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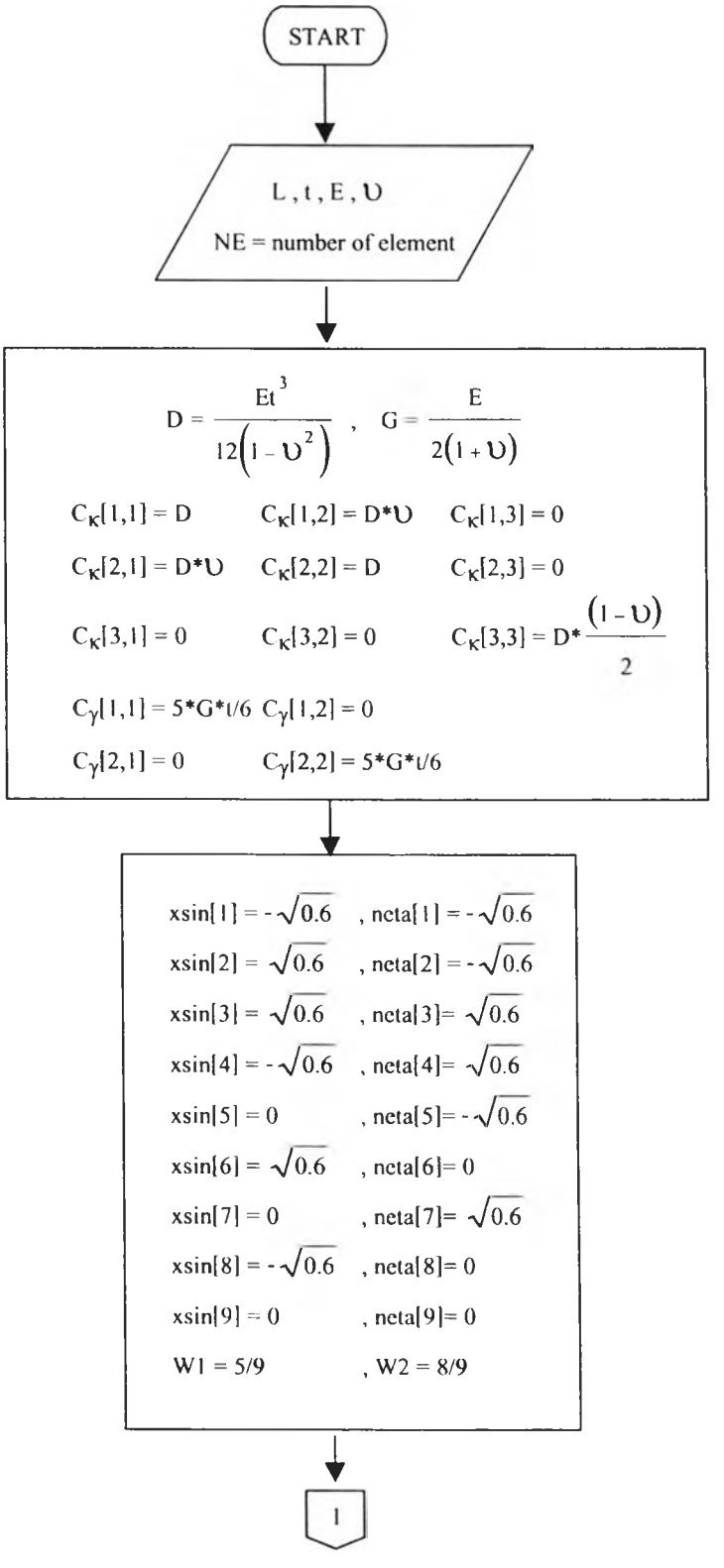
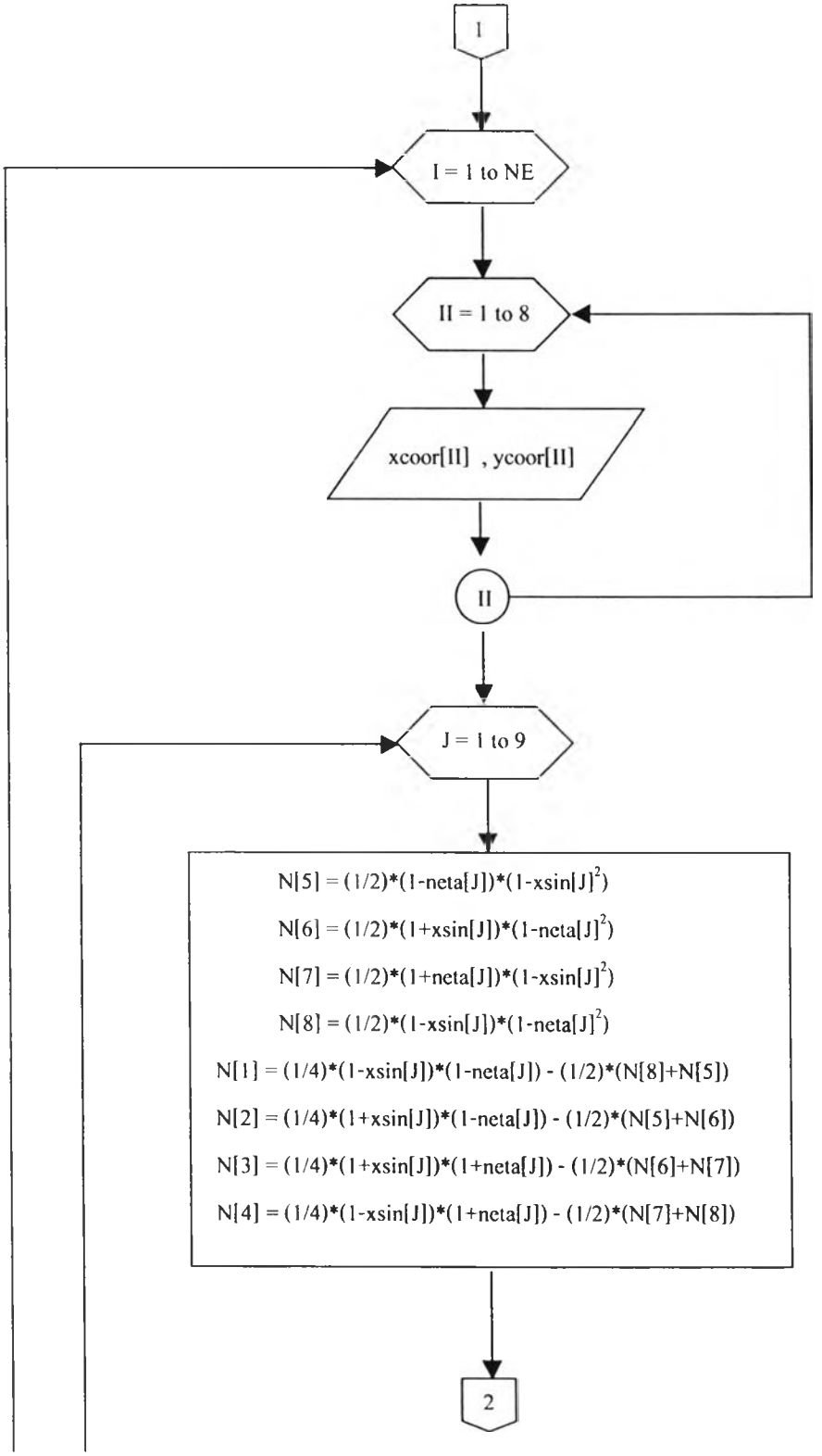
ภาคผนวก

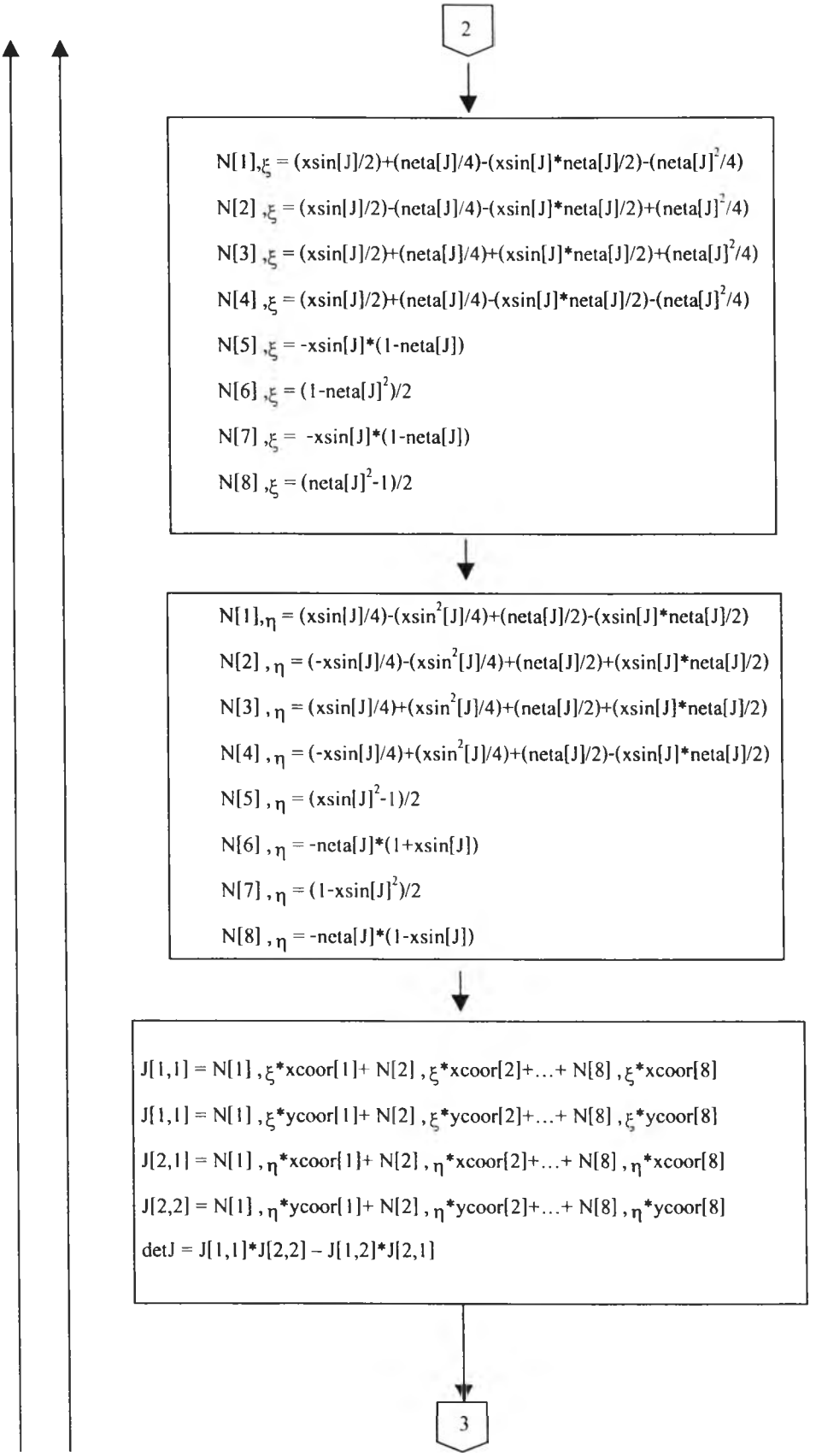
ภาคผนวก

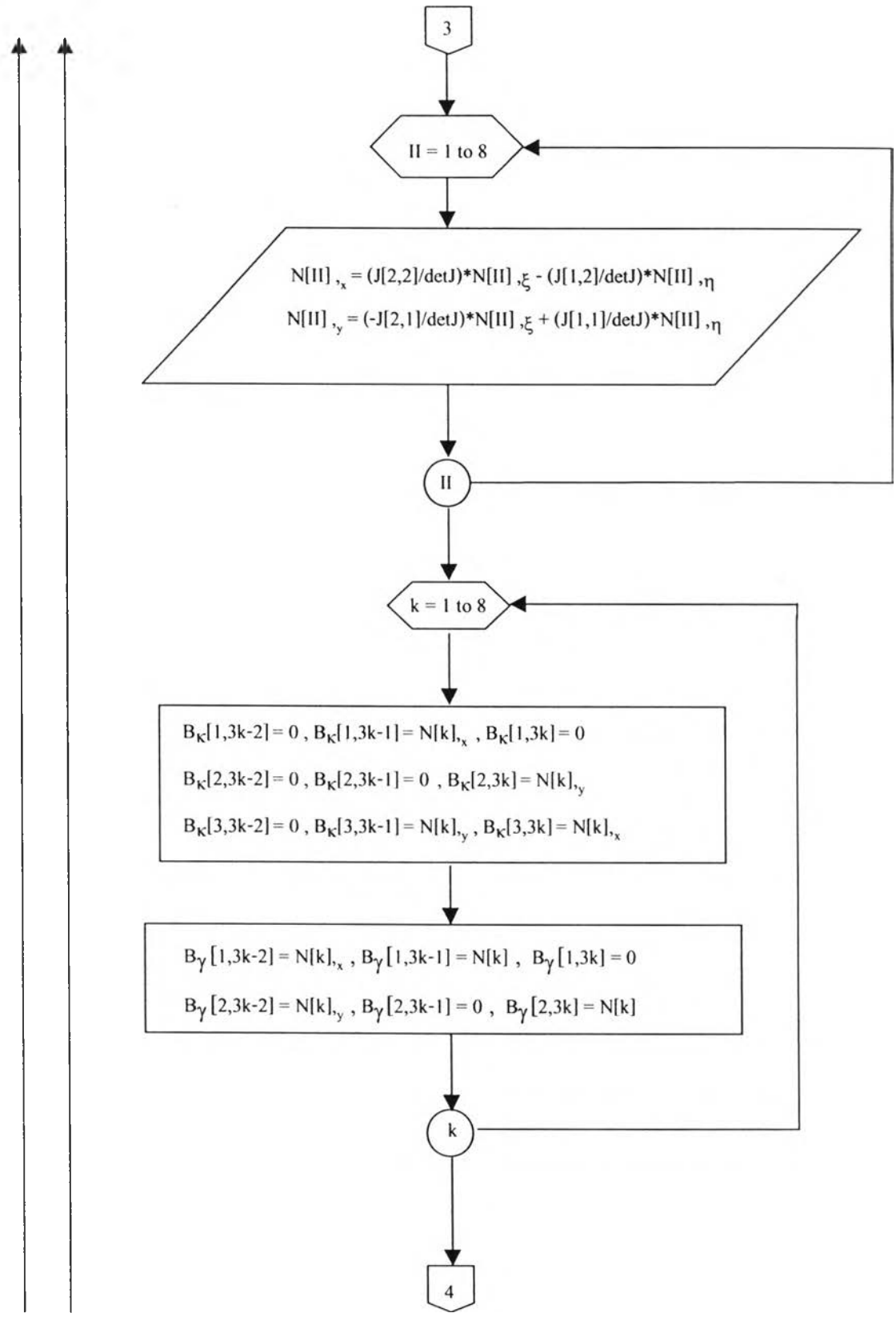
รายละเอียดผังงานการสร้างเมตริกซ์สติฟเนสของชิ้นส่วนต่าง ๆ

