CHAPTER II
BACKGROUND AND LITERATURE SURVEY

2.1 State of the Arts for Heat Exchanger Network Synthesis

Heat exchanger network (HEN) synthesis is one of the most extensively studied problems in industrial process synthesis. This is attributed to the importance of determining the energy costs for a process and improving the energy recovery in industrial sites. The first systematic method to consider energy recovery was the thermodynamic approach of the concept of pinch, introduced during the 1970s.

The first approaches in the 1960s and early 1970s treated the HEN synthesis problem without applying decomposition into sub-tasks. The limitations of optimization techniques were the bottleneck of the mathematical approaches at that time. For the synthesis problem of the HEN, the thermodynamic approach of pinch analysis was introduced by the work of Hohmann (1971) and Linnhoff and Flower (1978). As a result of the pinch concept, the single task approaches were shifted to procedures introducing techniques for decomposing the problem into three subtasks; minimum utility cost, minimum number of units and minimum investment cost network configurations. The main advantage of decomposing the HEN synthesis problem is that sub-problems can be treated in a much easier fashion than the original single-task problem. The sub-problems are the following

2.1.1 Minimum Utility Cost Target

Corresponds to the maximum energy recovery that can be achieved in a feasible HEN for a fixed heat recovery approach temperature (HRAT), allowing for the elimination of several non-energy efficient HEN structures. Minimum utility cost was first introduced by Hohmann (1971) and Linnhoff and Flower (1978) and later as an LP transportation model by Cerda et al. (1983), being an improvement of the LP transshipment model of Papoulias and Grossmann (1983).

2.1.2 Minimum Number of Units Target

Determines the match combination with the minimum number of units and their load distribution for a fixed utility cost. The MILP transportation model of Cerda and Westerberg (1983) and the MILP transshipment model of Papoulias and Grossmann (1983) are the most common, while the vertical heat transfer formulation of Gundersen and Grossmann (1990) and Gundersen, Duvold and Hashemi-Ahmady (1996) are also used.

2.1.3 Minimum Investment Cost Network Configurations

It is based on the heat load and match information of previous targets. Using the superstructure-based formulation, developed by Floudas et al. (1986), the NLP problem is formulated and optimized for the minimum total cost of the network. The objective function in this model is the investment cost of the heat exchangers that are postulated in a superstructure.

However, limitation of decomposition-based methods is that costs due to energy, units and area cannot be optimized simultaneously, and as a result the trade-offs are not taken into account appropriately. Thus, simultaneous heat exchanger network synthesis methods are taken place. The simultaneous approaches purpose to find the optimal network with or without some decomposed problem. The simultaneous optimization generally results in MINLP formulations, which assumptions exist to simplify these complex models.

Floudas and Ciric (1989) proposed a match-network hyperstructure model to simultaneously optimize all of the capital costs related to the heat exchanger network. This MINLP formulation is based on the combination of the transshipment model of Papoulias and Grossmann (1983) for match selection, and the minimum investment cost network configuration model of Floudas and Grossmann (1986) for determining the heat exchanger areas, temperatures and the flow rate in the network. The proposed simultaneous synthesis may still lead to suboptimal networks, since the value for HRAT must be specified before the design stage.

In 1990, Yee and Grossmann formulated another simultaneous synthesis where within each stage exchanges of heat can occur between each hot and cold stream. This model can simultaneously target for area and energy cost while properly accounting for the differences in heat transfer coefficients between the streams. The match-network hyperstructure model was then further modified by Ciric and Floudas (1991) to treat HRAT as an explicit optimization variable. This MINLP formulation included any decomposition into design targets and simultaneously optimizes trade-offs between energy, units and area. Ciric and Floudas (1991) also demonstrated the benefit of a simultaneous approach versus sequential methods.

In 1986, Floudas and Grossmann introduced a multiperiod MILP model for the minimum utilities cost and minimum number of match of target problems, based on Papoulias and Grossmann's (1983) transshipment model. In this model the changes in the pinch point and utility required at each time period are taken into account. Extensions were presented first by Floudas and Grossmann (1987), and NLP formulation based on a superstructure presentation of possible network topologies to derive automatically network configurations that feature minimum investment cost, minimum number of units, and minimum utility cost for each time period.

Ji and Bagajewicz (2001) introduced the rigorous procedure for the design of conventional atmospheric crude fractionation units. Part I aims to find the best scheme of a multipurpose crude distillation unit which can process the various crude. Heat demand-supply diagrams are used as a guide for optimal scheme instead of grand composite curves. Thus, the total energy consumption from stream, heater and cooler is clearly shown and this leads the process to be easily optimal. In part II, 2001, Soto and Bagajewicz attempted to design a multipurpose heat exchanger network that can handle in variety of crude. In order to overcome the smaller gap between hot and cold composite curves, models that fixed the heat recovery by using the minimum heat recovery approximation temperature (HRAT) and the exchanger minimum approach temperature (EMAT) was performed. In 2003, Part III, Soto and Bagajewicz established a model to determine a heat exchanger network with only two branches above and below desalter. The total annualized costs, operating cost and depreciation of capital, of solution limited to one or two branches are compared with the results of four branches. In this part, the present model is based on a transshipment model and the vertical heat exchange constraints combined with HRAT/EMAT. In addition, investment cost is not directly controlled by this model, but further indirectly controlled by limiting of the minimum unit numbers. The smaller number of units leads to minimal capital cost and energy consumption simultaneously.

New rigorous one-step MILP formulation for heat exchanger network synthesis was developed by Barbaro and Bagajewicz (2002). This methodology does neither rely on traditional supertargeting network design by the pinch technology, nor is a nonlinear model, but further use only one-step to optimize the solution. Cost-optimal networks, cost-effective solutions, can be obtained at once by using this model.

2.2 Basic Concepts for Using Mathematical Programming in Process Integration

Mathematical programming is a class of methods for solving constrained optimization problems. Since both continuous and binary variables can be used in the corresponding mathematical programming models, these methods are perfectly suited for typical design tasks encountered in process synthesis and process integration.

Generally, a mathematical programming model consists of an objective function (typically some economic criteria) and a set of equality constraints as well as inequality constraints. The general form is indicated below

$$\text{Min } f(x,y)$$

Subject to

$$g(x,y) \leq 0$$

$$h(x,y) = 0$$

where

$$x \in R^n$$

$$y \in [0,1]^m$$

It should be noticed that the variables x and y in general are vectors of variables, and that the constraints g and h similarly are vectors of functions. The objective function (f) is assumed to be a scalar.

The mathematical modeling of the systems lead to different types of formulations, such as Linear Programming (LP), Mixed Integer Linear Programming (MILP), Non-Linear Programming (NLP) and Mixed Integer Non-Linear Programming (MINLP) models.

If there are no binary variables, and all functions f, g and h are linear, we have the simplest class of problems, the Linear Programming (LP) models. Using the simplex algorithm, for example, LP models with hundreds of thousands variables and constraints can be solved in reasonable times with today's computer resources. If there are no binary variables, and at least one of the functions f, g and h are non-linear, we have a Non-Linear Programming (NLP) problem. These are generally much harder to solve, especially if the non-linearities are non-convex, because a local optimum may be found.

If there are binary variables in the model, and all functions f, g and h are linear, we have a Mixed Integer Linear Programming (MILP) problem. These can be solved to global optimality provided the number of binary variables does not cause a combinatorial explosion. Finally, if there are binary variables in the model, and at least one of the functions f, g and h are non-linear, we have the hardest class of problems, Mixed Integer Non-Linear Programming (MINLP) models. Unfortunately, most real design problems are of the MINLP type with significant problems related to computer time and local optima.

