2)}, \quad \tilde{F}^i_{(2)} = D\tilde{A}^{5i}_{(2)} = 4\mathcal{F}^{i5}_{(2)}. \quad \text{(C.41)}$$

VITAE

Patharadanai Nuchino was born in Mae Hong Son, Thailand, on 28 July 1991. His university education began in 2010 and has been supported by the Science Achievement Scholarship of Thailand (SAST). In 2014, he earned B.Sc. (1^{st} Class Honors, Gold Medal) in physics from Chaing Mai University. Then, he received M.Sc. in physics from Chulalongkorn University in 2016. After that, he has been continuously working for his Ph.D. at Chulalongkorn University. This book is his Ph.D. dissertation based on the following publications.

List of publications

1. Parinya Karndumri and Patharadanai Nuchino, “Supersymmetric solutions of matter-coupled 7D $N = 2$ gauged supergravity,” **Physical Review D** 98 (2018): 086012.

2. Parinya Karndumri and Patharadanai Nuchino, “Two-dimensional SCFTs from matter-coupled 7D $N = 2$ gauged supergravity,” **European Physical Journal C** 79 (2019): 652.

3. Parinya Karndumri and Patharadanai Nuchino, “Supersymmetric domain walls in $7D$ maximal gauged supergravity,” **European Physical Journal C** 79 (2019): 648.

4. Parinya Karndumri and Patharadanai Nuchino, “Supersymmetric solutions of 7D maximal gauged supergravity,” **Physical Review D** 101 (2020): 086012.

5. Parinya Karndumri and Patharadanai Nuchino, “Twisted compactifications of 6D field theories from maximal 7D gauged supergravity,” **European Physical Journal C** 80 (2020): 201.