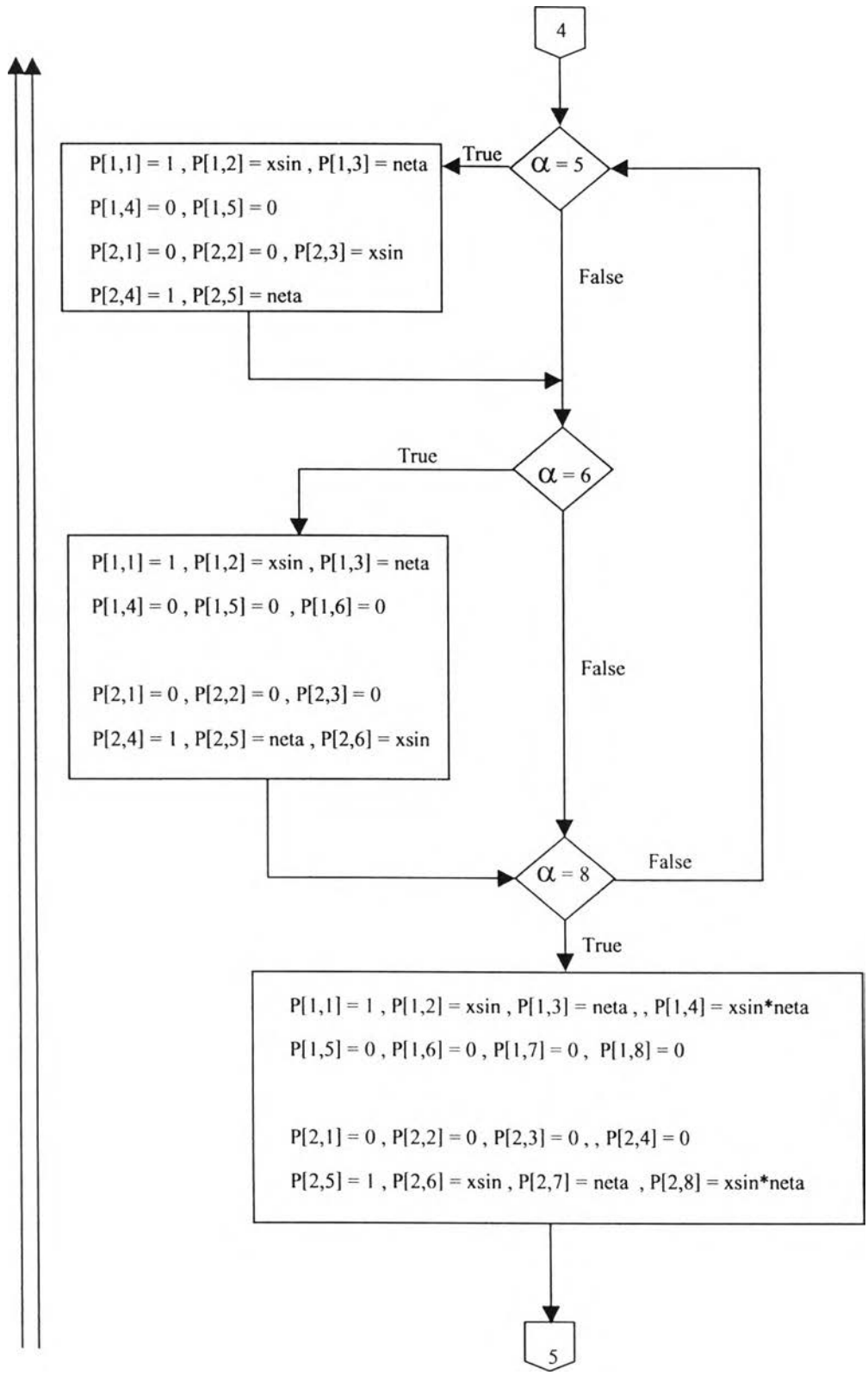
ก. ชิ้นส่วน PLAT8

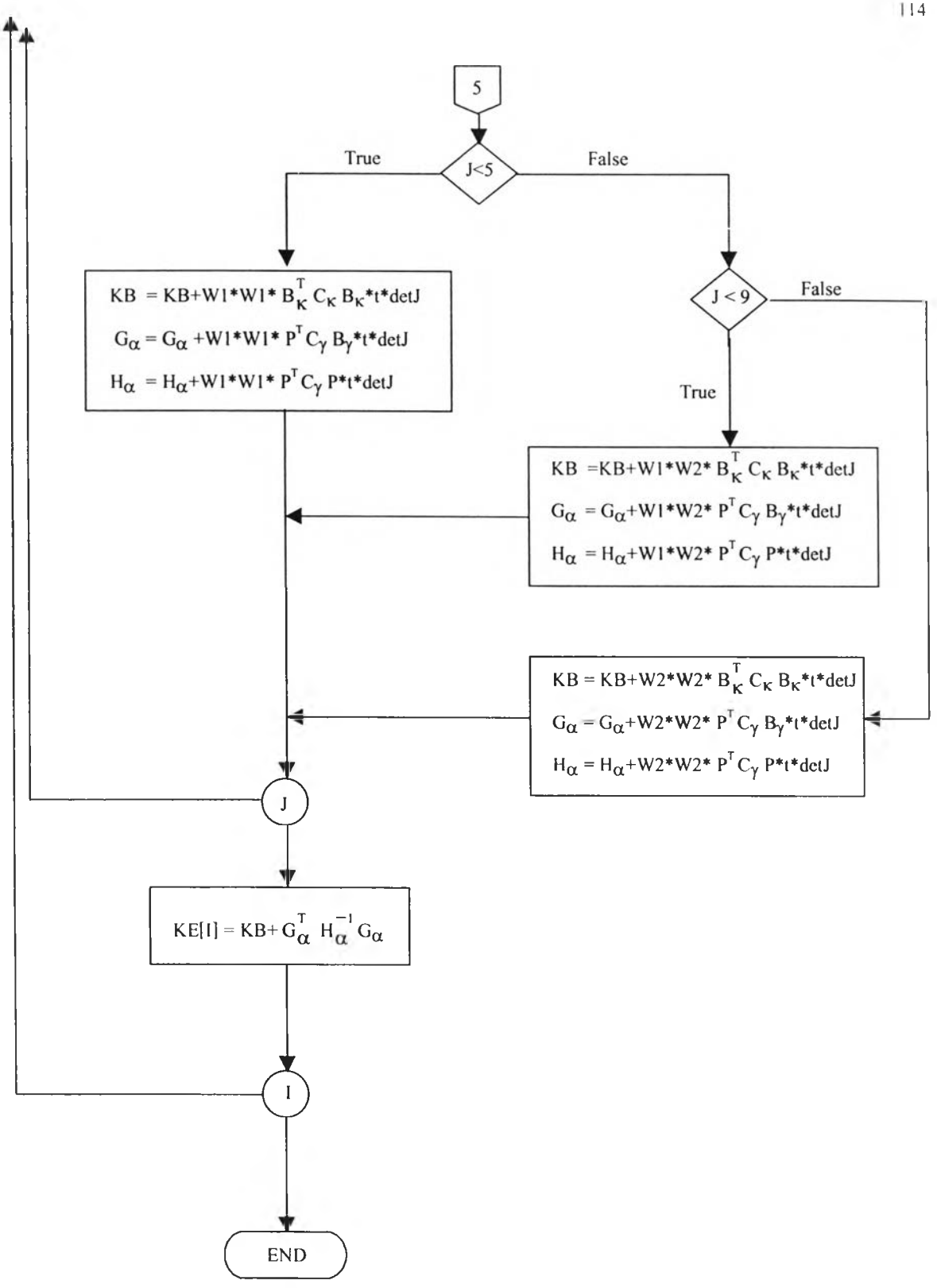



1
I = 1 to NE
II = 1 to 8
xcoor[II] , ycoor[II]
II
J = 1 to 9
N[5] = (1/2)*(1-neta[J])*(1-xsin[J]^2)
N[6] = (1/2)*(1+xsin[J])*(1-neta[J]^2)
N[7] = (1/2)*(1+neta[J])*(1-xsin[J]^2)
N[8] = (1/2)*(1-xsin[J])*(1-neta[J]^2)
N[1] = (1/4)*(1-xsin[J])*(1-neta[J]) - (1/2)*(N[8]+N[5])
N[2] = (1/4)*(1+xsin[J])*(1-neta[J]) - (1/2)*(N[5]+N[6])
N[3] = (1/4)*(1+xsin[J])*(1+neta[J]) - (1/2)*(N[6]+N[7])
N[4] = (1/4)*(1-xsin[J])*(1+neta[J]) - (1/2)*(N[7]+N[8])
2

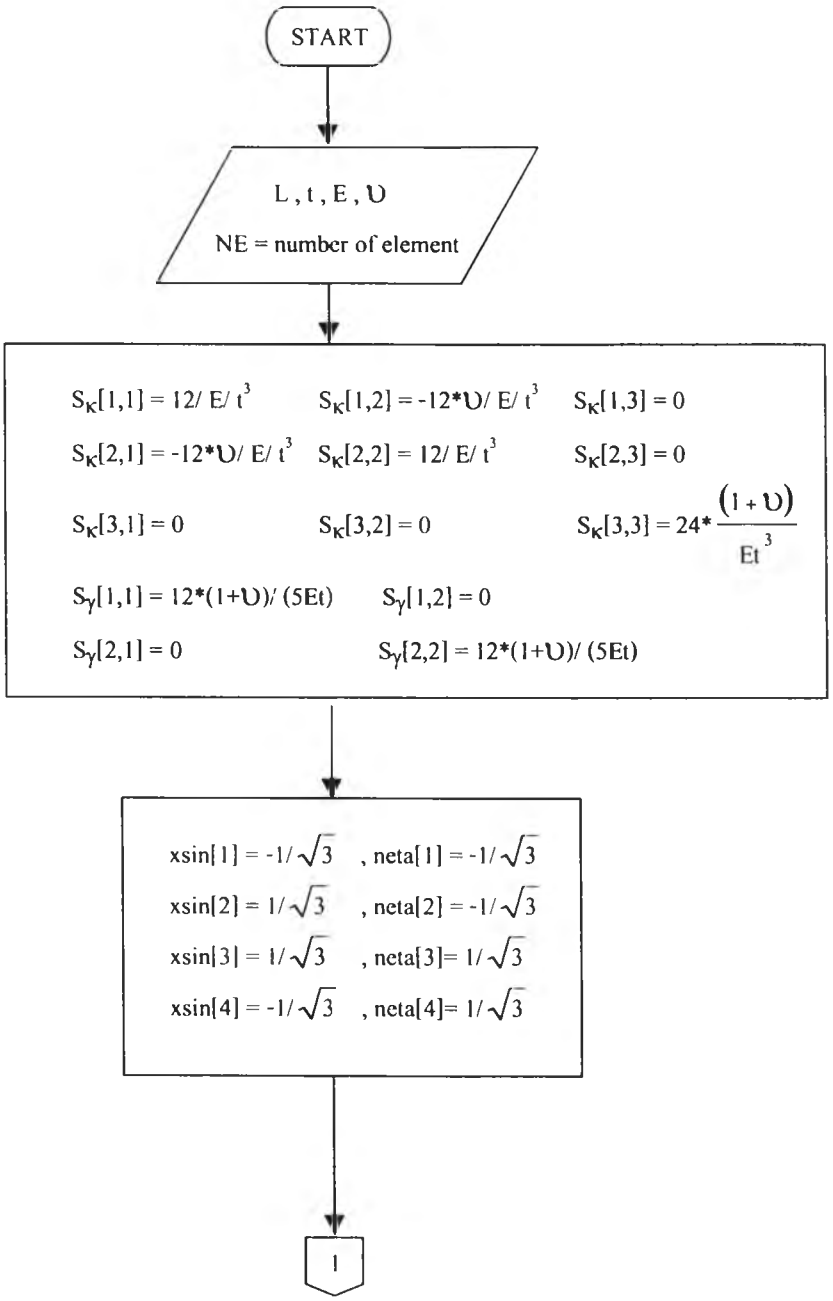

2
N[1],ξ = (xsin[J]/2)+(neta[J]/4)-(xsin[J]*neta[J]/2)-(neta[J]²/4)
N[2] ,ξ = (xsin[J]/2)-(neta[J]/4)-(xsin[J]*neta[J]/2)+(neta[J]²/4)
N[3] ,ξ = (xsin[J]/2)+(neta[J]/4)+(xsin[J]*neta[J]/2)+(neta[J]²/4)
N[4] ,ξ = (xsin[J]/2)+(neta[J]/4)-(xsin[J]*neta[J]/2)-(neta[J]²/4)
N[5] ,ξ = -xsin[J]*(1-neta[J])
N[6] ,ξ = (1-neta[J]²)/2
N[7] ,ξ = -xsin[J]*(1-neta[J])
N[8] ,ξ = (neta[J]²-1)/2
N[1],η = (xsin[J]/4)-(xsin²[J]/4)+(neta[J]/2)-(xsin[J]*neta[J]/2)
N[2] ,η = (-xsin[J]/4)-(xsin²[J]/4)+(neta[J]/2)+(xsin[J]*neta[J]/2)
N[3] ,η = (xsin[J]/4)+(xsin²[J]/4)+(neta[J]/2)+(xsin[J]*neta[J]/2)
N[4] ,η = (-xsin[J]/4)+(xsin²[J]/4)+(neta[J]/2)-(xsin[J]*neta[J]/2)
N[5] ,η = (xsin[J]²-1)/2
N[6] ,η = -neta[J]*(1+xsin[J])
N[7] ,η = (1-xsin[J]²)/2
N[8] ,η = -neta[J]*(1-xsin[J])
J[1,1] = N[1] ,ξ*xcoor[1]+ N[2] ,ξ*xcoor[2]+...+ N[8] ,ξ*xcoor[8]
J[1,1] = N[1] ,ξ*ycoor[1]+ N[2] ,ξ*ycoor[2]+...+ N[8] ,ξ*ycoor[8]
J[2,1] = N[1] ,η*xcoor[1]+ N[2] ,η*xcoor[2]+...+ N[8] ,η*xcoor[8]
J[2,2] = N[1] ,η*ycoor[1]+ N[2] ,η*ycoor[2]+...+ N[8] ,η*ycoor[8]
detJ = J[1,1]*J[2,2] – J[1,2]*J[2,1]
3

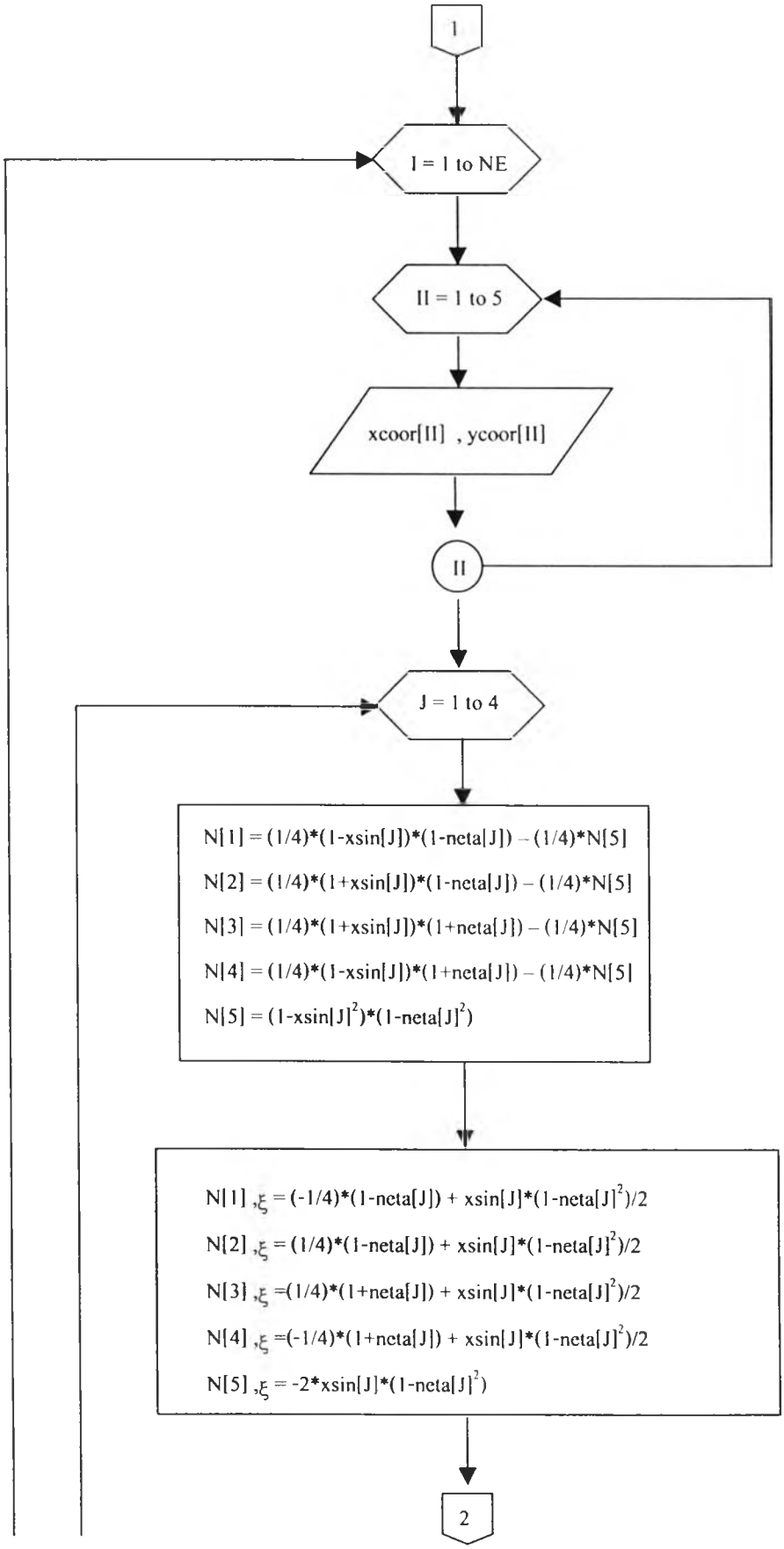

3
II = 1 to 8
N[II] ,x = (J[2,2]/detJ)*N[II] ,ξ - (J[1,2]/detJ)*N[II] ,η
N[II] ,y = (-J[2,1]/detJ)*N[II] ,ξ + (J[1,1]/detJ)*N[II] ,η
II
k = 1 to 8
Bκ[1,3k-2] = 0 , Bκ[1,3k-1] = N[k],x , Bκ[1,3k] = 0
Bκ[2,3k-2] = 0 , Bκ[2,3k-1] = 0 , Bκ[2,3k] = N[k],y
Bκ[3,3k-2] = 0 , Bκ[3,3k-1] = N[k],y , Bκ[3,3k] = N[k],x
Bγ[1,3k-2] = N[k],x , Bγ[1,3k-1] = N[k] , Bγ[1,3k] = 0
Bγ[2,3k-2] = N[k],y , Bγ[2,3k-1] = 0 , Bγ[2,3k] = N[k]
k
4

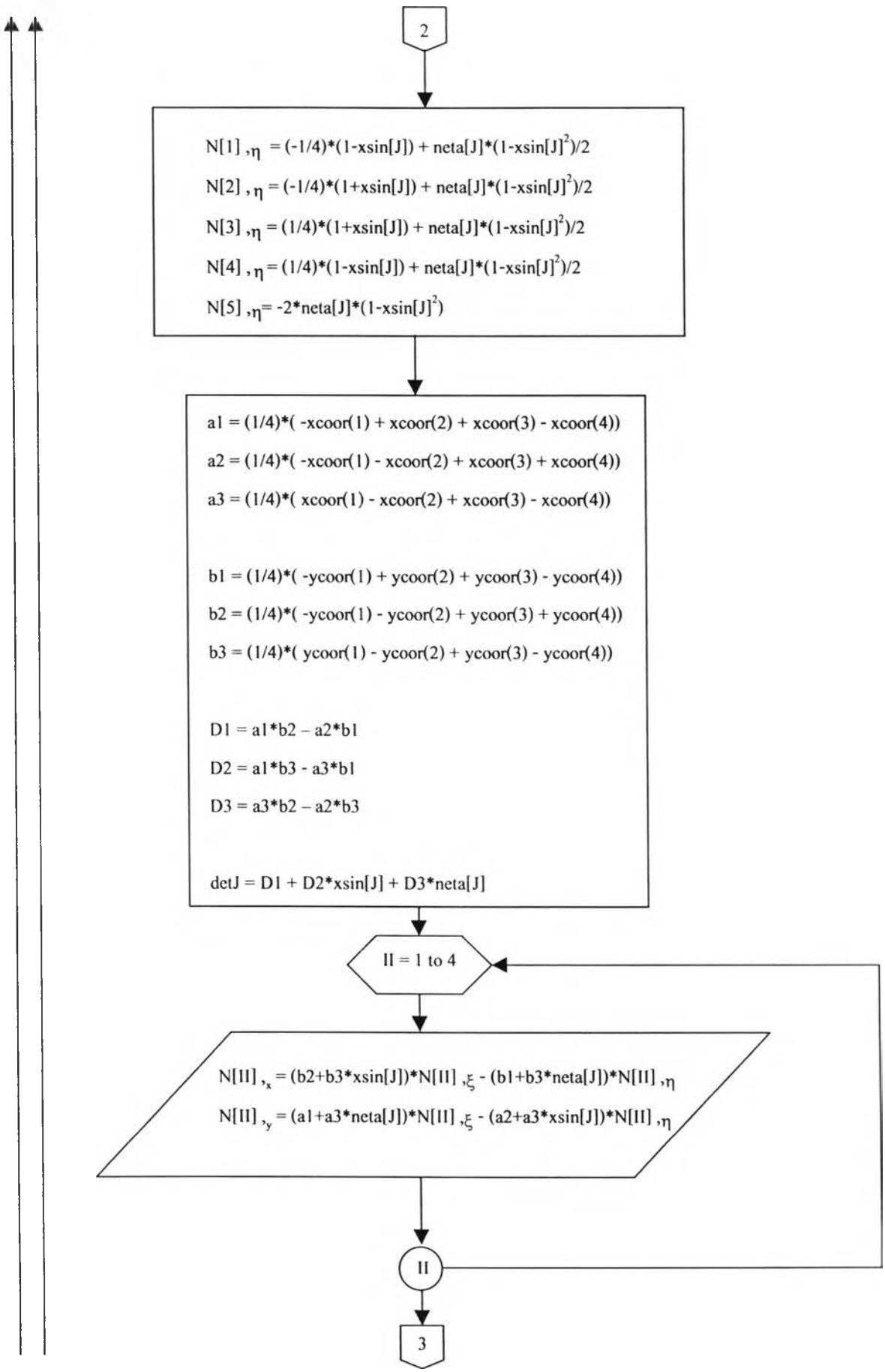

4
α = 5
True
False
P[1,1] = 1 , P[1,2] = xsin , P[1,3] = neta
P[1,4] = 0 , P[1,5] = 0
P[2,1] = 0 , P[2,2] = 0 , P[2,3] = xsin
P[2,4] = 1 , P[2,5] = neta
α = 6
True
False
P[1,1] = 1 , P[1,2] = xsin , P[1,3] = neta
P[1,4] = 0 , P[1,5] = 0 , P[1,6] = 0
P[2,1] = 0 , P[2,2] = 0 , P[2,3] = 0
P[2,4] = 1 , P[2,5] = neta , P[2,6] = xsin
α = 8
False
True
P[1,1] = 1 , P[1,2] = xsin , P[1,3] = neta , , P[1,4] = xsin*neta
P[1,5] = 0 , P[1,6] = 0 , P[1,7] = 0 , P[1,8] = 0
P[2,1] = 0 , P[2,2] = 0 , P[2,3] = 0 , , P[2,4] = 0
P[2,5] = 1 , P[2,6] = xsin , P[2,7] = neta , P[2,8] = xsin*neta
5