2.3 Model for Grass-Root Synthesis

This MILP model is based on the transportation transshipment scheme and it has the following features

- Counts heat exchangers units and shells
- Approximates the area required for each exchanger unit or shell
- Controls the total number of units

- Implicitly determines flow rates in splits
- Handles non-isothermal mixing
- Identifies bypasses in split situations when convenient
- Controls the temperature approximation (HRAT/EMAT of ΔT_{min}) when desired
- Can address block-design through the use of zones
- Allows multiple matches between two streams

2.4 Mathematical Model

2.4.1 Set Definitions

A set of several heat transfer zones is defined, namely $Z - \{z \mid z$ is a heat transfer zone$\}$

Use of zones can be used to separate the design in different sub-networks that are not interrelated, simplifying the network and the problem complexity. Next, the following sets are used to identify hot streams, cold streams, hot utilities and cold utilities.

H^z $= \{ i \mid i$ is a hot stream present in zone $z \}$

C^z $= \{ j \mid j$ is a cold stream present in zone $z \}$

$HU^z = \{ i \mid i$ is a heating utility present in zone $z \}$ $\quad (HU^z \subset H^z)$

$CU^z = \{ j \mid j$ is a heating utility present in zone $z \}$ $\quad (CU^z \subset C^z)$

Moreover, several temperature intervals are considered in each zone, in order to perform the heat balances and the area calculations. The different sets related to the temperature intervals are defined as

M^z $= \{ m \mid m$ is a temperature interval in zone z $\}$

M_i^z $= \{ m \mid m$ is a temperature interval belonging to zone z, in which hot stream i is presented $\}$

N_j^z = { n | n is a temperature interval belonging to zone z, in which cold stream j is presented }

H_m^z = { i | i is a hot stream present in temperature interval m in zone z }

C_n^z = { j | j is a cold stream present in temperature interval n in zone z }

m_i^0 = { m | m is the starting temperature interval for hot stream i }

n_j^0 = { n | n is the starting temperature interval for cold stream j }

m_i^f = { m | m is the final temperature interval for hot stream i }

n_j^f = { n | n is the final temperature interval for cold stream j }

The MILP model uses the temperature intervals to perform energy balances and mass flow balances. At each temperature interval, the variables $\hat{q}_{ijm}^{z,H}$ account for the overall heat exchanged in interval m of hot stream i and all the intervals of cold stream j, in zone z. Familiar with $\hat{q}_{ijm}^{z,H}$, the variables $\hat{q}_{ijn}^{z,C}$ are used to compute the overall heat received by cold stream j at interval n from all intervals of hot stream i. The variables $q_{im,jn}^{z,H}$ are used to formulate the heat transportation from interval to interval between both streams.

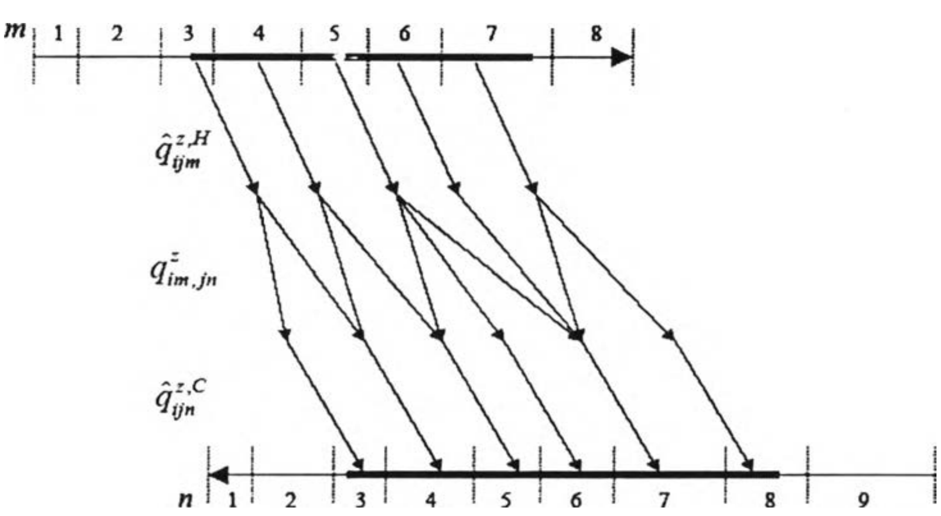


Figure 2.1 Basic scheme of the transportation/transshipment model.

A number of sets are introduced to define all possible sources and destinations for heat transfer in this transportation scheme.

P = { (i,j) | heat exchange match between hot stream i and cold stream j is permitted }

P^{H}_{im} = { j | heat transfer from hot stream i at interval m to cold stream j is permitted }

P^{C}_{jn} = { i | heat transfer from hot stream i to cold stream j at interval n is permitted }

Set P defines as allowed matching between hot and cold streams. In order not to againt the thermodynamically possible, permitted and forbidden heat exchange matches can be set up by the designer. Sets P^{H}_{im} and P^{C}_{jn} define as feasible heat transfer flows at each temperature interval.

Finally, the following sets allow the designer to manage additional features of the formulation.

NI^{H} = { i | non-isothermal mixing is permitted for hot stream i }

NI^{C} = { j | non-isothermal mixing is permitted for cold stream j }

S^{H} = { i | splits are allowed for hot stream i }

S^{C} = { j | splits are allowed for cold stream j }

B = { (i,j) | more than one heat exchanger unit is permitted between hot stream i and cold stream j }

The sets NI^{H} and NI^{C} are used to specify whether non-isothermal mixing of stream splits is permitted, while sets S^{H} and S^{C} establish the possibility of stream splits. Finally, set B is used to allow more than one heat exchanger match between two streams, as shown in Figure 2.2 for match (i_1,j_1). Thus, this model is able to distinguish situations where more than one heat exchanger unit is required to perform a heat exchange match. Next, the different equations of the model for grass-root design of heat exchanger networks are introduced.

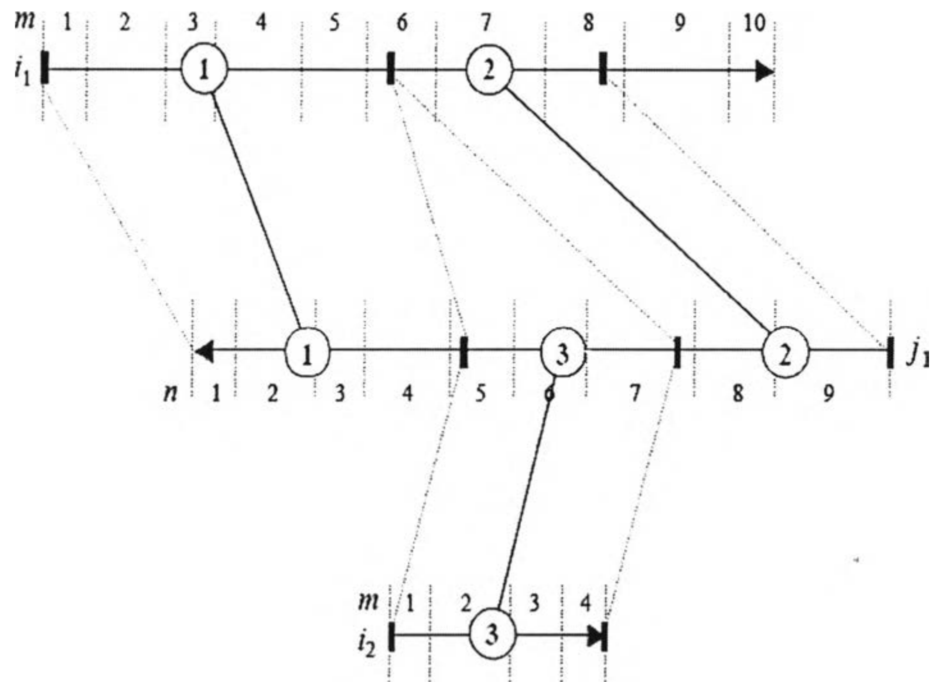


Figure 2.2 A case where more than one heat exchanger unit is required for a match (i,j).

2.4.2 Heat Balance Equations

The total heat available on each hot streams or the total heat demand of cold streams is equal to the heat transferred to the specific intervals. For heating and cooling utilities, these balances are described by the following equations.

Heat balance for heating utilities

$$F_i^H\left(T_m^U - T_m^L\right) = \sum_{\substack{n\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j\in C_n^z \\ j\in P_{im}^H \\ i\in P_{jn}^C}} q_{im,jn}^z \qquad z\in Z; m\in M^z; i\in H_m^z; i\in HU^z \qquad (2.1)$$

Heat balance for cooling utilities

$$F_j^C\left(T_n^U - T_n^L\right) = \sum_{\substack{m\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{i\in H_m^z \\ i\in P_{jn}^C \\ j\in P_{im}^H}} q_{im,jn}^z \qquad z\in Z; n\in M^z; j\in C_n^z; j\in CU^z \qquad (2.2)$$

The heat balances for process streams where only isothermal mixing of splits is considered are stated below.

Heat balance for hot process streams – $i \notin NI^H$

$$\Delta H_{im}^{z,H} = \sum_{\substack{n \in M^z \\ T_n^L < T_m^L}} \sum_{\substack{j \in C_n^z \\ j \in P_{im}^H \\ i \in P_{jn}^C}} q_{im,jn}^z \qquad z \in Z; m \in M^z; i \in H_m^z; i \notin HU^z; i \notin NI^H \quad (2.3)$$

Heat balance for cold process streams – $j \notin NI^C$

$$\Delta H_{jn}^{z,C} = \sum_{\substack{m \in M^Z \\ T_n^L < T_m^U}} \sum_{\substack{i \in H_m^z \\ i \in P_{jn}^C \\ j \in P_{im}^H}} q_{im,jn}^z \qquad z \in Z; n \in M^z; j \in C_n^z; j \notin CU^z; i \notin NI^C \quad (2.4)$$

The hot and cold cumulative heat transfer is defined in the next sets of equations. This cumulative transfer is introduced for presentation convenience because it is related to the equations that define the existence of heat exchangers in the different temperature intervals.

Cumulative heat transfer from hot stream i at interval m to cold stream j

$$\hat{q}_{ijm}^{z,H} = \sum_{\substack{n \in M^z; T_n^L < T_m^U \\ j \in C_n^z; i \in P_{jn}^C}} q_{im,jn}^z \qquad z \in Z; m \in M^z; i \in H_m^z; j \in C^z; j \in P_{im}^H \quad (2.5)$$

Cumulative heat transfer to cold stream j at interval n from hot stream i

$$\hat{q}_{ijn}^{z,C} = \sum_{\substack{m \in M^z; T_n^L < T_m^U \\ i \in H_m^z; j \in P_{im}^H}} q_{im,jn}^z \qquad z \in Z; n \in M^z; i \in H^z; j \in C_n^z; i \in P_{jn}^C \quad (2.6)$$

2.4.2.1 Heat Balance Equations for Streams Allowed to Have Non-Isothermal Split Mixing

A new variable ($\bar{q}$) is introduced to account for heat flows between intervals of the same stream that correspond to such mixing. Heat is artificially transferred from one interval to another within the same stream to account for non-isothermal mixing conditions. Figure 2.3 illustrates how this non-isothermal mixing of stream splits is taken into account.

Following the Figure 2.3, cold stream j has been split to exchange heat between stream i_1 and i_2 and non-isothermal mixing between these splits is allowed. This figure shows the upper portion, the split in the cold stream spans temperature intervals 3 and 8, while the lower portion spans from interval 5 to interval 8. However, the whole stream spans from interval 4 to interval 8 after mixing and the non-split part spans the rest of the intervals. In order to complete the non-isothermal mixing which allow one branch to reach a larger temperature as shown in the Figure 2.3, interval 3 get more heat than its demand $\left(\Delta H_{j3}^{z,C}\right)$ and transfer this surplus heat to interval 4 and 5. Interval 4 and 5 receive less heat than their demand from the hot streams, with the difference being transferred from interval 3 by the heat $\bar{q}$. The heat balance equations for non-isothermal mixing of split are shown as

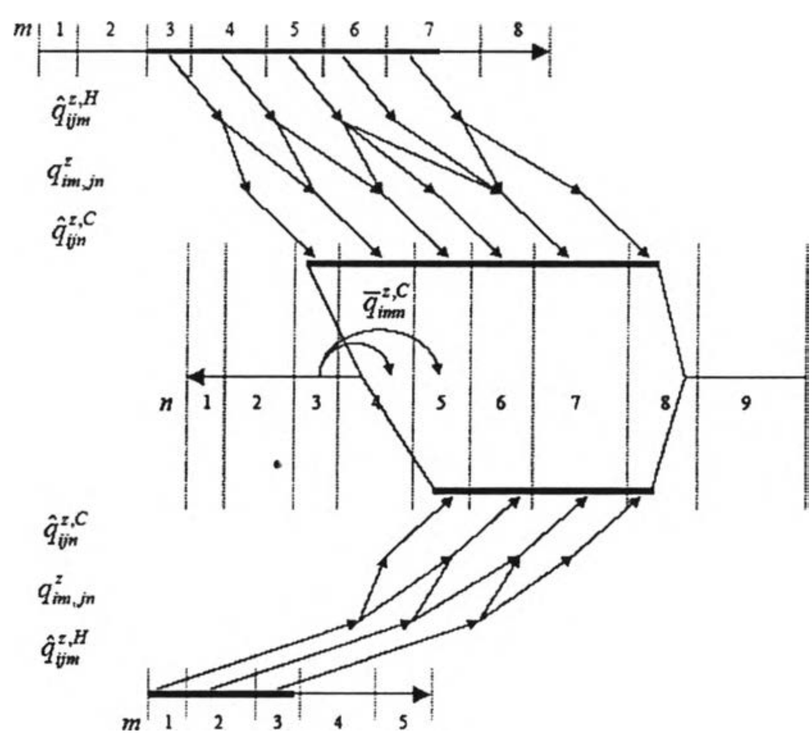


Figure 2.3 Non-isothermal split mixing.