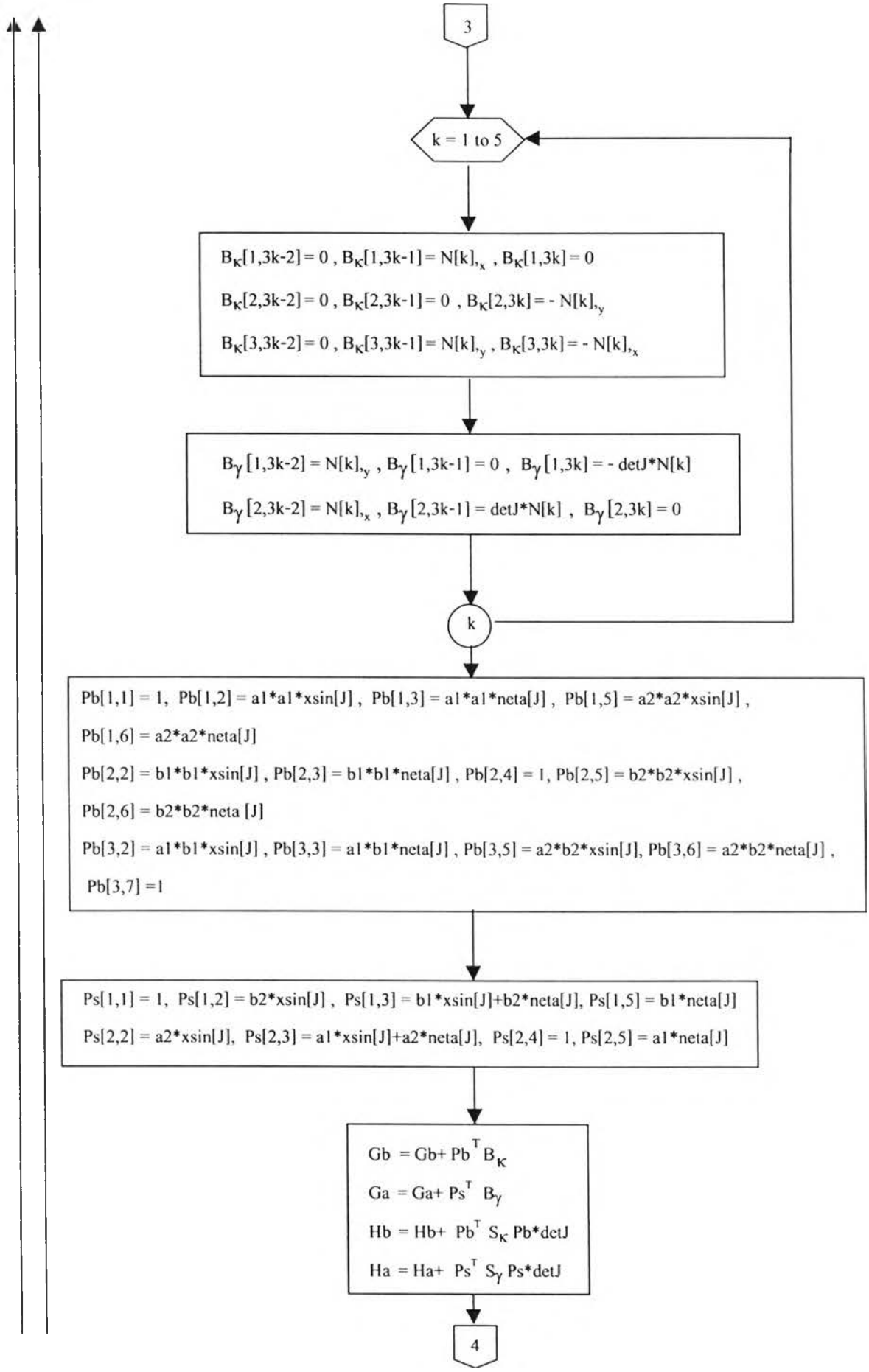

5
J<5
True
False
KB = KB+W1*W1* B_K^T C_K B_K*t*detJ
G_α = G_α +W1*W1* P^T C_γ B_γ*t*detJ
H_α = H_α+W1*W1* P^T C_γ P*t*detJ
J < 9
False
True
KB =KB+W1*W2* B_K^T C_K B_K*t*detJ
G_α = G_α+W1*W2* P^T C_γ B_γ*t*detJ
H_α = H_α+W1*W2* P^T C_γ P*t*detJ
KB = KB+W2*W2* B_K^T C_K B_K*t*detJ
G_α = G_α+W2*W2* P^T C_γ B_γ*t*detJ
H_α = H_α+W2*W2* P^T C_γ P*t*detJ
J
KE[I] = KB+ G_α^T H_α^{-1} G_α
I
END

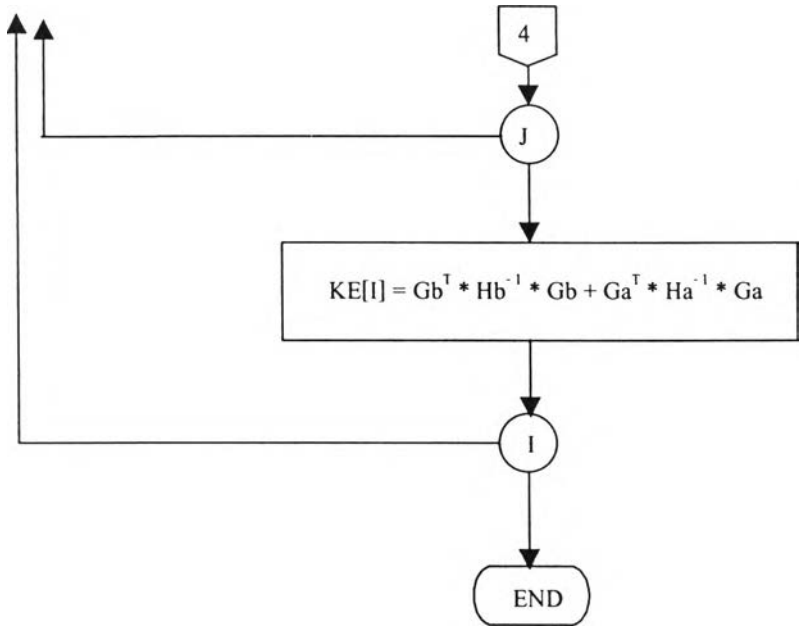
ข. ชิ้นส่วน HMPL5



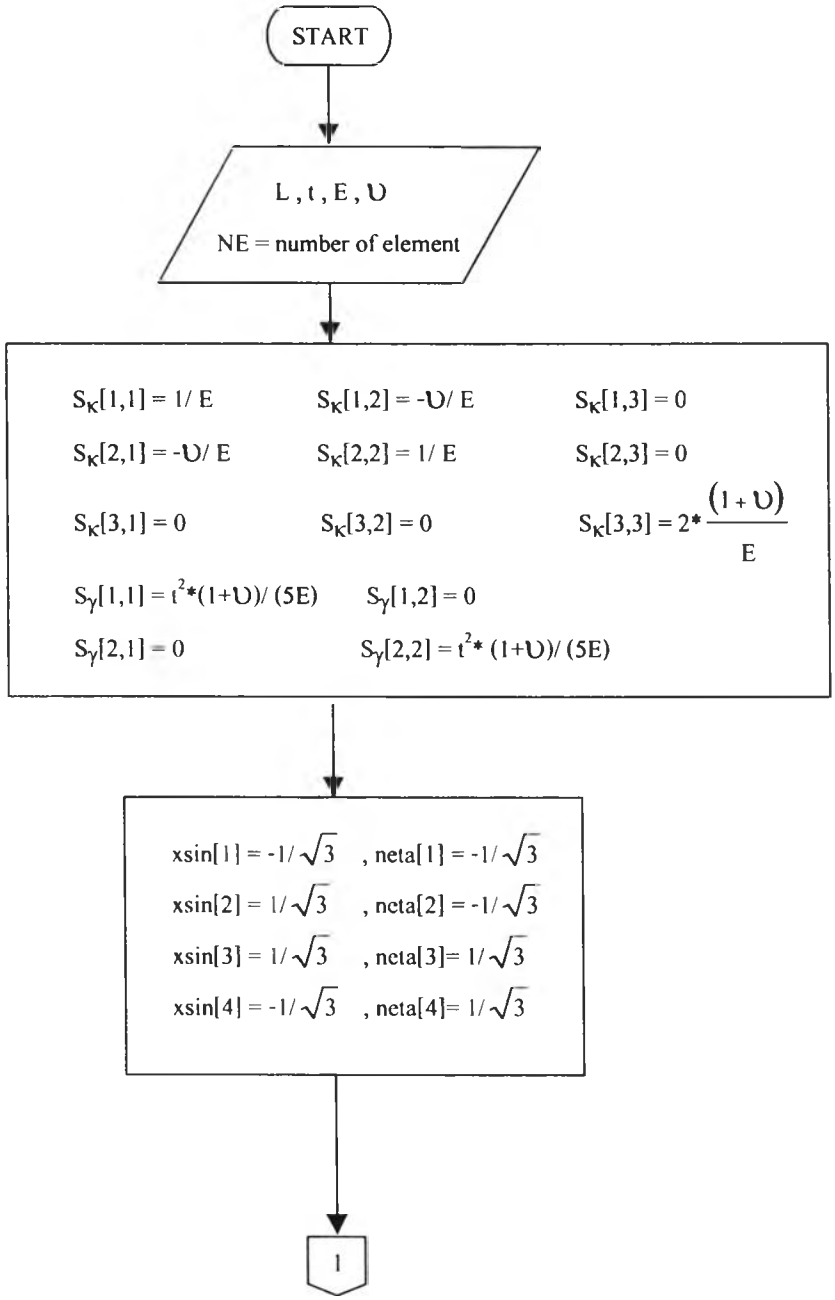

1
I = 1 to NE
II = 1 to 5
xcoor[II] , ycoor[II]
II
J = 1 to 4
N[1] = (1/4)*(1-xsin[J])*(1-neta[J]) – (1/4)*N[5]
N[2] = (1/4)*(1+xsin[J])*(1-neta[J]) – (1/4)*N[5]
N[3] = (1/4)*(1+xsin[J])*(1+neta[J]) – (1/4)*N[5]
N[4] = (1/4)*(1-xsin[J])*(1+neta[J]) – (1/4)*N[5]
N[5] = (1-xsin[J]^2)*(1-neta[J]^2)
N[1] ,ξ = (-1/4)*(1-neta[J]) + xsin[J]*(1-neta[J]^2)/2
N[2] ,ξ = (1/4)*(1-neta[J]) + xsin[J]*(1-neta[J]^2)/2
N[3] ,ξ =(1/4)*(1+neta[J]) + xsin[J]*(1-neta[J]^2)/2
N[4] ,ξ =(-1/4)*(1+neta[J]) + xsin[J]*(1-neta[J]^2)/2
N[5] ,ξ = -2*xsin[J]*(1-neta[J]^2)
2

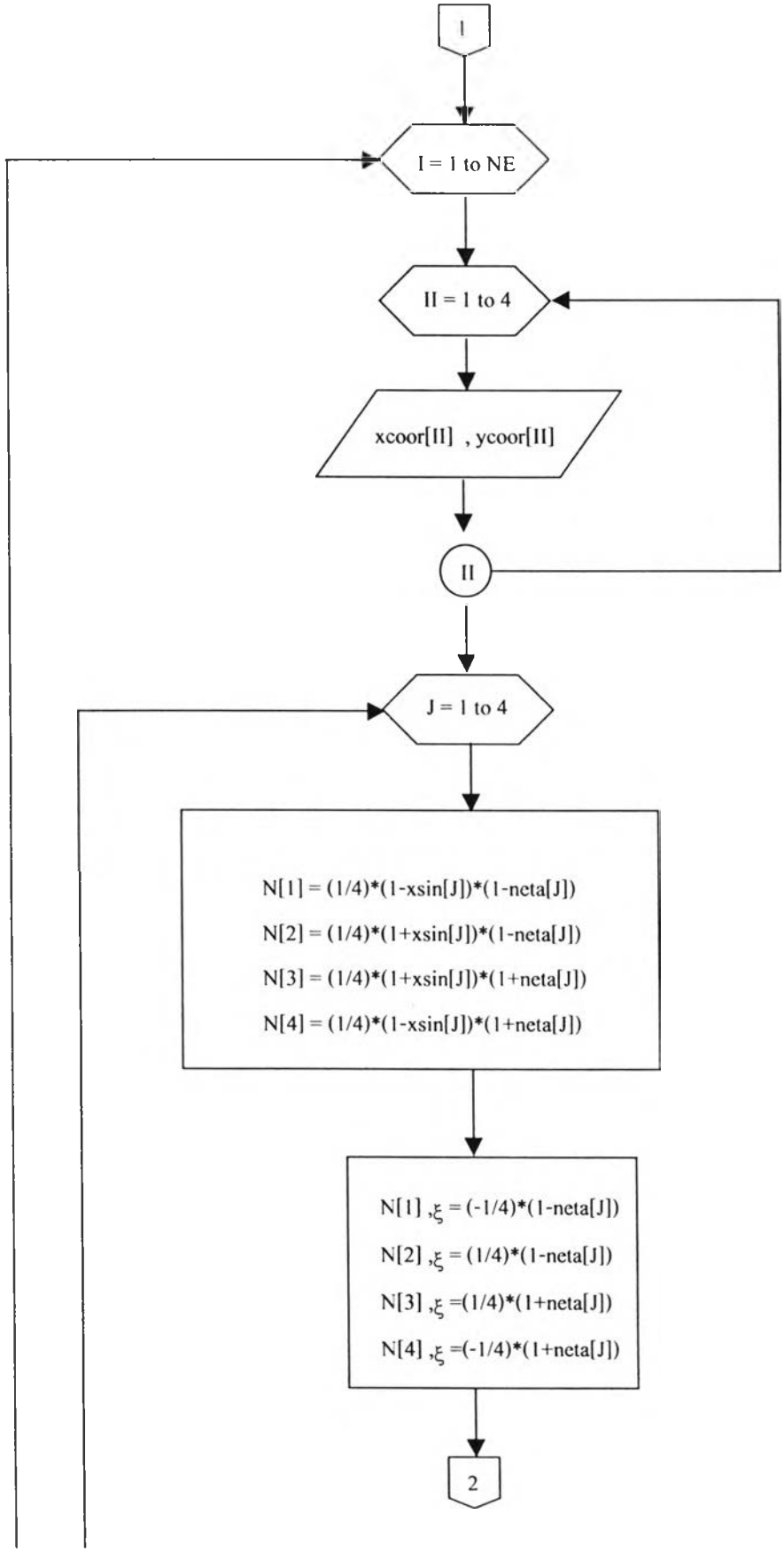

2
N[1] ,η = (-1/4)*(1-xsin[J]) + neta[J]*(1-xsin[J]^2)/2
N[2] ,η = (-1/4)*(1+xsin[J]) + neta[J]*(1-xsin[J]^2)/2
N[3] ,η = (1/4)*(1+xsin[J]) + neta[J]*(1-xsin[J]^2)/2
N[4] ,η = (1/4)*(1-xsin[J]) + neta[J]*(1-xsin[J]^2)/2
N[5] ,η = -2*neta[J]*(1-xsin[J]^2)
a1 = (1/4)*(-xcoor(1) + xcoor(2) + xcoor(3) - xcoor(4))
a2 = (1/4)*(-xcoor(1) - xcoor(2) + xcoor(3) + xcoor(4))
a3 = (1/4)*(xcoor(1) - xcoor(2) + xcoor(3) - xcoor(4))
b1 = (1/4)*(-ycoor(1) + ycoor(2) + ycoor(3) - ycoor(4))
b2 = (1/4)*(-ycoor(1) - ycoor(2) + ycoor(3) + ycoor(4))
b3 = (1/4)*(ycoor(1) - ycoor(2) + ycoor(3) - ycoor(4))
D1 = a1*b2 – a2*b1
D2 = a1*b3 - a3*b1
D3 = a3*b2 – a2*b3
detJ = D1 + D2*xsin[J] + D3*neta[J]
II = 1 to 4
N[II] ,x = (b2+b3*xsin[J])*N[II] ,ξ - (b1+b3*neta[J])*N[II] ,η
N[II] ,y = (a1+a3*neta[J])*N[II] ,ξ - (a2+a3*xsin[J])*N[II] ,η
II
3

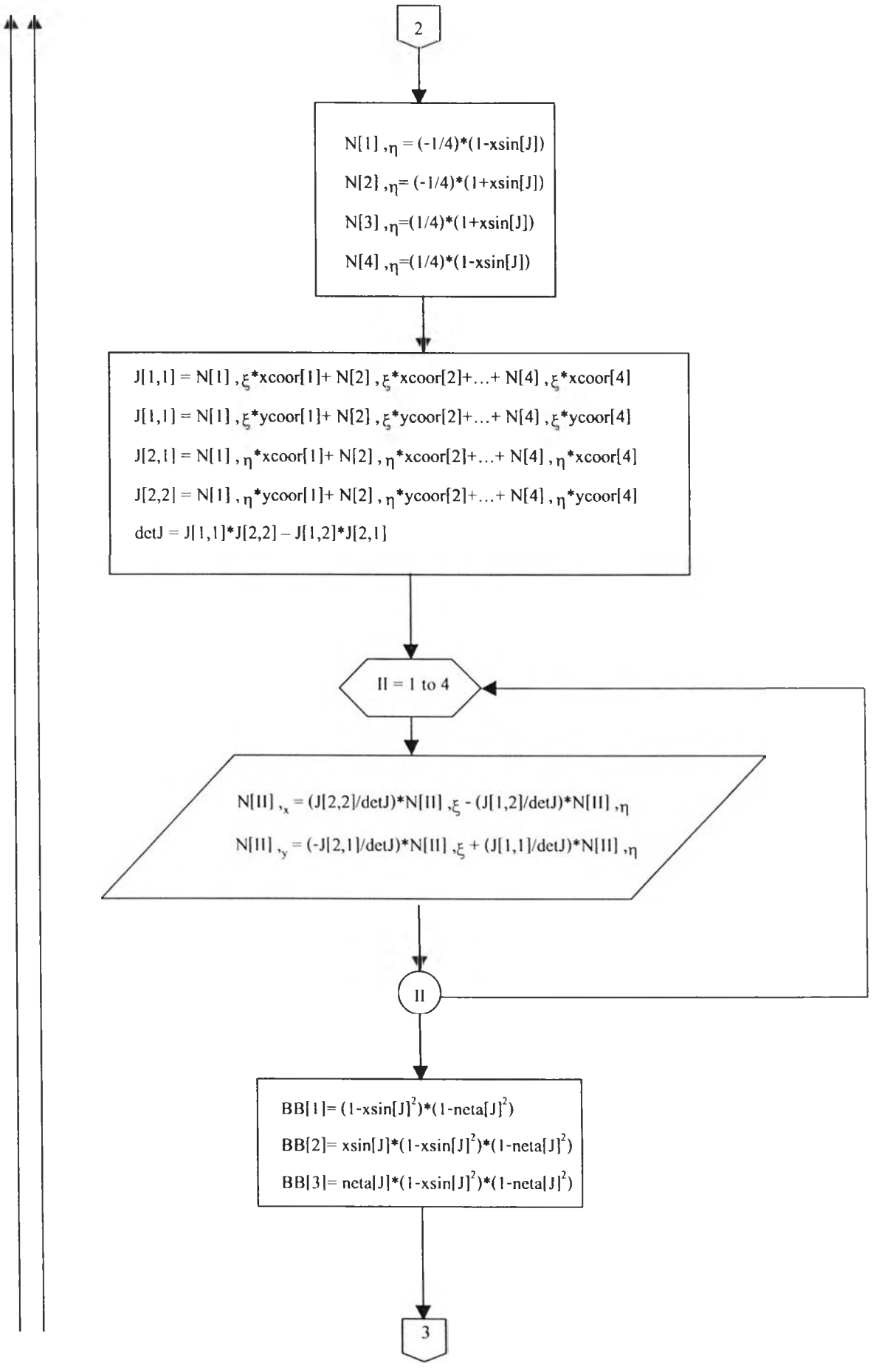

3
k = 1 to 5
$B_K[1,3k-2] = 0$, $B_K[1,3k-1] = N[k]_x$, $B_K[1,3k] = 0$
$B_K[2,3k-2] = 0$, $B_K[2,3k-1] = 0$, $B_K[2,3k] = - N[k]_y$
$B_K[3,3k-2] = 0$, $B_K[3,3k-1] = N[k]_y$, $B_K[3,3k] = - N[k]_x$
$B_\gamma[1,3k-2] = N[k]_y$, $B_\gamma[1,3k-1] = 0$, $B_\gamma[1,3k] = - detJ*N[k]$
$B_\gamma[2,3k-2] = N[k]_x$, $B_\gamma[2,3k-1] = detJ*N[k]$, $B_\gamma[2,3k] = 0$
k
Pb[1,1] = 1, Pb[1,2] = a1*a1*xsin[J] , Pb[1,3] = a1*a1*neta[J] , Pb[1,5] = a2*a2*xsin[J] ,
Pb[1,6] = a2*a2*neta[J]
Pb[2,2] = b1*b1*xsin[J] , Pb[2,3] = b1*b1*neta[J] , Pb[2,4] = 1, Pb[2,5] = b2*b2*xsin[J] ,
Pb[2,6] = b2*b2*neta [J]
Pb[3,2] = a1*b1*xsin[J] , Pb[3,3] = a1*b1*neta[J] , Pb[3,5] = a2*b2*xsin[J], Pb[3,6] = a2*b2*neta[J] ,
Pb[3,7] =1
Ps[1,1] = 1, Ps[1,2] = b2*xsin[J] , Ps[1,3] = b1*xsin[J]+b2*neta[J], Ps[1,5] = b1*neta[J]
Ps[2,2] = a2*xsin[J], Ps[2,3] = a1*xsin[J]+a2*neta[J], Ps[2,4] = 1, Ps[2,5] = a1*neta[J]
$Gb = Gb + Pb^T B_K$
$Ga = Ga + Ps^T B_\gamma$
$Hb = Hb + Pb^T S_K Pb*detJ$
$Ha = Ha + Ps^T S_\gamma Ps*detJ$
4