Heat balance for hot streams (non-isothermal mixing allowed)

$$\Delta H_{im}^{z,H} = \sum_{\substack{n\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j\in C_n^z \\ j\in P_{im}^H \\ i\in P_{jn}^C}} q_{im,jn}^z + \sum_{\substack{n\in M^z \\ n>m}} \sum_{i\in H_n^z} \bar{q}_{inm}^{z,H} - \sum_{\substack{n\in M^z \\ n<m}} \sum_{i\in H_n^z} \bar{q}_{inm}^{z,H} \qquad z\in Z; m\in M^z; i\in H_m^z; i\notin HU^z; i\in NI^H \tag{2.7}$$

Heat balance for cold streams (non-isothermal mixing allowed)

$$\Delta H_{jn}^{z,C} = \sum_{\substack{m\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{i\in H_m^z \\ i\in P_n^C \\ j\in P_m^H}} q_{im,jn}^z + \sum_{\substack{m\in M^z \\ m<n}} \sum_{j\in C_m^z} \bar{q}_{jmn}^{z,C} - \sum_{\substack{m\in M^z \\ m>n}} \sum_{j\in C_m^z} \bar{q}_{inm}^{z,C} \qquad z\in Z; n\in M^z; j\in C_n^z; j\notin CU^z; j\in NI^C \tag{2.8}$$

In addition, the condition that heat cannot be transferred within a stream if there is no heat transfer with other stream need to be established in the model. Consequently, these equations force $\bar{q}$ to be zero whenever there is no heat transferred with other streams.

Heat balance for hot streams – i $\in NI^H$

$$\sum_{\substack{n\in M^z \\ n<m}} \sum_{i\in H_n^z} \bar{q}_{inm}^{z,H} \le \sum_{\substack{n\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{j\in C_n^z ; j\in P_{im}^H \\ i\in P_{jn}^C}} q_{im,jn}^z \qquad z\in Z; m\in M^z; i\in H_m^z; i\notin HU^z; i\in NI^H \tag{2.9}$$

Heat balance for cold streams – i $\in NI^C$

$$\sum_{\substack{m\in M^z \\ m>n}} \sum_{j\in C_m^z} \bar{q}_{inm}^{z,C} \le \sum_{\substack{m\in M^z \\ T_n^L < T_m^U}} \sum_{\substack{i\in H_m^z ; i\in P_n^C \\ j\in P_m^H}} q_{im,jn}^z \qquad z\in Z; n\in M^z; j\in C_n^z; j\notin CU^z; j\in NI^C \tag{2.10}$$

2.4.3 Heat Exchanger Definition and Count

The model is defined as a consecutive series of heat exchange shells between a hot and a cold stream. For each temperature interval, heat transfer is accounted using the cumulative heat $(\hat{q})$, while the existence of a heat exchanger for a given interval is defined by a new variable (*Y*), which determines whether heat exchange takes place or not at that interval. In addition, two new variables (*K* and $\hat{K}$), which are closely related to the *Y* variables, are introduced in order to indicate whether a heat exchanger begins or ends at a specific interval. The use of these new variables to count units has been previously proposed by Bagajewicz and Rodera (1998) and later used by Bagajewicz and Soto (2001, 2003) and Ji and Bagajewicz (2002).

Even placing the multiple shells, this seems to be as a single heat exchanger. Nevertheless, there are cases where non-consecutive series of shells could be allowed. For those cases, different heat exchangers have to be defined for each series. In order to consider the possibility of multiple heat exchangers between the same pair of streams, the additional equations are required.

For the case where only one exchanger is allowed per match between streams i and j, $(i,j)\notin B$, then binary variable $Y_{ijm}^{z,H}$, and two continuous variables $K_{ijm}^{z,H}$, $\hat{K}_{ijm}^{z,H}$ are used. The binary variable $Y_{ijm}^{z,H}$, indicates that there is a match between stream i at interval m receiving heat from some intervals of stream j. In turn, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ indicate the beginning and end of a string of intervals for which the binary variable is active. Conversely, when $(i,j)\in B$, $Y_{ijm}^{z,H}$ is declared as continuous and $K_{ijm}^{z,H}$, $\hat{K}_{ijm}^{z,H}$ are set up as binary. The Y variables are probably greater or equal than one if a heat exchanger exists for the correspondent streams and interval. However, all variables $Y_{ijm}^{z,H}$, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ are getting to be zero when no heat exchanger exists matching streams i and j.

The following group of constraints is used to determine the existence of a heat exchanger for a given pair of streams and temperature intervals. When only one heat exchanger is allowed per match, constraint (2.15)–(2.19) and (2.20)–(2.24) are valid. The equation (2.25) applies further in cases where more than one exchanger is permitted. However, equations (2.15) and (2.20) only apply to the first and last interval of a hot stream, respectively, while the sets of equations (2.16)–(2.19) and (2.21)–(2.24) are used for all intervals.

Bounds on cumulative heat transfer for hot process streams

$$q_{ijm}^{L} Y_{ijm}^{z,H} \le \hat{q}_{ijm}^{z,H} \le \Delta H_{im}^{z,H} Y_{ijm}^{z,H} \qquad z\in Z\,; m\in M^{z}\,; i\in H_{m}^{z}\,; i\notin HU^{z}\,; j\in C^{z}\,; j\in P_{im}^{H} \qquad (2.11)$$

Bounds on cumulative heat transfer for cold process streams

$$q_{ijn}^{L} Y_{ijn}^{z,C} \le \hat{q}_{ijn}^{z,C} \le \Delta H_{jn}^{z,C} Y_{ijn}^{z,C} \qquad z \in Z; n \in M^{z}; i \in H^{z}; j \in C_{n}^{z}; j \notin CU^{z}; i \in P_{jn}^{C} \tag{2.12}$$

Bounds on cumulative heat transfer for heating utilities

$$q_{ijm}^{L} Y_{ijm}^{z,H} \le \hat{q}_{ijm}^{z,H} \le F_{i}^{U} \left(T_{m}^{U} - T_{m}^{L}\right) \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z}; i \in HU^{z}; j \in C^{z}; j \in P_{im}^{H} \tag{2.13}$$

Bounds on cumulative heat transfer for cooling utilities

$$q_{ijn}^{L} Y_{ijn}^{z,C} \le \hat{q}_{ijn}^{z,C} \le F_{j}^{U} \left(T_{n}^{U} - T_{n}^{L}\right) \qquad z \in Z; n \in M^{z}; i \in H^{z}; j \in C_{n}^{z}; j \in CU^{z}; i \in P_{jn}^{C} \tag{2.14}$$

Heat exchanger beginning for hot streams – $(i,j) \notin B$

$$K_{ijm}^{z,H} \ge Y_{ijm}^{z,H} \qquad z \in Z; m \in M^{z}; m = m_{i}^{0}; i \in H^{z}; j \in C^{z}; j \in P_{im}^{H}; (i,j) \notin B \tag{2.15}$$

$$K_{ijm}^{z,H} \le 2 - Y_{ijm}^{z,H} - Y_{ijm-1}^{z,H} \tag{2.16}$$

$$K_{ijm}^{z,H} \le Y_{ijm}^{z,H} \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z} \cap H_{m-1}^{z}; j \in C^{z}; j \in P_{im}^{H} \cap P_{im-1}^{H}; (i,j) \notin B \tag{2.17}$$

$$K_{ijm}^{z,H} \ge Y_{ijm}^{z,H} - Y_{ijm-1}^{z,H} \tag{2.18}$$

$$K_{ijm}^{z,H} \ge 0 \tag{2.19}$$

Heat exchanger ending for hot streams – $(i,j) \notin B$

$$\hat{K}_{ijm}^{z,H} \ge Y_{ijm}^{z,H} \qquad z \in Z; m \in M^{z}; m = m_{i}^{f}; i \in H^{z}; j \in C^{z}; j \in P_{im}^{H}; (i,j) \notin B \tag{2.20}$$