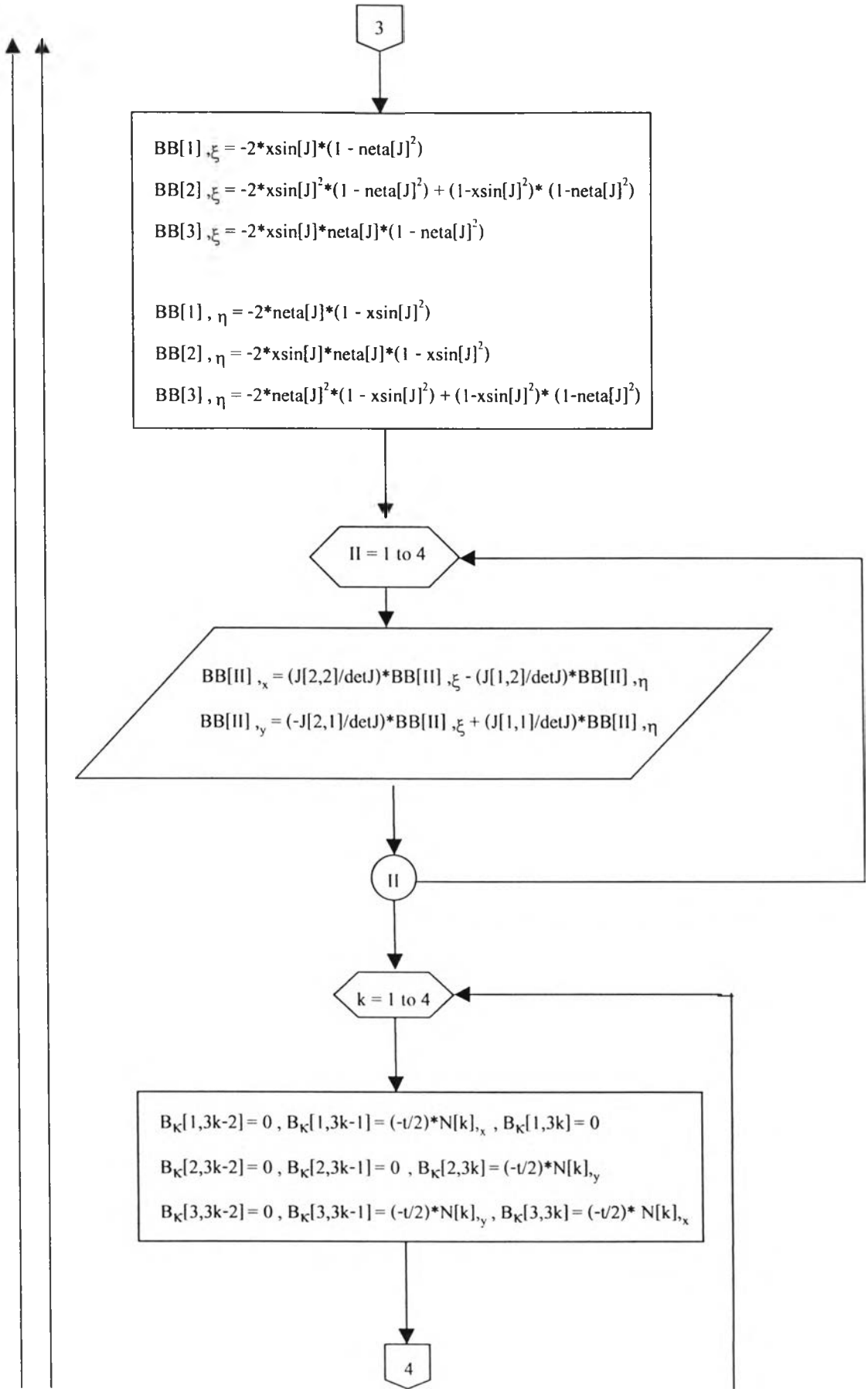

4
J
KE[I] = Gb^T * Hb^-1 * Gb + Ga^T * Ha^-1 * Ga
I
END

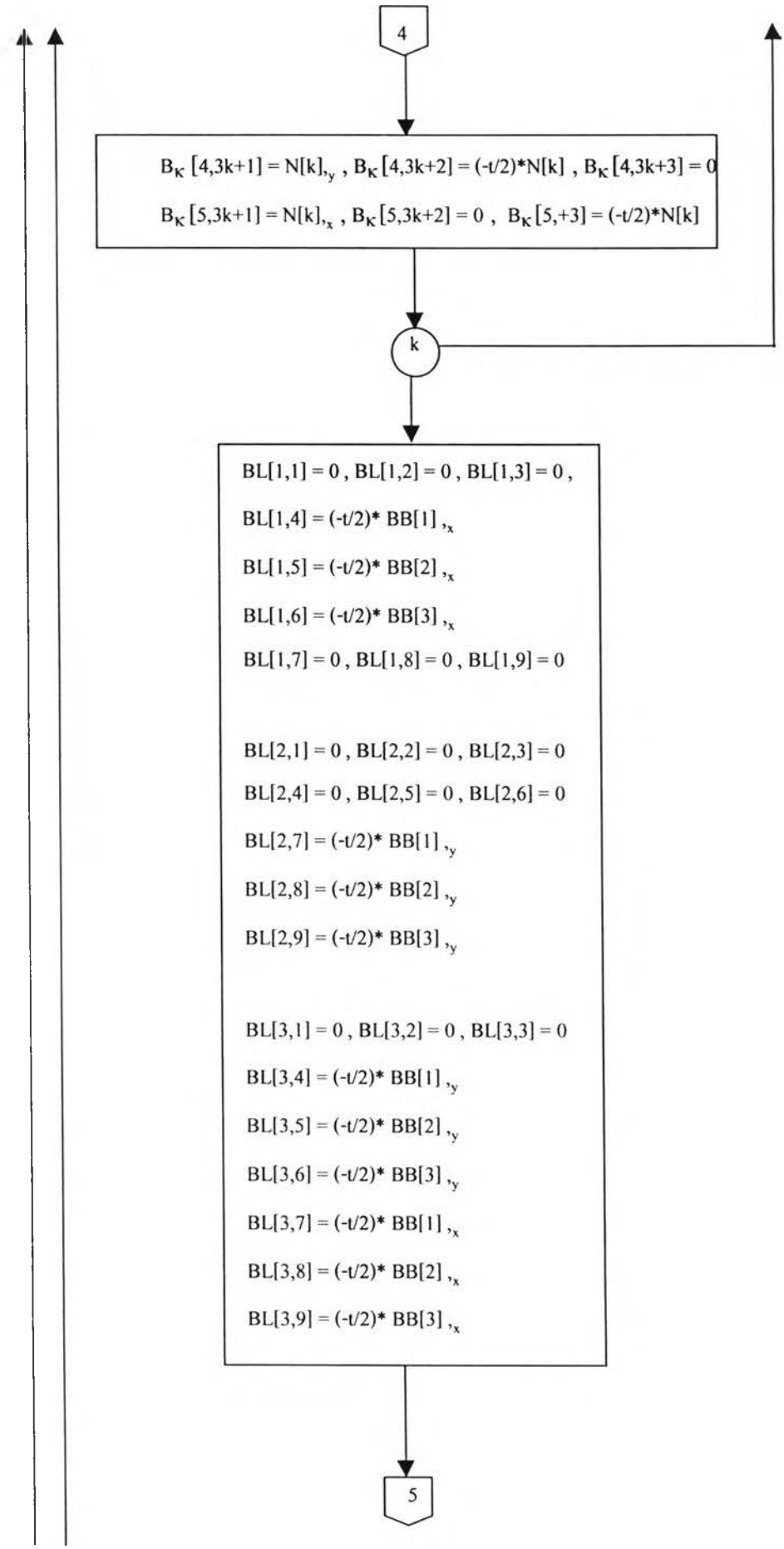
ค. ชิ้นส่วน BUBBLE4

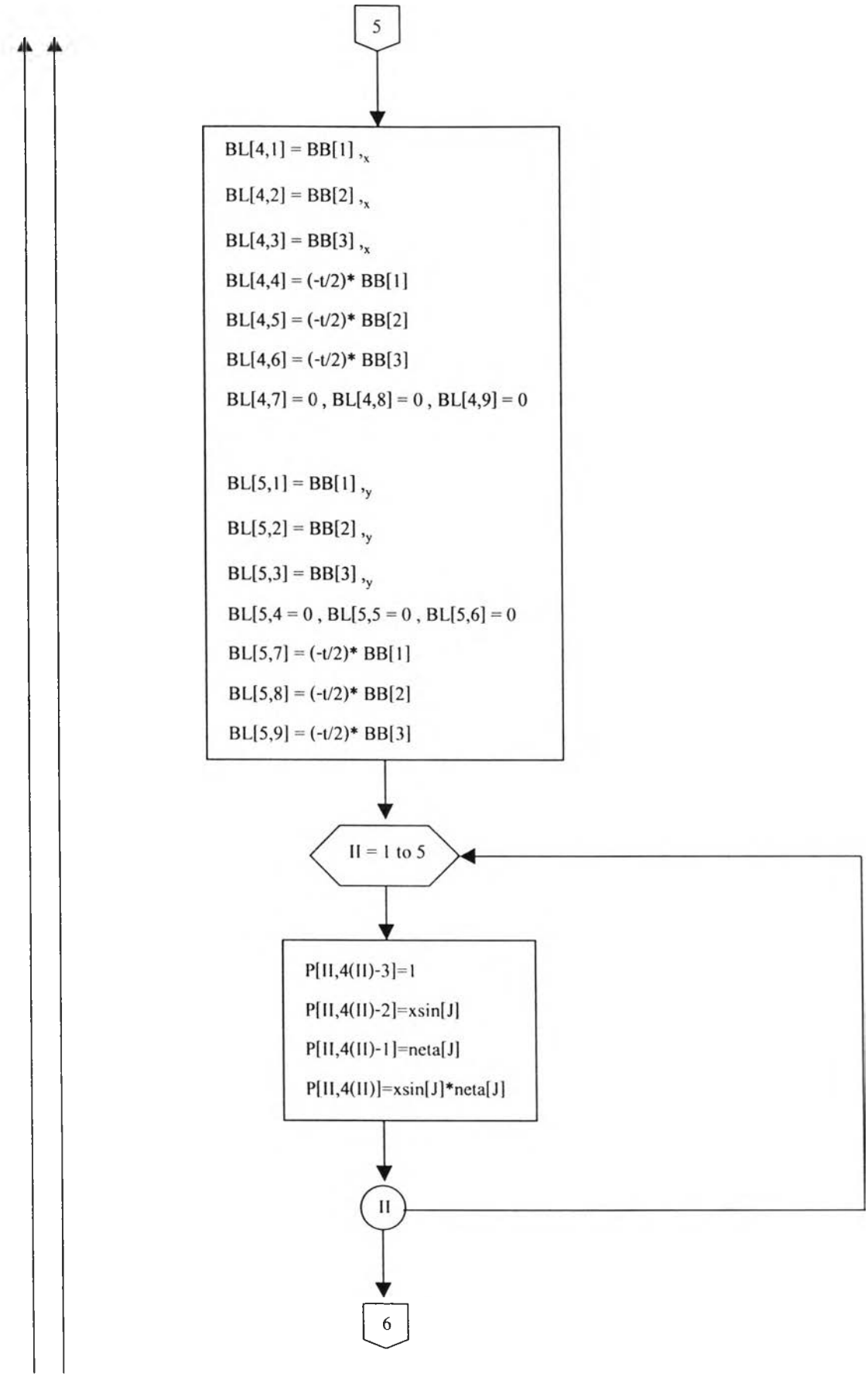


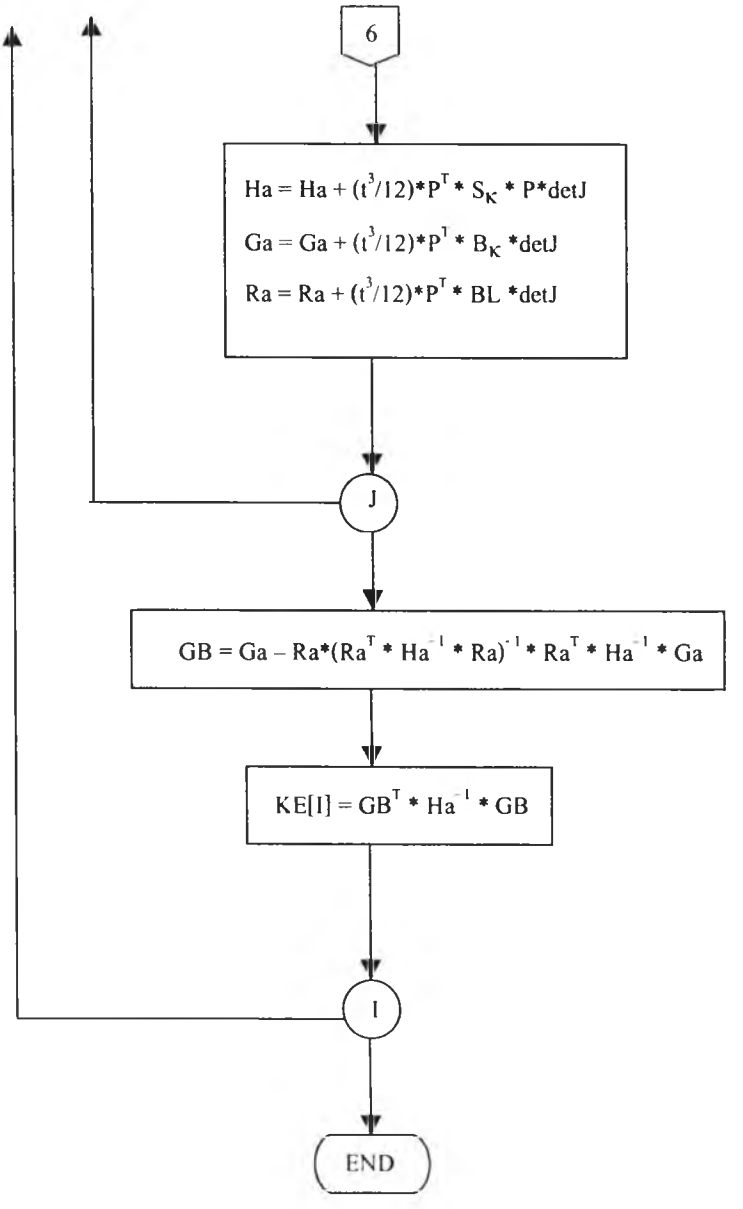

1
I = 1 to NE
II = 1 to 4
xcoor[II] , ycoor[II]
II
J = 1 to 4
N[1] = (1/4)*(1-xsin[J])*(1-neta[J])
N[2] = (1/4)*(1+xsin[J])*(1-neta[J])
N[3] = (1/4)*(1+xsin[J])*(1+neta[J])
N[4] = (1/4)*(1-xsin[J])*(1+neta[J])
N[1] ,ξ = (-1/4)*(1-neta[J])
N[2] ,ξ = (1/4)*(1-neta[J])
N[3] ,ξ =(1/4)*(1+neta[J])
N[4] ,ξ =(-1/4)*(1+neta[J])
2


2
N[1] ,η = (-1/4)*(1-xsin[J])
N[2] ,η= (-1/4)*(1+xsin[J])
N[3] ,η=(1/4)*(1+xsin[J])
N[4] ,η=(1/4)*(1-xsin[J])
J[1,1] = N[1] ,ξ*xcoor[1]+ N[2] ,ξ*xcoor[2]+...+ N[4] ,ξ*xcoor[4]
J[1,1] = N[1] ,ξ*ycoor[1]+ N[2] ,ξ*ycoor[2]+...+ N[4] ,ξ*ycoor[4]
J[2,1] = N[1] ,η*xcoor[1]+ N[2] ,η*xcoor[2]+...+ N[4] ,η*xcoor[4]
J[2,2] = N[1] ,η*ycoor[1]+ N[2] ,η*ycoor[2]+...+ N[4] ,η*ycoor[4]
detJ = J[1,1]*J[2,2] – J[1,2]*J[2,1]
II = 1 to 4
N[II] ,x = (J[2,2]/detJ)*N[II] ,ξ - (J[1,2]/detJ)*N[II] ,η
N[II] ,y = (-J[2,1]/detJ)*N[II] ,ξ + (J[1,1]/detJ)*N[II] ,η
II
BB[1]= (1-xsin[J]2)*(1-neta[J]2)
BB[2]= xsin[J]*(1-xsin[J]2)*(1-neta[J]2)
BB[3]= neta[J]*(1-xsin[J]2)*(1-neta[J]2)
3

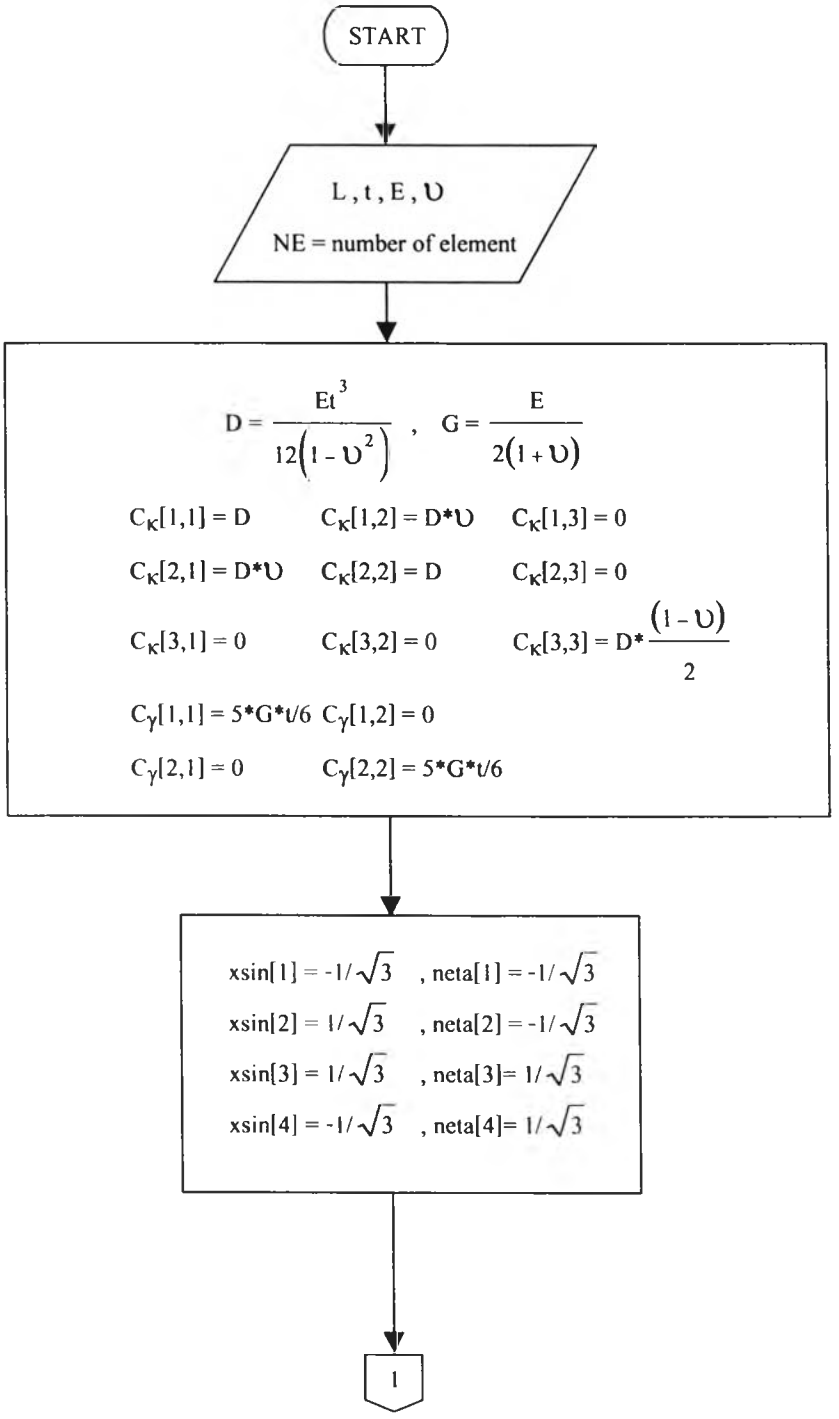

3
BB[1] ,ξ = -2*xsin[J]*(1 - neta[J]^2)
BB[2] ,ξ = -2*xsin[J]^2*(1 - neta[J]^2) + (1-xsin[J]^2)* (1-neta[J]^2)
BB[3] ,ξ = -2*xsin[J]*neta[J]*(1 - neta[J]^2)
BB[1] , η = -2*neta[J]*(1 - xsin[J]^2)
BB[2] , η = -2*xsin[J]*neta[J]*(1 - xsin[J]^2)
BB[3] , η = -2*neta[J]^2*(1 - xsin[J]^2) + (1-xsin[J]^2)* (1-neta[J]^2)
II = 1 to 4
BB[II] ,x = (J[2,2]/detJ)*BB[II] ,ξ - (J[1,2]/detJ)*BB[II] ,η
BB[II] ,y = (-J[2,1]/detJ)*BB[II] ,ξ + (J[1,1]/detJ)*BB[II] ,η
II
k = 1 to 4
Bκ[1,3k-2] = 0 , Bκ[1,3k-1] = (-t/2)*N[k],x , Bκ[1,3k] = 0
Bκ[2,3k-2] = 0 , Bκ[2,3k-1] = 0 , Bκ[2,3k] = (-t/2)*N[k],y
Bκ[3,3k-2] = 0 , Bκ[3,3k-1] = (-t/2)*N[k],y , Bκ[3,3k] = (-t/2)* N[k],x
4

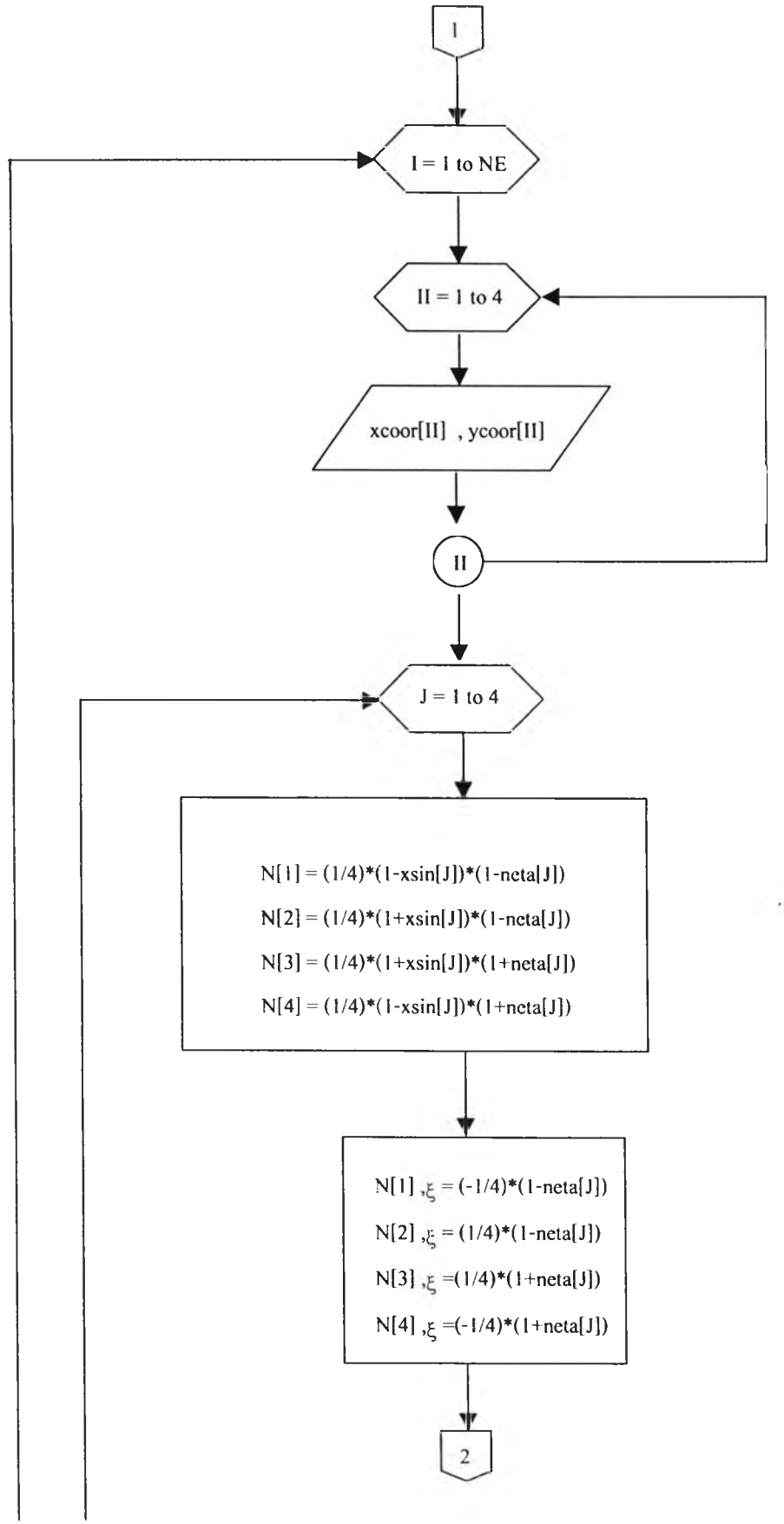

4
$B_K[4,3k+1] = N[k]_{,y}$, $B_K[4,3k+2] = (-t/2)*N[k]$, $B_K[4,3k+3] = 0$
$B_K[5,3k+1] = N[k]_{,x}$, $B_K[5,3k+2] = 0$, $B_K[5,+3] = (-t/2)*N[k]$
k
BL[1,1] = 0 , BL[1,2] = 0 , BL[1,3] = 0 ,
BL[1,4] = (-t/2)* BB[1] ,x
BL[1,5] = (-t/2)* BB[2] ,x
BL[1,6] = (-t/2)* BB[3] ,x
BL[1,7] = 0 , BL[1,8] = 0 , BL[1,9] = 0
BL[2,1] = 0 , BL[2,2] = 0 , BL[2,3] = 0
BL[2,4] = 0 , BL[2,5] = 0 , BL[2,6] = 0
BL[2,7] = (-t/2)* BB[1] ,y
BL[2,8] = (-t/2)* BB[2] ,y
BL[2,9] = (-t/2)* BB[3] ,y
BL[3,1] = 0 , BL[3,2] = 0 , BL[3,3] = 0
BL[3,4] = (-t/2)* BB[1] ,y
BL[3,5] = (-t/2)* BB[2] ,y
BL[3,6] = (-t/2)* BB[3] ,y
BL[3,7] = (-t/2)* BB[1] ,x
BL[3,8] = (-t/2)* BB[2] ,x
BL[3,9] = (-t/2)* BB[3] ,x
5

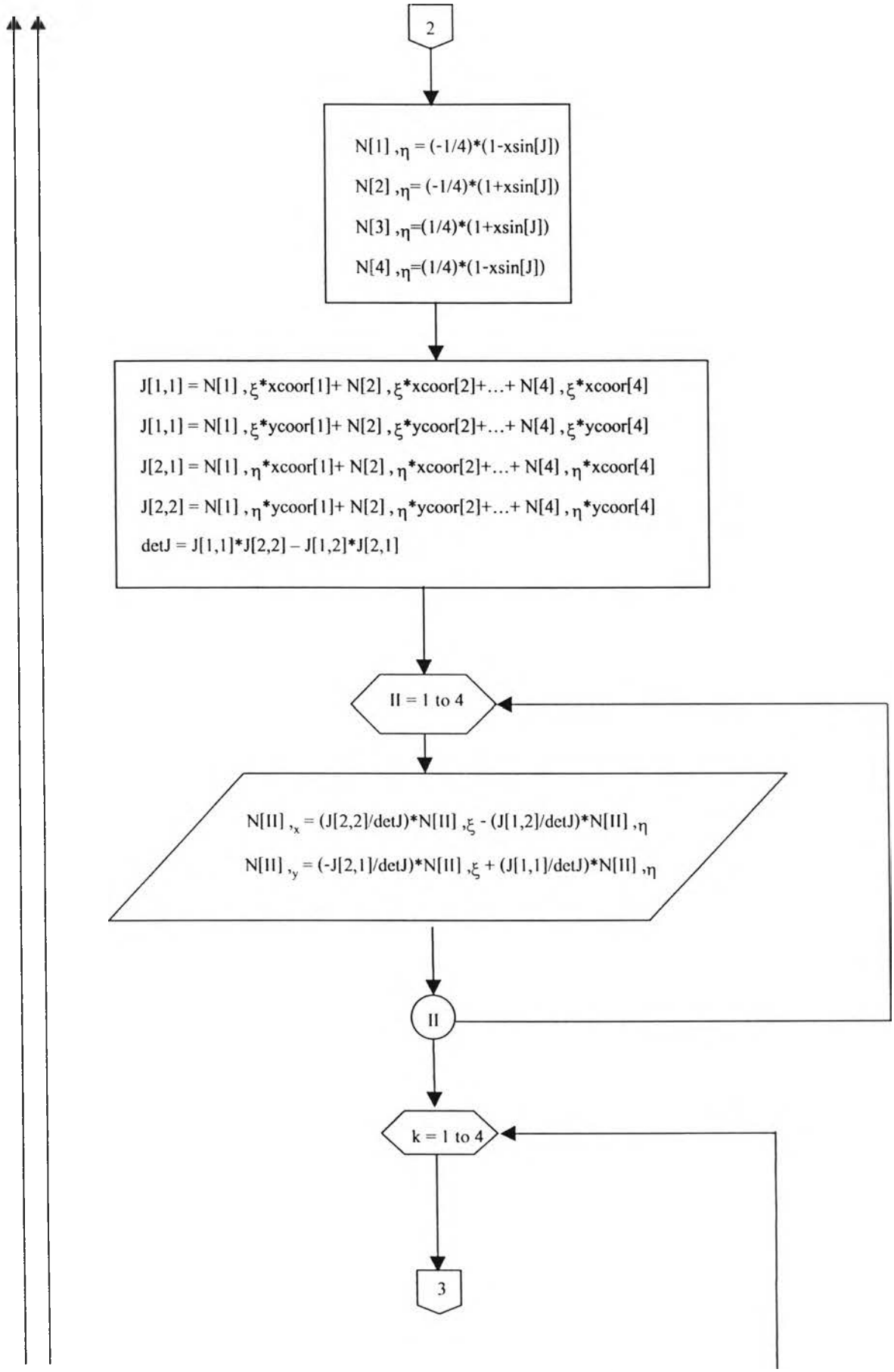

5
BL[4,1] = BB[1] ,x
BL[4,2] = BB[2] ,x
BL[4,3] = BB[3] ,x
BL[4,4] = (-t/2)* BB[1]
BL[4,5] = (-t/2)* BB[2]
BL[4,6] = (-t/2)* BB[3]
BL[4,7] = 0 , BL[4,8] = 0 , BL[4,9] = 0
BL[5,1] = BB[1] ,y
BL[5,2] = BB[2] ,y
BL[5,3] = BB[3] ,y
BL[5,4 = 0 , BL[5,5 = 0 , BL[5,6] = 0
BL[5,7] = (-t/2)* BB[1]
BL[5,8] = (-t/2)* BB[2]
BL[5,9] = (-t/2)* BB[3]
II = 1 to 5
P[II,4(II)-3]=1
P[II,4(II)-2]=xsin[J]
P[II,4(II)-1]=neta[J]
P[II,4(II)]=xsin[J]*neta[J]
II
6

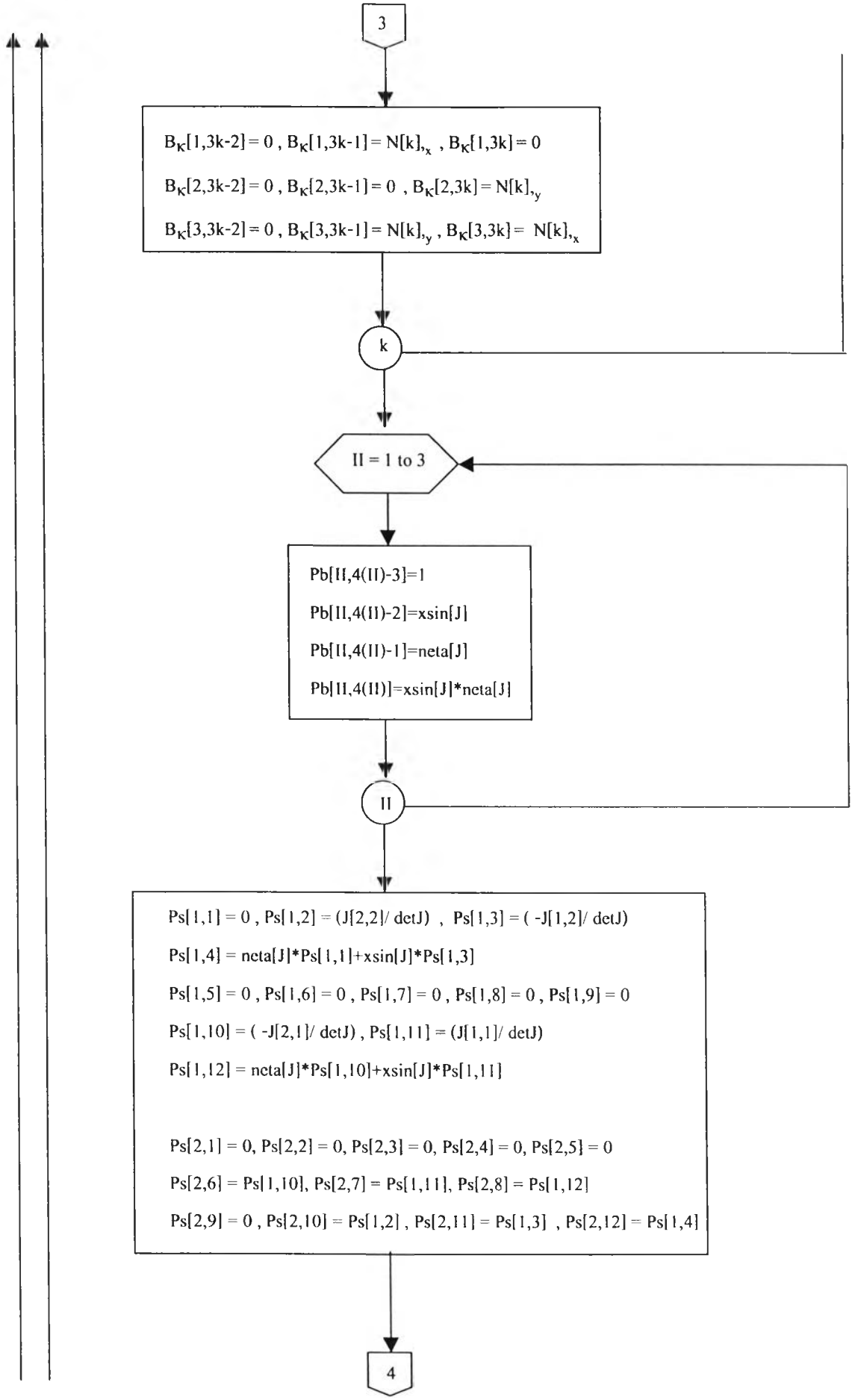

6
Ha = Ha + (t^3/12)*P^T * S_K * P*detJ
Ga = Ga + (t^3/12)*P^T * B_K *detJ
Ra = Ra + (t^3/12)*P^T * BL *detJ
J
GB = Ga – Ra*(Ra^T * Ha^-1 * Ra)^-1 * Ra^T * Ha^-1 * Ga
KE[I] = GB^T * Ha^-1 * GB
I
END