$$\hat{K}_{ijm}^{z,H} \le 2 - Y_{ijm}^{z,H} - Y_{ijm+1}^{z,H} \tag{2.21}$$

$$\hat{K}_{ijm}^{z,H} \le Y_{ijm}^{z,H} \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z} \cap H_{m+1}^{z}; j \in C^{z}; j \in P_{im}^{H} \cap P_{im+1}^{H}; (i,j) \notin B \tag{2.22}$$

$$\hat{K}_{ijm}^{z,H} \ge Y_{ijm}^{z,H} - Y_{ijm+1}^{z,H} \tag{2.23}$$

$$\hat{K}_{ijm}^{z,H} \ge 0 \tag{2.24}$$

Heat exchanger existence on hot streams - $(i,j) \in B$

$$Y_{ijm}^{z,H} = \sum_{\substack{l \in M_{i}^{z} \\ l \le m \\ j \in P_{il}^{H}}} K_{ijl}^{z,H} - \sum_{\substack{l \in M_{i}^{z} \\ l \le m-1 \\ j \in P_{il}^{H}}} \hat{K}_{ijl}^{z,H} \qquad z \in Z; m \in M^{z}; i \in H_{m}^{z}; j \in C^{z}; j \in P_{im}^{H}; (i,j) \in B \tag{2.25}$$

The example shown in Figure 2.4 for a match $(i,j) \notin B$, only one heat exchanger is allowed, will explain how the previous sets of constraints work. The hot side of heat exchanger spans from interval 3 to 8 of stream i, heat transferred to cold stream j is not shown. Since, only one heat exchanger is permitted for this match, variables $Y_{ijm}^{z,H}$ are defined as binary while $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ are continuous. The values for all variables are given in Table 2.1. These numbers correspond to the set of constraints in (2.15)-(2.19) and (2.20)-(2.24).

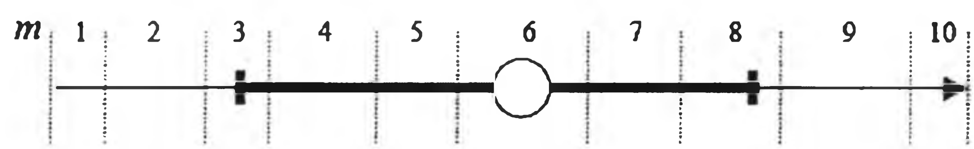


Figure 2.4 Heat exchanger definition when $(i,j) \notin B$.

Table 2.1 Values of $Y_{ijm}^{z,H}$, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ variables when $(i,j) \notin B$

m	$Y_{ijm}^{z,H}$	$K_{ijm}^{z,H}$	$\hat{K}_{ijm}^{z,H}$
1	0	0	0
2	0	0	0
3	1	1	0
4	1	0	0
5	1	0	0
6	1	0	0
7	1	0	0
8	1	0	1
9	0	0	0
10	0	0	0

Following Figure 2.4, whenever $Y_{ijm}^{z,H} = 0$ then it follows that $K_{ijm}^{z,H} = 0$ and $\hat{K}_{ijm}^{z,H} = 0$, explain in constraint (2.17) and (2.22). At any interval where $Y_{ijm-1}^{z,H} = 1$, constraint (2.18) becomes trivial and thus $K_{ijm}^{z,H}$ is getting to be zero because when $Y_{ijm}^{z,H} = 1$, constraint (2.16) gives $K_{ijm}^{z,H}$ to zero.

The possibility of allowing two heat exchangers between the same pair of streams is considered. In Figure 2.5, there are two heat exchangers between the shown hot stream and a certain cold stream, $(i,j) \in B$. Both exchangers are placed in series for the hot stream without any other unit in between. Then, the constraint (2.25) is used for defining heat exchangers existence. Additionally, variables $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ are declared as binary while $Y_{ijm}^{z,H}$ are stated as continuous which the values of these variables are shown in Table 2.2.

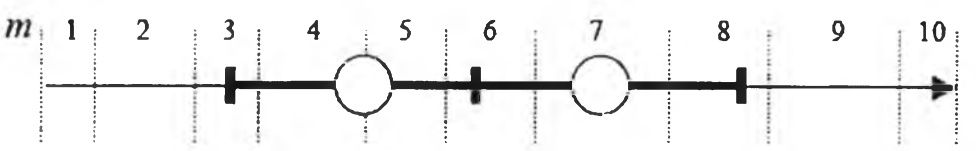


Figure 2.5 Heat exchanger definition when $(i,j) \in B$.

Table 2.2 Values of $Y_{ijm}^{z,H}$, $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ variables when $(i,j) \in B$

m	Y_{ijm}	K_{ijm}	$\hat{K}_{ijm}$
1	0	0	0
2	0	0	0
3	1	1	0
4	1	0	0
5	1	0	0
6	2	1	1
7	1	0	0
8	1	0	1
9	0	0	0
10	0	0	0

Whenever a heat exchanger begins or ends, the binary variables $K_{ijm}^{z,H}$ and $\hat{K}_{ijm}^{z,H}$ are set to one. Then constraint (2.25) leads the values of $Y_{ijm}^{z,H}$ equal to one for all intervals m between the beginning and end of a heat exchanger. Note that, when a heat exchanger between the same pair of stream ends and another one begins in the same interval (interval 6 for this example) then $Y_{ijm}^{z,H}$ is equal to two. Since $Y_{ijm}^{z,H}=2$ is not feasible if the Y are declared as binary variables and constraints (2.15) and (2.16) are used, this is why a different set of equations and variable declarations is required when $(i,j) \in B$, at a cost of increasing the number of binary variables.

A similar set of equations is used to define the location of a heat exchanger for cold streams. These expressions are presented next without further explanation.

Heat exchanger beginning for cold streams - $(i,j) \notin B$

$$K_{ijn}^{z,C} \geq Y_{ijn}^{z,C} \qquad z \in Z; n \in M^z; n = n_j^0; i \in H^z; j \in C_n^z; i \in P_{jn}^C; (i,j) \notin B \tag{2.26}$$

$$\left.\begin{aligned} & K_{ijn}^{z,C} \leq 2 - Y_{ijn}^{z,C} - Y_{ijn-1}^{z,C} && (2.27)\\ & K_{ijn}^{z,C} \leq Y_{ijn}^{z,C} && (2.28)\\ & K_{ijn}^{z,C} \geq Y_{ijn}^{z,C} - Y_{ijn-1}^{z,C} && (2.29)\\ & K_{ijn}^{z,C} \geq 0 && (2.30) \end{aligned}\right\} \quad z \in Z; n \in M^z; i \in H^z; j \in C_n^z \cap C_{n-1}^z; i \in P_{jn}^C \cap P_{jn-1}^C; (i,j) \notin B$$

Heat exchanger ending for cold streams - $(i,j) \notin B$

$$\hat{K}_{ijn}^{z,C} \geq Y_{ijn}^{z,C} \qquad z \in Z; n \in M^z; n = n_j^0; i \in H^z; j \in C_n^z; i \in P_{jn}^C; (i,j) \notin B \tag{2.31}$$

$$\left.\begin{aligned} & \hat{K}_{ijn}^{z,C} \leq 2 - Y_{ijn}^{z,C} - Y_{ijn-1}^{z,C} && (2.32)\\ & \hat{K}_{ijn}^{z,C} \leq Y_{ijn}^{z,C} && (2.33)\\ & \hat{K}_{ijn}^{z,C} \geq Y_{ijn}^{z,C} - Y_{ijn-1}^{z,C} && (2.34)\\ & \hat{K}_{ijn}^{z,C} \geq 0 && (2.35) \end{aligned}\right\} \quad z \in Z; n \in M^z; i \in H^z; j \in C_n^z \cap C_{n-1}^z; i \in P_{jn}^C \cap P_{jn-1}^C; (i,j) \notin B$$

Heat exchanger existence on cold streams - $(i,j) \in B$

$$Y_{ijn}^{z,C} = \sum_{\substack{l \in N_j^z \\ l \leq n \\ i \in P_{jl}^C}} K_{ijl}^{z,C} - \sum_{\substack{l \in N_j^z \\ l \leq n-1 \\ i \in P_{jl}^C}} \hat{K}_{ijl}^{z,C} \qquad z \in Z; n \in M^z; i \in H^z; j \in C_n^z; i \in P_{jn}^C; (i,j) \in B \tag{2.36}$$

Lastly, by counting the number of beginnings or endings of heat exchanger, the number of heat exchanger units between a given pair of streams, E_{ij}^z, can be figured out. The beginnings number is calculated by equation (2.37) to (2.38) and equation (2.39) to (2.40) is used to generate the endings number. For the last equation, (2.42), the number of shell, U_{ij}^z, need to be greater or equal to the number of heat exchanger units, E_{ij}^{z}. Because a single heat exchanger does not mean only one shell, the shell number should be need to satisfy the required area for each match.

Number of heat exchangers between hot stream i and cold stream j – $(i,j) \notin B$

$$E_{ij}^{z} = \sum_{m \in M_i^z\,;\, j \in P_{im}^H} K_{ijm}^{z,H} \qquad (2.37)$$

$$E_{ij}^{z} = \sum_{n \in N_j^z\,;\, i \in P_{jn}^C} K_{ijn}^{z,C} \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P \qquad (2.38)$$

$$E_{ij}^{z} = \sum_{m \in M_i^z\,;\, j \in P_{im}^H} \hat{K}_{ijm}^{z,H} \qquad (2.39)$$

$$E_{ij}^{z} = \sum_{n \in N_j^z\,;\, i \in P_{jn}^C} \hat{K}_{ijn}^{z,C} \qquad (2.40)$$

$$E_{ij}^{z} \le 1 \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B \qquad (2.41)$$

$$E_{ij}^{z} \le E_{ij}^{z,\max} \qquad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \in B \qquad (2.42)$$

However, each shell number will be counted as a separate heat exchanger whenever the condition of more than one exchanger is presented. The constraints for this situation are shown below.

Number of heat exchangers between hot stream i and cold stream j - $(i,j) \in B$

$$U_{ij}^{z} = \sum_{m \in M_i^z;\, j \in P_{im}^H} K_{ijm}^{z,H} \qquad (2.43)$$

$$U_{ij}^{z} = \sum_{m \in M_i^z;\, j \in P_{im}^H} \hat{K}_{ijm}^{z,H} \qquad (2.44)$$

$$U_{ij}^{z} = \sum_{n \in N_j^z;\, i \in P_{jn}^C} K_{ijn}^{z,C} \qquad (2.45)$$

$$U_{ij}^{z} = \sum_{n \in N_j^z;\, i \in P_{jn}^C} \hat{K}_{ijn}^{z,C} \qquad (2.46)$$

2.4.4 Heat Transfer Consistency

To explain the heat load of each exchanger unit for multiple heat exchange, heat transfer consistency constraints are necessary to be addressed. When heat exchanges from hot stream to cold stream with two exchangers exist in series, for example in Figure 2.6, the cumulative heat of hot stream in interval 6, $\hat{q}_{ij6}^{z,H}$ is transfer to the cold stream in interval 5 and the heat left of hot stream, $\tilde{q}_{ij6}^{z,H}$, is sent into interval 8 of cold stream. The amount of heat that is transferred to the next heat exchanger in series, $\tilde{q}_{ijm}^{z,H}$, is used to calculate the heat load and area calculations in each heat exchanger. Table 2.3 expressed the values of the variables involved in heat load calculation which are the heat exchanger existence, beginning and ending of each heat exchanger unit and the value of $\tilde{q}$. Another variable need to initiate is called $X_{im,jn}^{z}$ which used to find out the ending interval for each heat exchanger connected in sequence for match (i,j). So, the value of $X_{im,jn}^{z}$ will be zero whenever m and n are cold-end intervals and be higher than zero in all other situations.

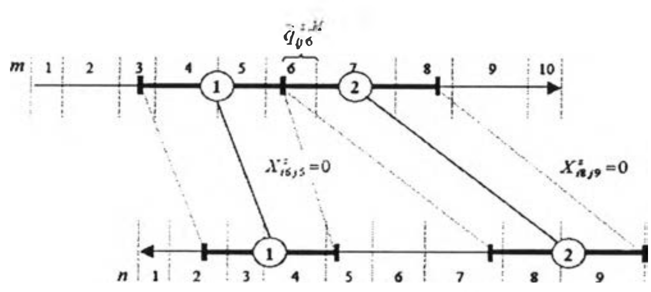


Figure 2.6 Heat transfer consistency example when $(i,j) \in B$.

Table 2.3 Values of variables $K_{ijm}^{z,H}$, $\hat{K}_{ijm}^{z,H}$, $Y_{ijm}^{z,H}$ and $\widetilde{q}_{ijm}^{z,H}$ when $(i,j) \in B$

$Load \; Unit\ 1 \quad = \hat{q}_{ij3}^{z,H} + \hat{q}_{ij4}^{z,H} + \hat{q}_{ij5}^{z,H} + \hat{q}_{ij6}^{z,H} - \widetilde{q}_{ij6}^{z,H} = \hat{q}_{ij2}^{z,C} + \hat{q}_{ij3}^{z,C} + \hat{q}_{ij4}^{z,C} + \hat{q}_{ij5}^{z,C}$

M	Y_m	K_m	$\hat{K}_m$	$\widetilde{q}_m$
1	0	0	0	0
2	0	0	0	0
3	1	1	0	0
4	1	0	0	0
5	1	0	0	0
6	2	1	1	≥0
7	1	0	0	0
8	1	0	1	0
9	0	0	0	0
10	0	0	0	0

$Load \; Units\ 1+2 \quad = \hat{q}_{ij3}^{z,H} + \hat{q}_{ij4}^{z,H} + \hat{q}_{ij5}^{z,H} + \hat{q}_{ij6}^{z,H} + \hat{q}_{ij7}^{z,H} + \hat{q}_{ij8}^{z,H} = \hat{q}_{ij2}^{z,C} + \hat{q}_{ij3}^{z,C} + \hat{q}_{ij4}^{z,C} + \hat{q}_{ij5}^{z,C} + \hat{q}_{ij7}^{z,C} + \hat{q}_{ij8}^{z,C} + \hat{q}_{ij9}^{z,C}$

n	Y_n	K_n	$\hat{K}_n$	$\widetilde{q}_m$
1	0	0	0	0
2	1	1	0	0
3	1	0	0	0
4	1	0	0	0
5	1	0	1	0
6	0	0	0	0
7	1	1	0	0
8	1	0	0	0
9	1	0	1	0

The heat transfer consistency constraints for multiple heat exchangers are expressed here.