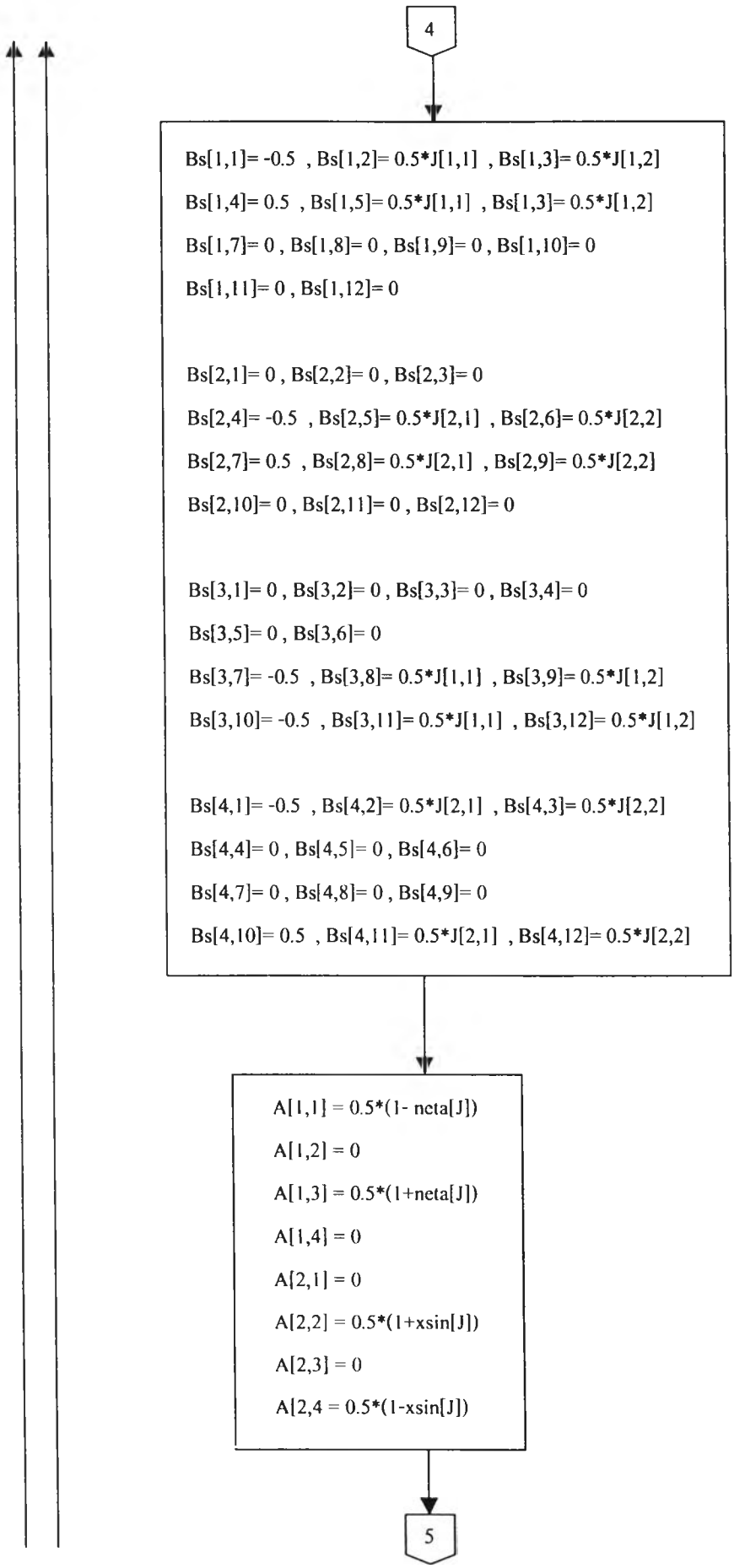
ง. ชิ้นส่วน MiSP4

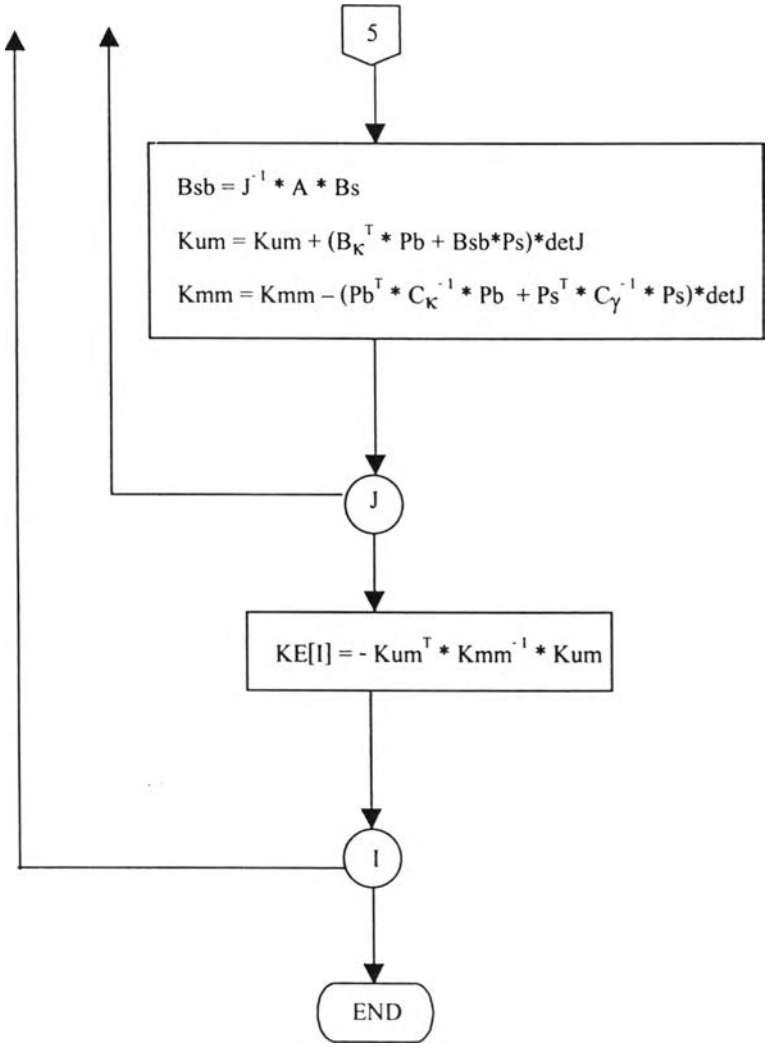



1
I = 1 to NE
II = 1 to 4
xcoor[II] , ycoor[II]
II
J = 1 to 4
N[1] = (1/4)*(1-xsin[J])*(1-neta[J])
N[2] = (1/4)*(1+xsin[J])*(1-neta[J])
N[3] = (1/4)*(1+xsin[J])*(1+neta[J])
N[4] = (1/4)*(1-xsin[J])*(1+neta[J])
N[1] ,ξ = (-1/4)*(1-neta[J])
N[2] ,ξ = (1/4)*(1-neta[J])
N[3] ,ξ =(1/4)*(1+neta[J])
N[4] ,ξ =(-1/4)*(1+neta[J])
2

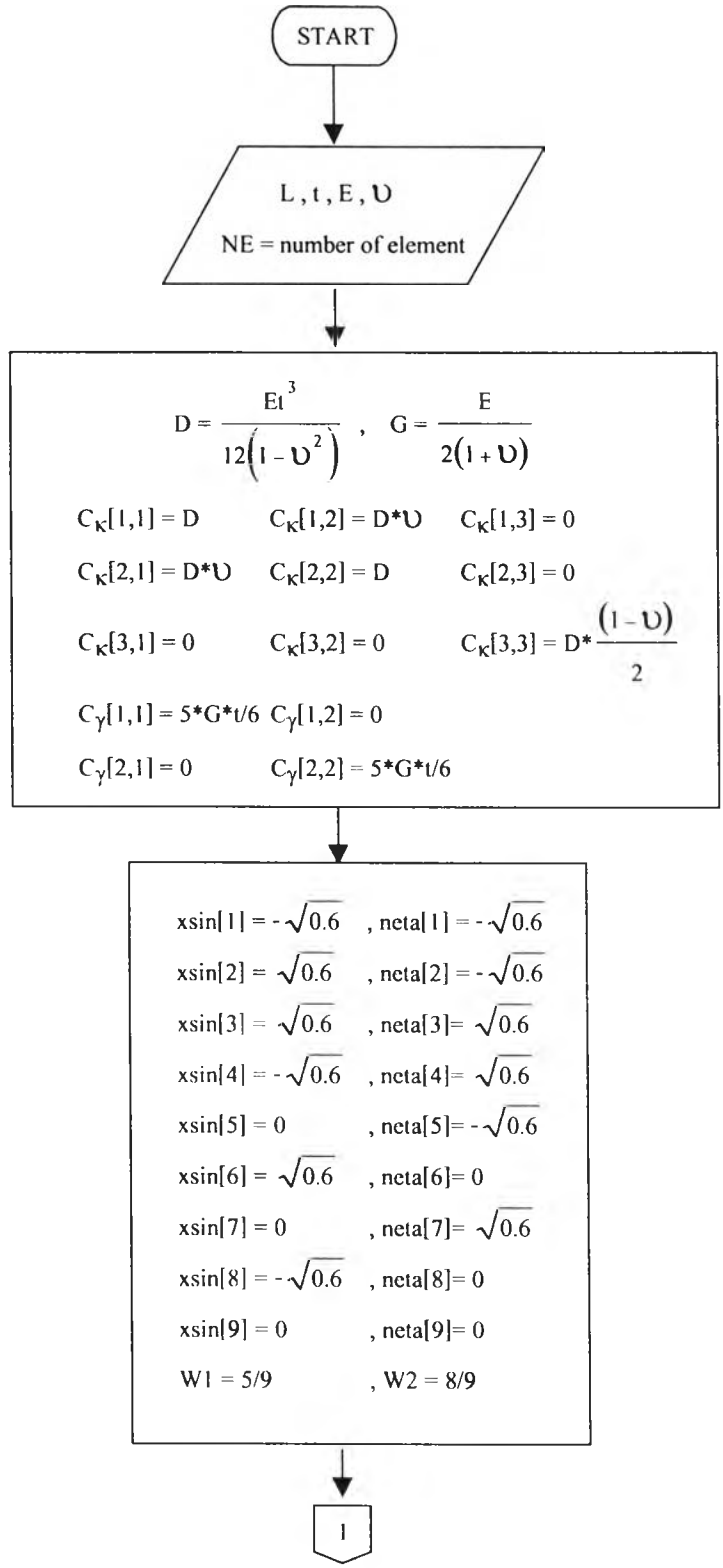

2
N[1] ,η = (-1/4)*(1-xsin[J])
N[2] ,η= (-1/4)*(1+xsin[J])
N[3] ,η=(1/4)*(1+xsin[J])
N[4] ,η=(1/4)*(1-xsin[J])
J[1,1] = N[1] ,ξ*xcoor[1]+ N[2] ,ξ*xcoor[2]+...+ N[4] ,ξ*xcoor[4]
J[1,1] = N[1] ,ξ*ycoor[1]+ N[2] ,ξ*ycoor[2]+...+ N[4] ,ξ*ycoor[4]
J[2,1] = N[1] ,η*xcoor[1]+ N[2] ,η*xcoor[2]+...+ N[4] ,η*xcoor[4]
J[2,2] = N[1] ,η*ycoor[1]+ N[2] ,η*ycoor[2]+...+ N[4] ,η*ycoor[4]
detJ = J[1,1]*J[2,2] – J[1,2]*J[2,1]
II = 1 to 4
N[II] ,x = (J[2,2]/detJ)*N[II] ,ξ - (J[1,2]/detJ)*N[II] ,η
N[II] ,y = (-J[2,1]/detJ)*N[II] ,ξ + (J[1,1]/detJ)*N[II] ,η
II
k = 1 to 4
3

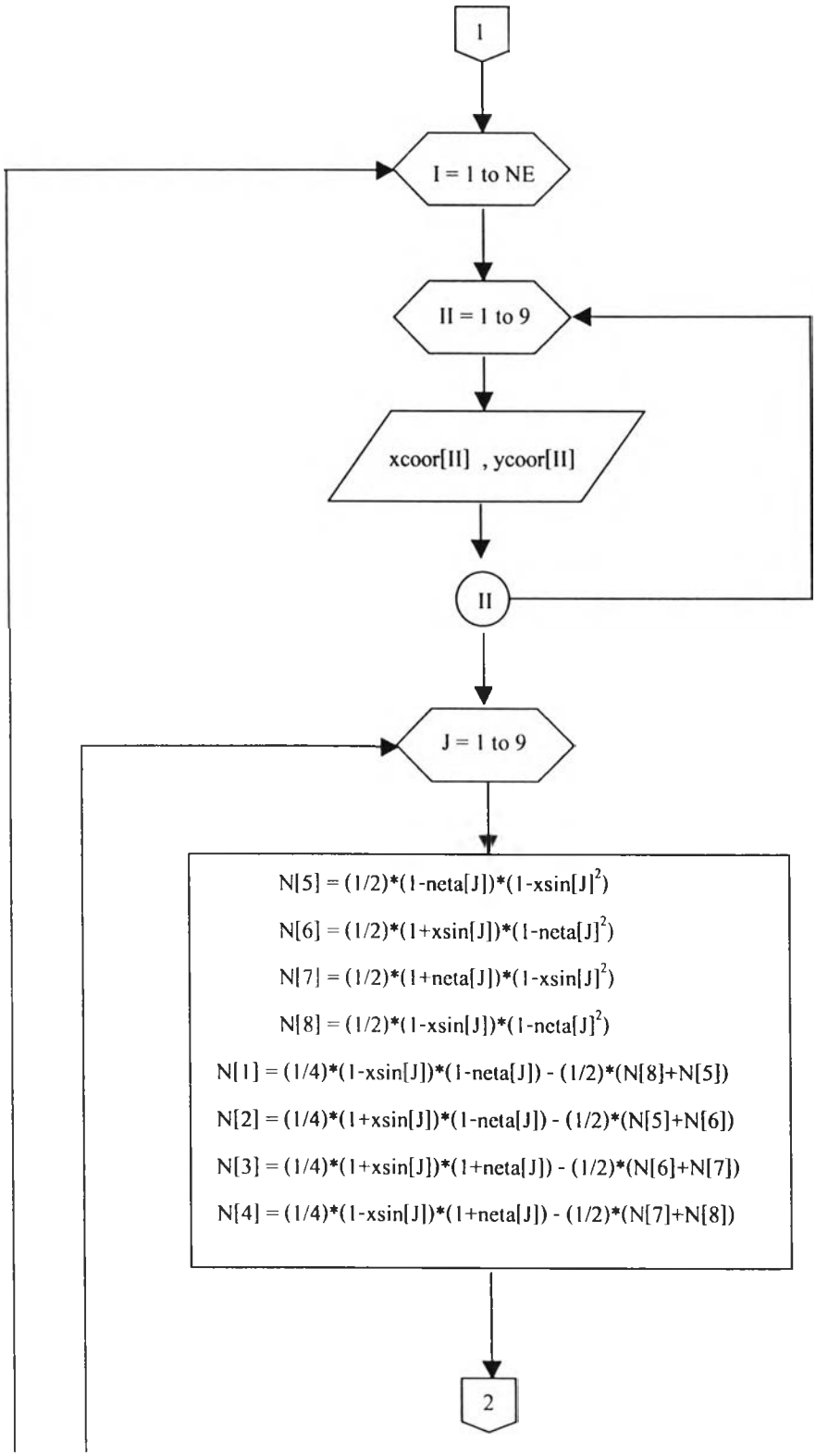

3
$B_K[1,3k-2] = 0$, $B_K[1,3k-1] = N[k]_x$, $B_K[1,3k] = 0$
$B_K[2,3k-2] = 0$, $B_K[2,3k-1] = 0$, $B_K[2,3k] = N[k]_y$
$B_K[3,3k-2] = 0$, $B_K[3,3k-1] = N[k]_y$, $B_K[3,3k] = N[k]_x$
k
II = 1 to 3
Pb[II,4(II)-3]=1
Pb[II,4(II)-2]=xsin[J]
Pb[II,4(II)-1]=neta[J]
Pb[II,4(II)]=xsin[J]*neta[J]
II
Ps[1,1] = 0 , Ps[1,2] = (J[2,2]/ detJ) , Ps[1,3] = (-J[1,2]/ detJ)
Ps[1,4] = neta[J]*Ps[1,1]+xsin[J]*Ps[1,3]
Ps[1,5] = 0 , Ps[1,6] = 0 , Ps[1,7] = 0 , Ps[1,8] = 0 , Ps[1,9] = 0
Ps[1,10] = (-J[2,1]/ detJ) , Ps[1,11] = (J[1,1]/ detJ)
Ps[1,12] = neta[J]*Ps[1,10]+xsin[J]*Ps[1,11]
Ps[2,1] = 0, Ps[2,2] = 0, Ps[2,3] = 0, Ps[2,4] = 0, Ps[2,5] = 0
Ps[2,6] = Ps[1,10], Ps[2,7] = Ps[1,11], Ps[2,8] = Ps[1,12]
Ps[2,9] = 0 , Ps[2,10] = Ps[1,2] , Ps[2,11] = Ps[1,3] , Ps[2,12] = Ps[1,4]
4

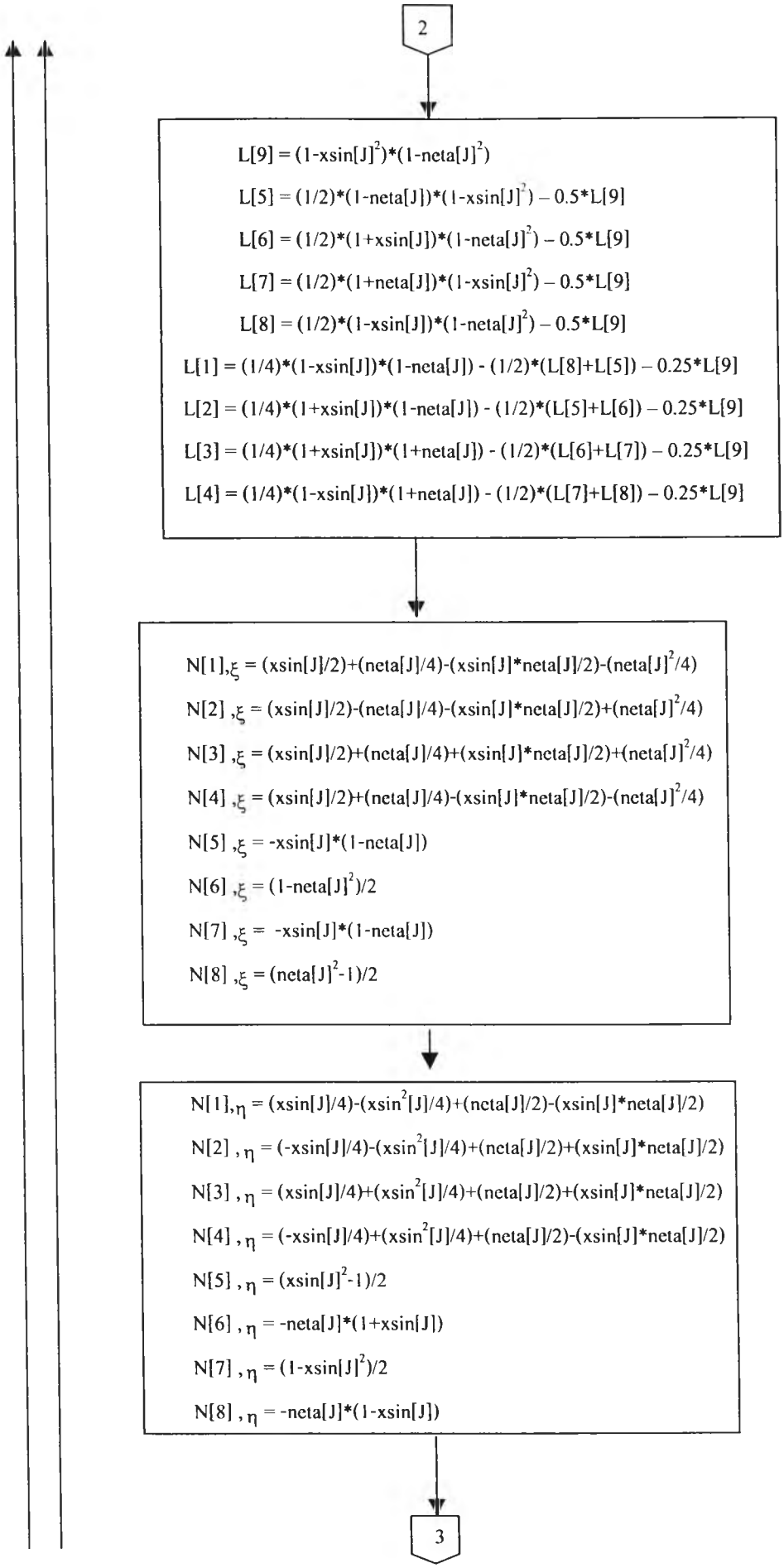

4
Bs[1,1]= -0.5 , Bs[1,2]= 0.5*J[1,1] , Bs[1,3]= 0.5*J[1,2]
Bs[1,4]= 0.5 , Bs[1,5]= 0.5*J[1,1] , Bs[1,3]= 0.5*J[1,2]
Bs[1,7]= 0 , Bs[1,8]= 0 , Bs[1,9]= 0 , Bs[1,10]= 0
Bs[1,11]= 0 , Bs[1,12]= 0
Bs[2,1]= 0 , Bs[2,2]= 0 , Bs[2,3]= 0
Bs[2,4]= -0.5 , Bs[2,5]= 0.5*J[2,1] , Bs[2,6]= 0.5*J[2,2]
Bs[2,7]= 0.5 , Bs[2,8]= 0.5*J[2,1] , Bs[2,9]= 0.5*J[2,2]
Bs[2,10]= 0 , Bs[2,11]= 0 , Bs[2,12]= 0
Bs[3,1]= 0 , Bs[3,2]= 0 , Bs[3,3]= 0 , Bs[3,4]= 0
Bs[3,5]= 0 , Bs[3,6]= 0
Bs[3,7]= -0.5 , Bs[3,8]= 0.5*J[1,1] , Bs[3,9]= 0.5*J[1,2]
Bs[3,10]= -0.5 , Bs[3,11]= 0.5*J[1,1] , Bs[3,12]= 0.5*J[1,2]
Bs[4,1]= -0.5 , Bs[4,2]= 0.5*J[2,1] , Bs[4,3]= 0.5*J[2,2]
Bs[4,4]= 0 , Bs[4,5]= 0 , Bs[4,6]= 0
Bs[4,7]= 0 , Bs[4,8]= 0 , Bs[4,9]= 0
Bs[4,10]= 0.5 , Bs[4,11]= 0.5*J[2,1] , Bs[4,12]= 0.5*J[2,2]
A[1,1] = 0.5*(1- neta[J])
A[1,2] = 0
A[1,3] = 0.5*(1+neta[J])
A[1,4] = 0
A[2,1] = 0
A[2,2] = 0.5*(1+xsin[J])
A[2,3] = 0
A[2,4 = 0.5*(1-xsin[J])
5

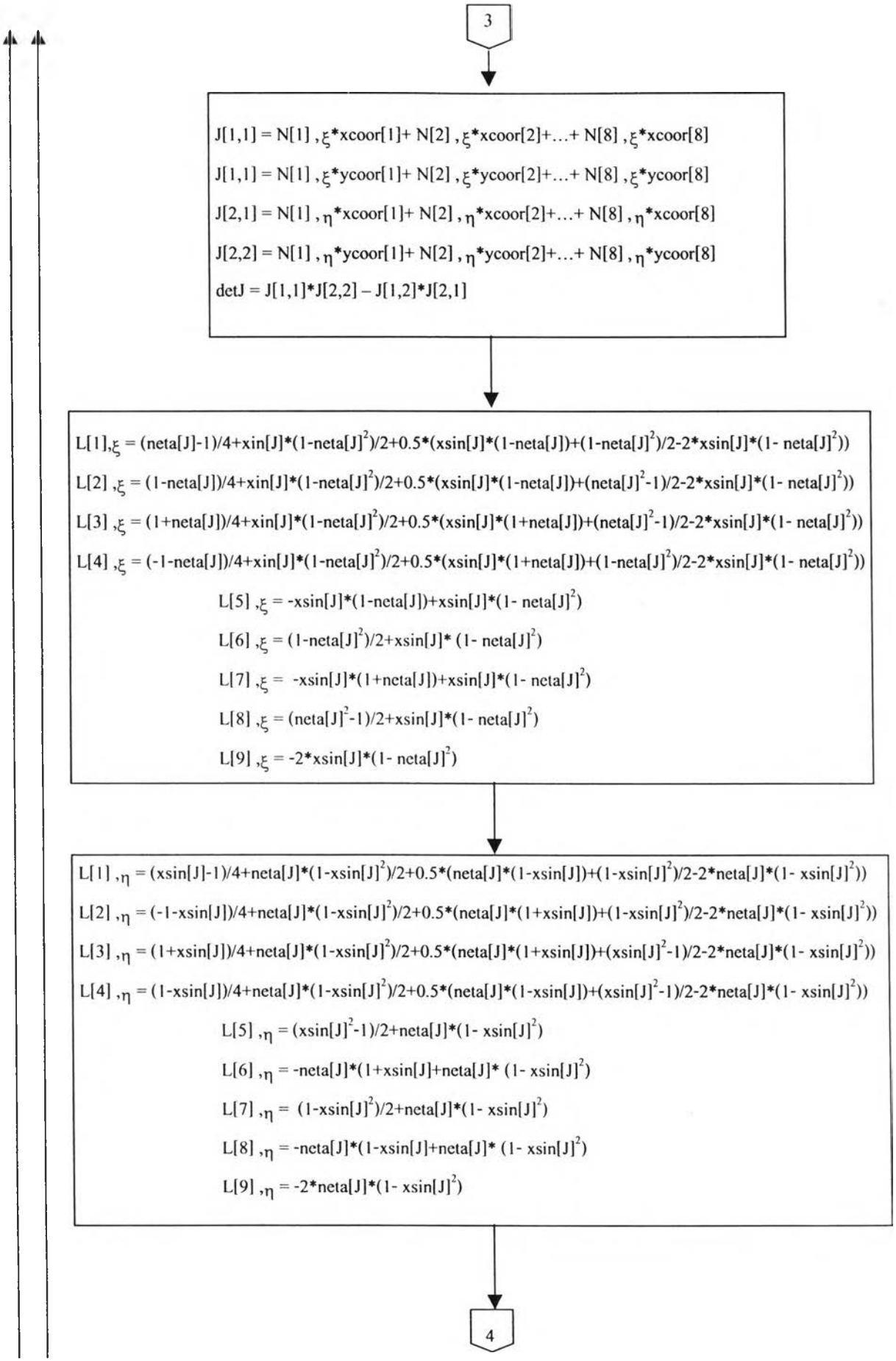

5
Bsb = J^-1 * A * Bs
Kum = Kum + (B_K^T * Pb + Bsb*Ps)*detJ
Kmm = Kmm – (Pb^T * C_K^-1 * Pb + Ps^T * C_γ^-1 * Ps)*detJ
J
KE[I] = - Kum^T * Kmm^-1 * Kum
I
END

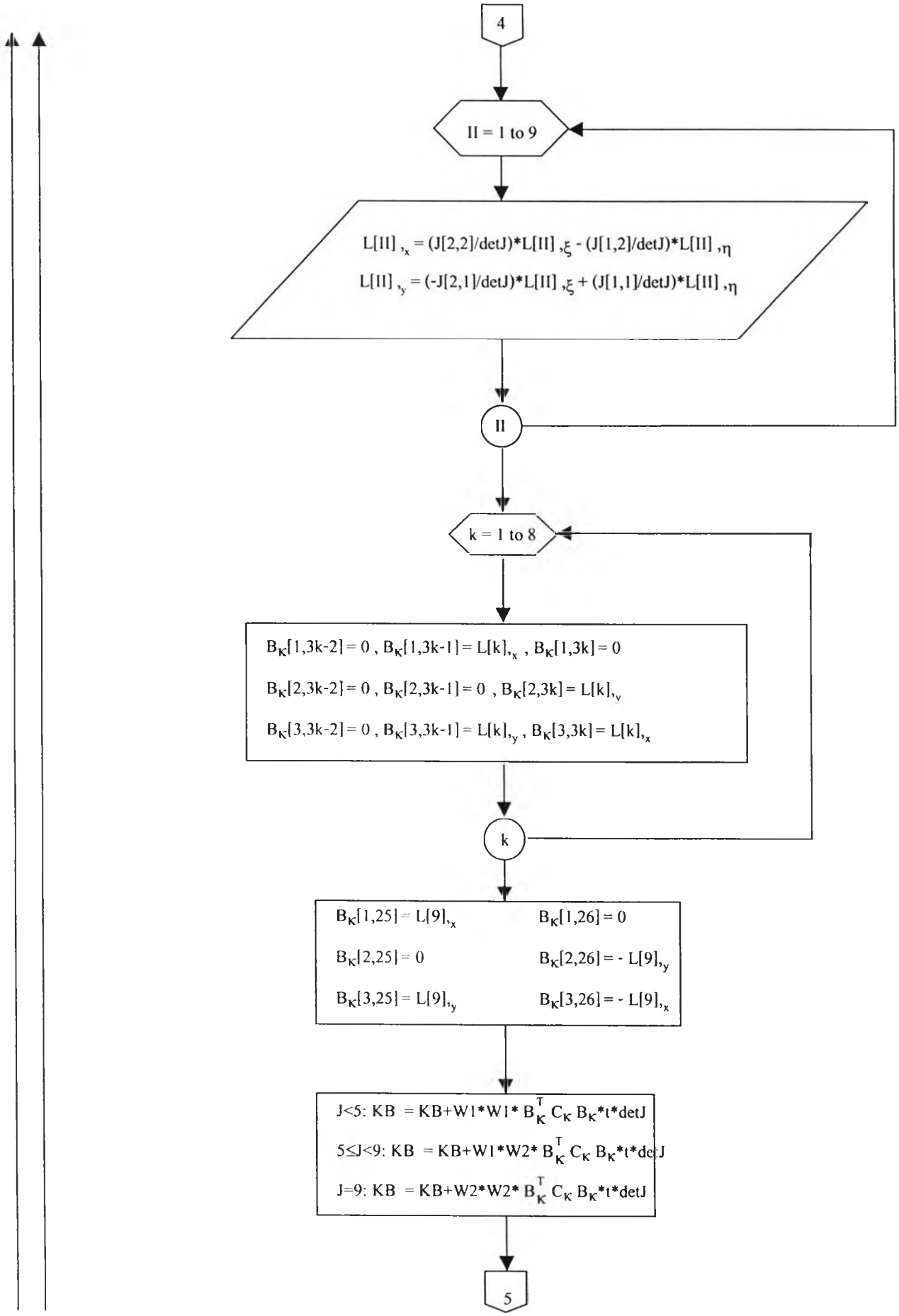
จ. ชิ้นส่วน PLAT8HR

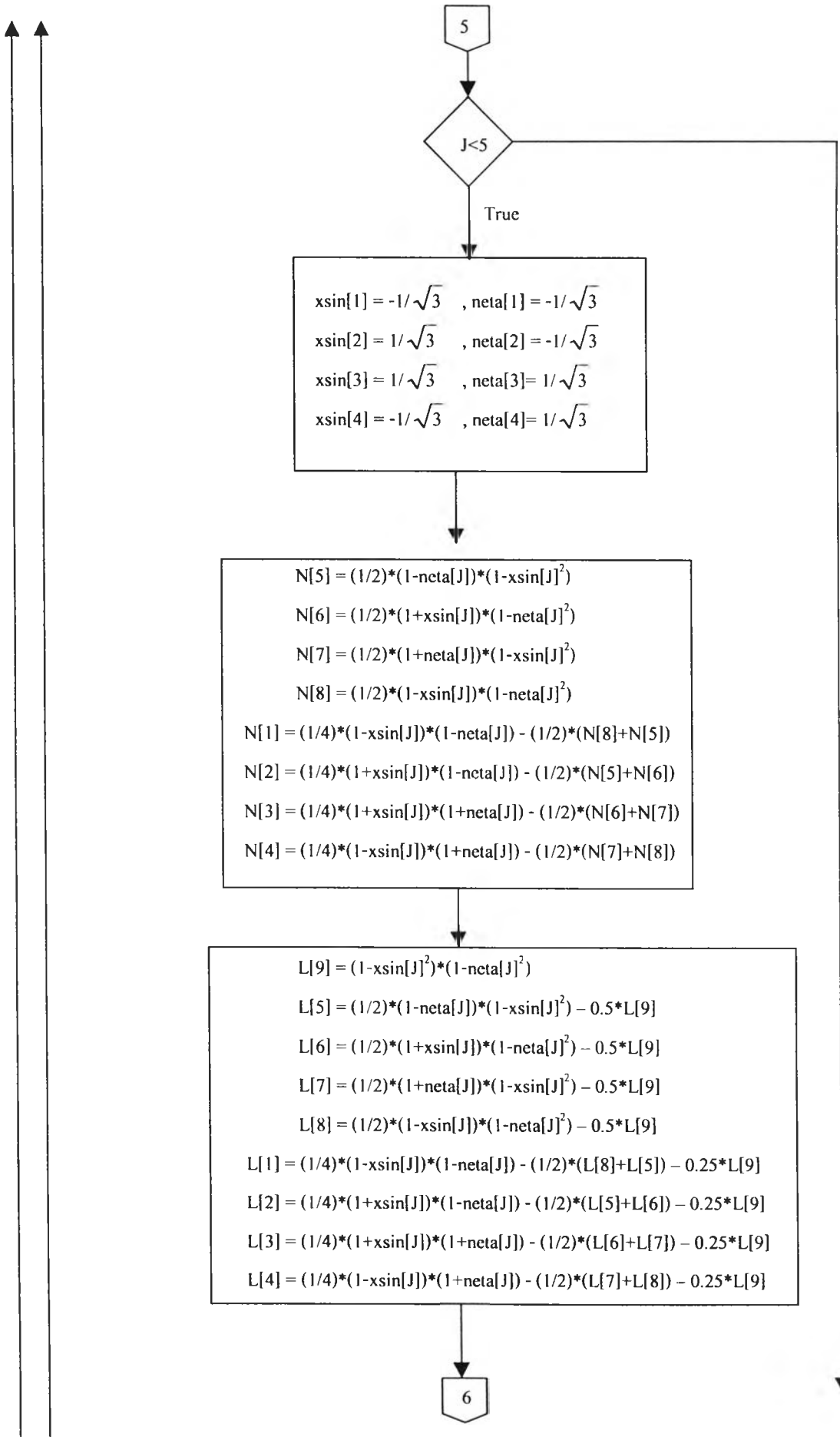


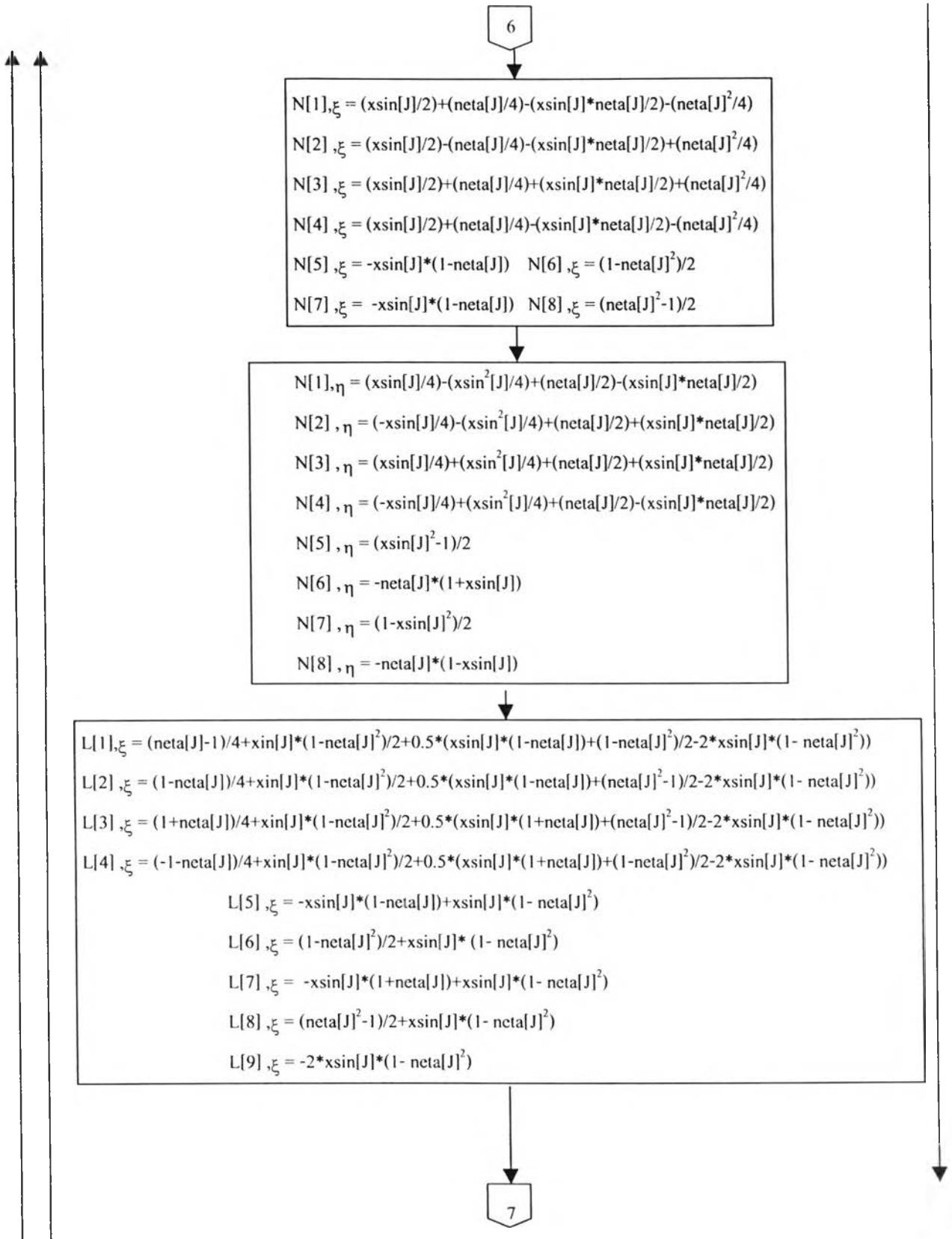

1
I = 1 to NE
II = 1 to 9
xcoor[II] , ycoor[II]
II
J = 1 to 9
N[5] = (1/2)*(1-neta[J])*(1-xsin[J]^2)
N[6] = (1/2)*(1+xsin[J])*(1-neta[J]^2)
N[7] = (1/2)*(1+neta[J])*(1-xsin[J]^2)
N[8] = (1/2)*(1-xsin[J])*(1-neta[J]^2)
N[1] = (1/4)*(1-xsin[J])*(1-neta[J]) - (1/2)*(N[8]+N[5])
N[2] = (1/4)*(1+xsin[J])*(1-neta[J]) - (1/2)*(N[5]+N[6])
N[3] = (1/4)*(1+xsin[J])*(1+neta[J]) - (1/2)*(N[6]+N[7])
N[4] = (1/4)*(1-xsin[J])*(1+neta[J]) - (1/2)*(N[7]+N[8])
2