Heat transfer consistency for multiple heat exchangers between the same pair of streams

$$\sum_{\substack{l \in M_i^z \\ l \le m}} \hat{q}_{ijl}^{z,H} - \widetilde{q}_{ijm}^{z,H} \le \sum_{\substack{l \in N_j^z \\ l \le n}} \hat{q}_{ijl}^{z,C} - \widetilde{q}_{ijm}^{z,C} + 4X_{im,jn}^{z} \, Max \left\{ \sum_{\substack{l \in M_i^z \\ l \le m \\ j \in P_{il}^H}} \Delta H_{il}^{z,H} ; \sum_{\substack{l \in M_i^z \\ l \le n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C} \right\} \tag{2.47}$$

$$\sum_{\substack{l \in M_i^z \\ l \le m}} \hat{q}_{ijl}^{z,H} - \widetilde{q}_{ijm}^{z,H} \ge \sum_{\substack{l \in N_j^z \\ l \le n}} \hat{q}_{ijl}^{z,C} - \widetilde{q}_{ijm}^{z,C} - 4X_{im,jn}^{z} \, Max \left\{ \sum_{\substack{l \in M_i^z \\ l \le m \\ j \in P_{il}^H}} \Delta H_{il}^{z,H} ; \sum_{\substack{l \in M_i^z \\ l \le n \\ i \in P_{jl}^C}} \Delta H_{jl}^{z,C} \right\} \tag{2.48}$$

$$X_{im,jn}^{z} = 2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C} + \frac{1}{4} \sum_{\substack{l \in N_j^z \\ l \le n}} \hat{K}_{ijl}^{z,C} - \frac{1}{4} \sum_{\substack{l \in M_i^z \\ l \le m}} \hat{K}_{ijl}^{z,H} \tag{2.49}$$

$$z \in Z ; m,n \in M^z ; \; T_n^L \le T_m^U ; (i,j) \in B ; \; i \in H_m^z ; j \in C_n^z ; \; i \in P_{jn}^C ; j \in P_{im}^H \quad (2.47\text{–}2.49)$$

$$\sum_{\substack{l \in M_i^z \\ l \le m \\ j \in P_{il}^H}} \hat{K}_{ijl}^{z,H} - \sum_{\substack{l \in N_j^z \\ l \le n \\ i \in P_{jl}^C}} \hat{K}_{ijl}^{z,C} \ge 0 \qquad z \in Z ; m,n \in M^z ; T_n^L < T_m^U ; T_n^L \ge T_m^L ; \; (i,j) \in B ; \; i \in H_m^z ; j \in C_n^z ; i \in P_{jn}^C ; j \in P_{im}^H \tag{2.50}$$

$$\sum_{\substack{l \in M_i^z \\ l \le m \\ j \in P_{il}^H}} \left(K_{ijl}^{z,H} - \hat{K}_{ijl}^{z,H} \right) \le 1 \tag{4.51}$$

$$\sum_{\substack{l \in N_j^z \\ l \le n \\ i \in P_{jl}^C}} \left(K_{ijl}^{z,C} - \hat{K}_{ijl}^{z,C} \right) \le 1 \tag{4.52}$$

$$z \in Z ; m \in M^z ; (i,j) \in B ; \; i \in H_m^z ; j \in P_{im}^H \quad (4.51\text{–}4.52)$$

$$\left.\begin{aligned}
&\tilde{q}_{ijm}^{z,H} \le \hat{q}_{ijm}^{z,H} && (4.53)\\
&\tilde{q}_{ijm}^{z,H} \le K_{ijm}^{z,H}\Delta H_{im}^{z,H} && (4.54)\\
&\tilde{q}_{ijm}^{z,H} \le \hat{K}_{ijm}^{z,H}\Delta H_{im}^{z,H} && (4.55)\\
&\tilde{q}_{ijm}^{z,H} \ge 0 && (4.56)
\end{aligned}\right\} z\in Z;\ m\in M^{z};\ (i,j)\in B;\ i\in H_{m}^{z};\ j\in P_{im}^{H}$$

$$\left.\begin{aligned}
&\tilde{q}_{ijn}^{z,C} \le \hat{q}_{ijn}^{z,C} && (4.57)\\
&\tilde{q}_{ijn}^{z,C} \le K_{ijn}^{z,C}\Delta H_{in}^{z,C} && (4.58)\\
&\tilde{q}_{ijn}^{z,C} \le \hat{K}_{ijn}^{z,C}\Delta H_{in}^{z,C} && (4.59)\\
&\tilde{q}_{ijn}^{z,C} \ge 0 && (4.60)
\end{aligned}\right\} z\in Z;\ n\in M^{z};\ (i,j)\in B;\ j\in C_{n}^{z};\ i\in P_{jn}^{C}$$

Main constraints for the heat transfer consistency are the equation (2.47) to (2.49). All these constraints show that whatever calculated from hot or cold stream, the heat load of heat exchanger also be the same. In addition, in case where there is the cold-end interval, $X_{im,jn}^{z}=0$, the equation (2.47) and (2.48) become an equality as

$$\sum_{\substack{l\in M_i^z\\ l\le m}} \hat{q}_{ijl}^{z,H} - \tilde{q}_{ijn}^{z,H} = \sum_{\substack{l\in N_j^z\\ l\le n}} \hat{q}_{ijl}^{z,C} - \tilde{q}_{ijm}^{z,C}$$

For example in Figure 2.6, at interval 6 of hot stream and interval 5 for cold stream, the constraint (2.47) and (2.48) will be summary to

$$\hat{q}_{ij3}^{z,H} + \hat{q}_{ij4}^{z,H} + \hat{q}_{ij5}^{z,H} + \hat{q}_{ij6}^{z,H} - \tilde{q}_{ij6}^{z,H} = \hat{q}_{ij2}^{z,C} + \hat{q}_{ij3}^{z,C} + \hat{q}_{ij4}^{z,C} + \hat{q}_{ij5}^{z,C} - \tilde{q}_{ij5}^{z,C}$$

And the heat exchanger does not start at interval 5 of cold stream, so the value of $\tilde{q}_{ij5}^{z,C}$ is zero. This lead the equation become

$$\hat{q}_{ij3}^{z,H} + \hat{q}_{ij4}^{z,H} + \hat{q}_{ij5}^{z,H} + \hat{q}_{ij6}^{z,H} - \tilde{q}_{ij6}^{z,H} = \hat{q}_{ij2}^{z,C} + \hat{q}_{ij3}^{z,C} + \hat{q}_{ij4}^{z,C} + \hat{q}_{ij5}^{z,C}$$

The next constraint, (2.50), is produced to make sure that there is feasible temperature difference between hot and cold stream at the cold-end, that is the hot stream temperature is forced to be higher than the cold stream temperature at the cold-end of the heat exchanger. Figure 2.7 will show more clear in description. Following constraints, (2.51)-(2.52), are used to describe that a new exchanger can only start, in the same interval with the first one sequentially, when the previous exchanger has ended. Last sets of constraint, (2.53) to (2.60), are used to specify the value of variable $\widetilde{q}$. This variable is created to be zero for all intervals except the connection interval between two exchangers which continuous constructed in series, first heat exchanger ends and the second exchanger starts in the same interval.