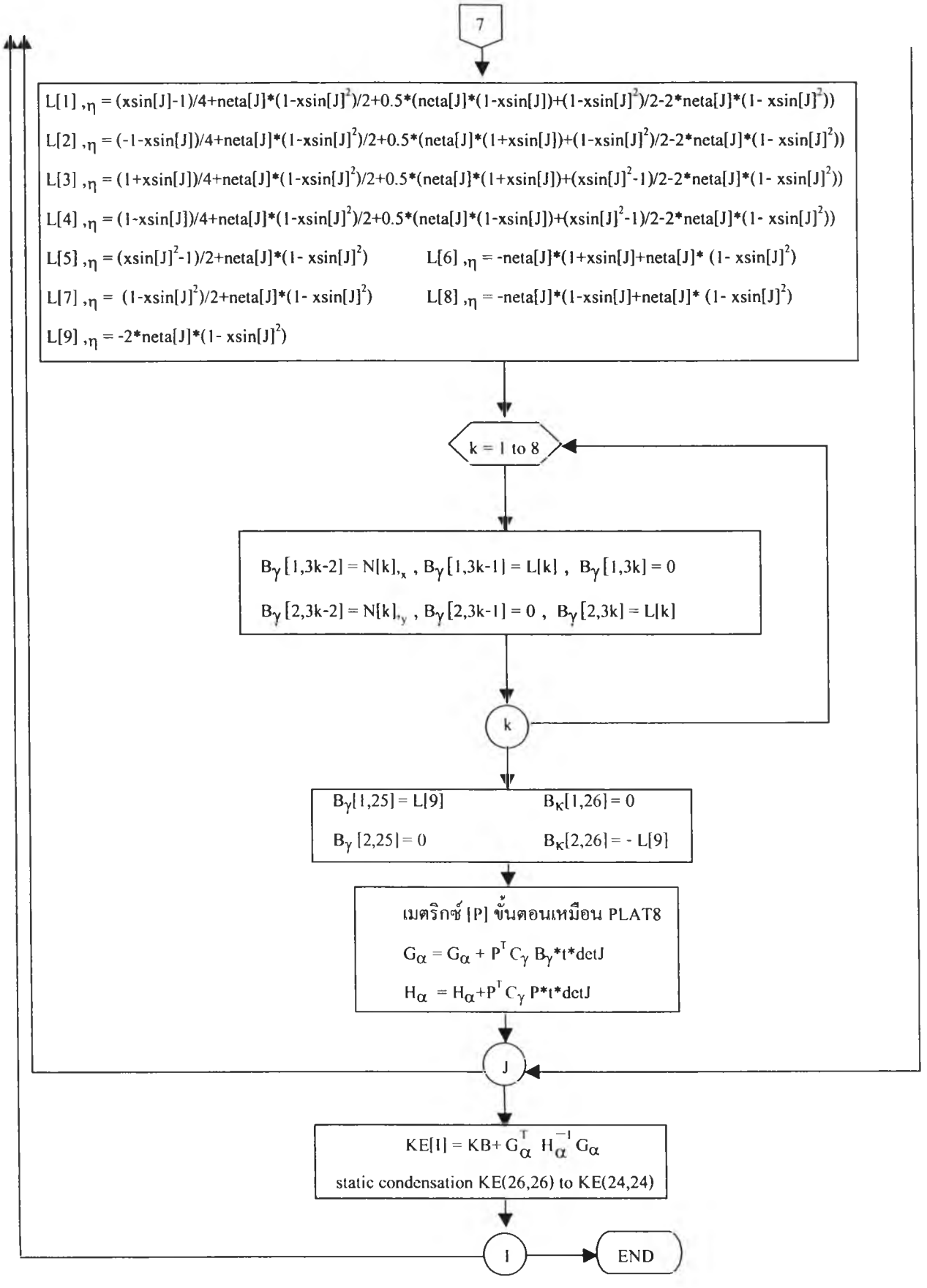

2
L[9] = (1-xsin[J]^2)*(1-neta[J]^2)
L[5] = (1/2)*(1-neta[J])*(1-xsin[J]^2) – 0.5*L[9]
L[6] = (1/2)*(1+xsin[J])*(1-neta[J]^2) – 0.5*L[9]
L[7] = (1/2)*(1+neta[J])*(1-xsin[J]^2) – 0.5*L[9]
L[8] = (1/2)*(1-xsin[J])*(1-neta[J]^2) – 0.5*L[9]
L[1] = (1/4)*(1-xsin[J])*(1-neta[J]) - (1/2)*(L[8]+L[5]) – 0.25*L[9]
L[2] = (1/4)*(1+xsin[J])*(1-neta[J]) - (1/2)*(L[5]+L[6]) – 0.25*L[9]
L[3] = (1/4)*(1+xsin[J])*(1+neta[J]) - (1/2)*(L[6]+L[7]) – 0.25*L[9]
L[4] = (1/4)*(1-xsin[J])*(1+neta[J]) - (1/2)*(L[7]+L[8]) – 0.25*L[9]
N[1],ξ = (xsin[J]/2)+(neta[J]/4)-(xsin[J]*neta[J]/2)-(neta[J]^2/4)
N[2] ,ξ = (xsin[J]/2)-(neta[J]/4)-(xsin[J]*neta[J]/2)+(neta[J]^2/4)
N[3] ,ξ = (xsin[J]/2)+(neta[J]/4)+(xsin[J]*neta[J]/2)+(neta[J]^2/4)
N[4] ,ξ = (xsin[J]/2)+(neta[J]/4)-(xsin[J]*neta[J]/2)-(neta[J]^2/4)
N[5] ,ξ = -xsin[J]*(1-neta[J])
N[6] ,ξ = (1-neta[J]^2)/2
N[7] ,ξ = -xsin[J]*(1-neta[J])
N[8] ,ξ = (neta[J]^2-1)/2
N[1],η = (xsin[J]/4)-(xsin^2[J]/4)+(neta[J]/2)-(xsin[J]*neta[J]/2)
N[2] ,η = (-xsin[J]/4)-(xsin^2[J]/4)+(neta[J]/2)+(xsin[J]*neta[J]/2)
N[3] ,η = (xsin[J]/4)+(xsin^2[J]/4)+(neta[J]/2)+(xsin[J]*neta[J]/2)
N[4] ,η = (-xsin[J]/4)+(xsin^2[J]/4)+(neta[J]/2)-(xsin[J]*neta[J]/2)
N[5] ,η = (xsin[J]^2-1)/2
N[6] ,η = -neta[J]*(1+xsin[J])
N[7] ,η = (1-xsin[J]^2)/2
N[8] ,η = -neta[J]*(1-xsin[J])
3


3
J[1,1] = N[1] ,ξ*xcoor[1]+ N[2] ,ξ*xcoor[2]+...+ N[8] ,ξ*xcoor[8]
J[1,1] = N[1] ,ξ*ycoor[1]+ N[2] ,ξ*ycoor[2]+...+ N[8] ,ξ*ycoor[8]
J[2,1] = N[1] ,η*xcoor[1]+ N[2] ,η*xcoor[2]+...+ N[8] ,η*xcoor[8]
J[2,2] = N[1] ,η*ycoor[1]+ N[2] ,η*ycoor[2]+...+ N[8] ,η*ycoor[8]
detJ = J[1,1]*J[2,2] – J[1,2]*J[2,1]
L[1],ξ = (neta[J]-1)/4+xin[J]*(1-neta[J]^2)/2+0.5*(xsin[J]*(1-neta[J])+(1-neta[J]^2)/2-2*xsin[J]*(1- neta[J]^2))
L[2] ,ξ = (1-neta[J])/4+xin[J]*(1-neta[J]^2)/2+0.5*(xsin[J]*(1-neta[J])+(neta[J]^2-1)/2-2*xsin[J]*(1- neta[J]^2))
L[3] ,ξ = (1+neta[J])/4+xin[J]*(1-neta[J]^2)/2+0.5*(xsin[J]*(1+neta[J])+(neta[J]^2-1)/2-2*xsin[J]*(1- neta[J]^2))
L[4] ,ξ = (-1-neta[J])/4+xin[J]*(1-neta[J]^2)/2+0.5*(xsin[J]*(1+neta[J])+(1-neta[J]^2)/2-2*xsin[J]*(1- neta[J]^2))
L[5] ,ξ = -xsin[J]*(1-neta[J])+xsin[J]*(1- neta[J]^2)
L[6] ,ξ = (1-neta[J]^2)/2+xsin[J]* (1- neta[J]^2)
L[7] ,ξ = -xsin[J]*(1+neta[J])+xsin[J]*(1- neta[J]^2)
L[8] ,ξ = (neta[J]^2-1)/2+xsin[J]*(1- neta[J]^2)
L[9] ,ξ = -2*xsin[J]*(1- neta[J]^2)
L[1] ,η = (xsin[J]-1)/4+neta[J]*(1-xsin[J]^2)/2+0.5*(neta[J]*(1-xsin[J])+(1-xsin[J]^2)/2-2*neta[J]*(1- xsin[J]^2))
L[2] ,η = (-1-xsin[J])/4+neta[J]*(1-xsin[J]^2)/2+0.5*(neta[J]*(1+xsin[J])+(1-xsin[J]^2)/2-2*neta[J]*(1- xsin[J]^2))
L[3] ,η = (1+xsin[J])/4+neta[J]*(1-xsin[J]^2)/2+0.5*(neta[J]*(1+xsin[J])+(xsin[J]^2-1)/2-2*neta[J]*(1- xsin[J]^2))
L[4] ,η = (1-xsin[J])/4+neta[J]*(1-xsin[J]^2)/2+0.5*(neta[J]*(1-xsin[J])+(xsin[J]^2-1)/2-2*neta[J]*(1- xsin[J]^2))
L[5] ,η = (xsin[J]^2-1)/2+neta[J]*(1- xsin[J]^2)
L[6] ,η = -neta[J]*(1+xsin[J]+neta[J]* (1- xsin[J]^2)
L[7] ,η = (1-xsin[J]^2)/2+neta[J]*(1- xsin[J]^2)
L[8] ,η = -neta[J]*(1-xsin[J]+neta[J]* (1- xsin[J]^2)
L[9] ,η = -2*neta[J]*(1- xsin[J]^2)
4


4
II = 1 to 9
L[II] ,x = (J[2,2]/detJ)*L[II] ,ξ - (J[1,2]/detJ)*L[II] ,η
L[II] ,y = (-J[2,1]/detJ)*L[II] ,ξ + (J[1,1]/detJ)*L[II] ,η
II
k = 1 to 8
BK[1,3k-2] = 0 , BK[1,3k-1] = L[k],x , BK[1,3k] = 0
BK[2,3k-2] = 0 , BK[2,3k-1] = 0 , BK[2,3k] = L[k],y
BK[3,3k-2] = 0 , BK[3,3k-1] = L[k],y , BK[3,3k] = L[k],x
k
BK[1,25] = L[9],x
BK[1,26] = 0
BK[2,25] = 0
BK[2,26] = - L[9],y
BK[3,25] = L[9],y
BK[3,26] = - L[9],x
J<5: KB = KB+W1*W1* BK^T CK BK*t*detJ
5≤J<9: KB = KB+W1*W2* BK^T CK BK*t*detJ
J=9: KB = KB+W2*W2* BK^T CK BK*t*detJ
5


5
J<5
True
xsin[1] = -1/√3 , neta[1] = -1/√3
xsin[2] = 1/√3 , neta[2] = -1/√3
xsin[3] = 1/√3 , neta[3]= 1/√3
xsin[4] = -1/√3 , neta[4]= 1/√3
N[5] = (1/2)*(1-neta[J])*(1-xsin[J]^2)
N[6] = (1/2)*(1+xsin[J])*(1-neta[J]^2)
N[7] = (1/2)*(1+neta[J])*(1-xsin[J]^2)
N[8] = (1/2)*(1-xsin[J])*(1-neta[J]^2)
N[1] = (1/4)*(1-xsin[J])*(1-neta[J]) - (1/2)*(N[8]+N[5])
N[2] = (1/4)*(1+xsin[J])*(1-neta[J]) - (1/2)*(N[5]+N[6])
N[3] = (1/4)*(1+xsin[J])*(1+neta[J]) - (1/2)*(N[6]+N[7])
N[4] = (1/4)*(1-xsin[J])*(1+neta[J]) - (1/2)*(N[7]+N[8])
L[9] = (1-xsin[J]^2)*(1-neta[J]^2)
L[5] = (1/2)*(1-neta[J])*(1-xsin[J]^2) – 0.5*L[9]
L[6] = (1/2)*(1+xsin[J])*(1-neta[J]^2) – 0.5*L[9]
L[7] = (1/2)*(1+neta[J])*(1-xsin[J]^2) – 0.5*L[9]
L[8] = (1/2)*(1-xsin[J])*(1-neta[J]^2) – 0.5*L[9]
L[1] = (1/4)*(1-xsin[J])*(1-neta[J]) - (1/2)*(L[8]+L[5]) – 0.25*L[9]
L[2] = (1/4)*(1+xsin[J])*(1-neta[J]) - (1/2)*(L[5]+L[6]) – 0.25*L[9]
L[3] = (1/4)*(1+xsin[J])*(1+neta[J]) - (1/2)*(L[6]+L[7]) – 0.25*L[9]
L[4] = (1/4)*(1-xsin[J])*(1+neta[J]) - (1/2)*(L[7]+L[8]) – 0.25*L[9]
6


6
N[1],ξ = (xsin[J]/2)+(neta[J]/4)-(xsin[J]*neta[J]/2)-(neta[J]²/4)
N[2] ,ξ = (xsin[J]/2)-(neta[J]/4)-(xsin[J]*neta[J]/2)+(neta[J]²/4)
N[3] ,ξ = (xsin[J]/2)+(neta[J]/4)+(xsin[J]*neta[J]/2)+(neta[J]²/4)
N[4] ,ξ = (xsin[J]/2)+(neta[J]/4)-(xsin[J]*neta[J]/2)-(neta[J]²/4)
N[5] ,ξ = -xsin[J]*(1-neta[J]) N[6] ,ξ = (1-neta[J]²)/2
N[7] ,ξ = -xsin[J]*(1-neta[J]) N[8] ,ξ = (neta[J]²-1)/2
N[1],η = (xsin[J]/4)-(xsin²[J]/4)+(neta[J]/2)-(xsin[J]*neta[J]/2)
N[2] ,η = (-xsin[J]/4)-(xsin²[J]/4)+(neta[J]/2)+(xsin[J]*neta[J]/2)
N[3] ,η = (xsin[J]/4)+(xsin²[J]/4)+(neta[J]/2)+(xsin[J]*neta[J]/2)
N[4] ,η = (-xsin[J]/4)+(xsin²[J]/4)+(neta[J]/2)-(xsin[J]*neta[J]/2)
N[5] ,η = (xsin[J]²-1)/2
N[6] ,η = -neta[J]*(1+xsin[J])
N[7] ,η = (1-xsin[J]²)/2
N[8] ,η = -neta[J]*(1-xsin[J])
L[1],ξ = (neta[J]-1)/4+xin[J]*(1-neta[J]²)/2+0.5*(xsin[J]*(1-neta[J])+(1-neta[J]²)/2-2*xsin[J]*(1- neta[J]²))
L[2] ,ξ = (1-neta[J])/4+xin[J]*(1-neta[J]²)/2+0.5*(xsin[J]*(1-neta[J])+(neta[J]²-1)/2-2*xsin[J]*(1- neta[J]²))
L[3] ,ξ = (1+neta[J])/4+xin[J]*(1-neta[J]²)/2+0.5*(xsin[J]*(1+neta[J])+(neta[J]²-1)/2-2*xsin[J]*(1- neta[J]²))
L[4] ,ξ = (-1-neta[J])/4+xin[J]*(1-neta[J]²)/2+0.5*(xsin[J]*(1+neta[J])+(1-neta[J]²)/2-2*xsin[J]*(1- neta[J]²))
L[5] ,ξ = -xsin[J]*(1-neta[J])+xsin[J]*(1- neta[J]²)
L[6] ,ξ = (1-neta[J]²)/2+xsin[J]* (1- neta[J]²)
L[7] ,ξ = -xsin[J]*(1+neta[J])+xsin[J]*(1- neta[J]²)
L[8] ,ξ = (neta[J]²-1)/2+xsin[J]*(1- neta[J]²)
L[9] ,ξ = -2*xsin[J]*(1- neta[J]²)
7


7
L[1] ,η = (xsin[J]-1)/4+neta[J]*(1-xsin[J]²)/2+0.5*(neta[J]*(1-xsin[J])+(1-xsin[J]²)/2-2*neta[J]*(1- xsin[J]²))
L[2] ,η = (-1-xsin[J])/4+neta[J]*(1-xsin[J]²)/2+0.5*(neta[J]*(1+xsin[J])+(1-xsin[J]²)/2-2*neta[J]*(1- xsin[J]²))
L[3] ,η = (1+xsin[J])/4+neta[J]*(1-xsin[J]²)/2+0.5*(neta[J]*(1+xsin[J])+(xsin[J]²-1)/2-2*neta[J]*(1- xsin[J]²))
L[4] ,η = (1-xsin[J])/4+neta[J]*(1-xsin[J]²)/2+0.5*(neta[J]*(1-xsin[J])+(xsin[J]²-1)/2-2*neta[J]*(1- xsin[J]²))
L[5] ,η = (xsin[J]²-1)/2+neta[J]*(1- xsin[J]²)
L[6] ,η = -neta[J]*(1+xsin[J]+neta[J]* (1- xsin[J]²)
L[7] ,η = (1-xsin[J]²)/2+neta[J]*(1- xsin[J]²)
L[8] ,η = -neta[J]*(1-xsin[J]+neta[J]* (1- xsin[J]²)
L[9] ,η = -2*neta[J]*(1- xsin[J]²)
k = 1 to 8
Bγ[1,3k-2] = N[k],x , Bγ[1,3k-1] = L[k] , Bγ[1,3k] = 0
Bγ[2,3k-2] = N[k],y , Bγ[2,3k-1] = 0 , Bγ[2,3k] = L[k]
k
Bγ[1,25] = L[9]
Bγ[2,25] = 0
Bκ[1,26] = 0
Bκ[2,26] = - L[9]
เมตริกซ์ [P] ขั้นตอนเหมือน PLAT8
Gα = Gα + PᵀCγ Bγ*t*detJ
Hα = Hα+PᵀCγ P*t*detJ
J
KE[I] = KB+ G_α^T H_α^-1 Gα
static condensation KE(26,26) to KE(24,24)
I
END

ประวัติผู้เขียนวิทยานิพนธ์

นายภาคภูมิ วานิชกมลนันท์ เกิดเมื่อวันที่ 23 เดือนมิถุนายน พุทธศักราช 2516 มีภูมิลำเนาอยู่บ้านเลขที่ 38 หมู่ 8 ตำบลหนองผึ้ง อำเภอสารภี จังหวัดเชียงใหม่ สำเร็จการศึกษาปริญญาตรีวิศวกรรมศาสตรบัณฑิต สาขาวิศวกรรมโยธา คณะวิศวกรรมศาสตร์ จากสถาบันเทคโนโลยีพระจอมเกล้าธนบุรี ในปีการศึกษา 2536 หลังจากนั้นเข้าทำงานที่บริษัท อรุณชัยเสรี คอนซัลติ้งเอนจิเนียร์ จำกัด และต่อมาเข้าศึกษาในหลักสูตรวิศวกรรมศาสตรมหาบัณฑิตที่ภาควิชาวิศวกรรมโยธา คณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย ในปีการศึกษา 2540