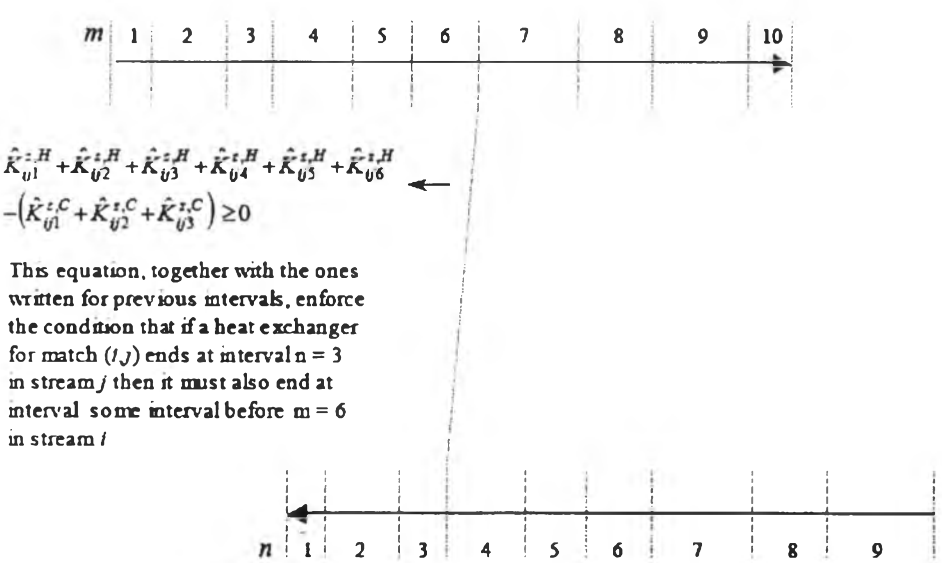


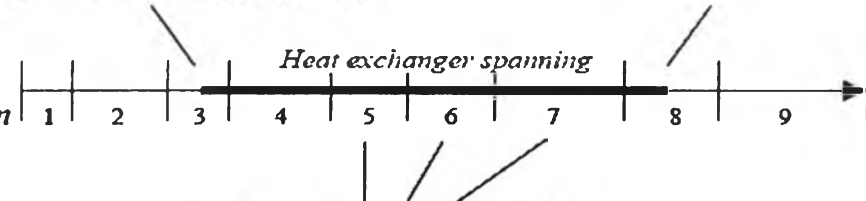
Figure 2.7 Integer cut for heat exchanger end when $(i,j)\in B$.

2.4.5 <u>Flow Rate Consistency Within Heat Exchangers</u>

The assumption that constant flow rate passed through heat exchanger is applied to the MILP model. The next equation group expresses the consistency of flow rate within a heat exchanger. In Figure 2.8 depicts an example of heat exchanger which exchange heat during the interval 3 to interval 8 of hot stream i with the cold stream j. Next, new word need to be introduced, they are called "extreme intervals" which are the intervals 3 and 8 for this example while "exchanger-internal intervals" are referred to the retired intervals which are the interval 4 to 7.

Let explain more details for this example where allow only one exchanger for match, $(i,j)\in B$. For the exchanger-internal intervals, interval 4 to 7, the flow rate can be consistently established as the ratio of the cumulative heat transfer, the heat capacity and the interval temperature difference. In contrast, this equation can not be used for the extreme intervals because the real temperature difference between upper and lower bound of interval are not the same as normal range, it is smaller. Consequently, flow rate for the interval 3 and 8 can be solved by the inequality constraints as mention in Figure2.8.

$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U-T_m^L)}\geq\frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U-T_{m-1}^L)} \qquad \frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U-T_m^L)}\leq\frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U-T_{m-1}^L)}$$



$$\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U-T_m^L)}=\frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U-T_{m-1}^L)}$$

Figure 2.8 Flow rate consistency equations.

The equations used for classify which interval is exchanger-internals or extreme intervals are introduced couple with the variable α. Actually, it is defined as continuous but the following constraints enforces it to be one when the interval is exchanger-internal and zero for all others.

Definition of exchanger-internal intervals for hot streams

$$\alpha_{ijm}^{z,H}\leq 1-K_{ijm}^{z,H}-K_{ijm-1}^{z,H} \tag{2.61}$$

$$\alpha_{ijm}^{z,H}\leq 1-\hat{K}_{ijm}^{z,H}-\hat{K}_{ijm-1}^{z,H} \tag{2.62}$$

$$\alpha_{ijm}^{z,H}\geq Y_{ijm}^{z,H}-K_{ijm}^{z,H}-K_{ijm-1}^{z,H}-\hat{K}_{ijm}^{z,H}-\hat{K}_{ijm-1}^{z,H} \tag{2.63}$$

$$\alpha_{ijm}^{z,H}\geq 0 \tag{2.64}$$

$$z\in Z;\ m\in M^z;\ i\in H_m^z\cap H_{m-1}^z;\ j\in P_{im}^H\cap P_{im-1}^H;\ i\in S^H,\ j\in C^z$$

At exchanger-internal interval, there is no exchanger begins or ends, so $K_{ijm}^{z,H}$, $K_{ijm-1}^{z,H}$, $\hat{K}_{ijm}^{z,H}$, $\hat{K}_{ijm-1}^{z,H}$ are all zero and $Y_{ijm}^{z,H}=1$. The constraint (2.63) gives the value of $\alpha_{ijm}^{z,H}$ to be one. On the other hand, for the extreme intervals, at least one of $K_{ijm}^{z,H}$, $K_{ijm-1}^{z,H}$, $\hat{K}_{ijm}^{z,H}$, $\hat{K}_{ijm-1}^{z,H}$ will be equal to one or $Y_{ijm}^{z,H}=0$. So, $\alpha_{ijm}^{z,H}$ will become to zero.

However, there is another condition, which effect to these constraint equations. When splitting stream flow rate is allowed, the flow rate consistency equation will be

Flow rate consistency for hot streams in exchanger-internal intervals-$i\in S^H$, $(i,j)\notin B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U-T_m^L)}\le\frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U-T_{m-1}^L)}+\left(1-\alpha_{ijm}^{z,H}\right)\cdot F_i && (2.65)\\
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U-T_m^L)}\ge\frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U-T_{m-1}^L)}-\left(1-\alpha_{ijm}^{z,H}\right)\cdot F_i && (2.66)
\end{aligned}\right\}\ \begin{aligned}&z\in Z;m\in M^z;i\in H_m^z\cap H_{m-1}^z\\&i\in S^H;j\in C^z;j\in P_{im}^H\cap P_{im-1}^H\end{aligned}$$

Flow rate consistency for hot streams in extreme intervals - $i\in S^H$, $(i,j)\notin B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U-T_m^L)}\ge\frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U-T_{m-1}^L)}-\left(1+\hat{K}_{ijm-1}^{z,H}+\hat{K}_{ijm}^{z,H}-K_{ijm-1}^{z,H}\right)\cdot F_i && (2.67)\\
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U-T_m^L)}\le\frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U-T_{m-1}^L)}+\left(1+K_{ijm-1}^{z,H}+K_{ijm}^{z,H}-\hat{K}_{ijm}^{z,H}\right)\cdot F_i && (2.68)
\end{aligned}\right\}\ \begin{aligned}&z\in Z;m\in M^z\\&i\in H_m^z\cap H_{m-1}^z\\&j\in P_{im}^H\cap P_{im-1}^H\\&i\in S^H;j\in C^z;(i,j)\notin B\end{aligned}$$

- For the exchanger-internal interval, $\alpha=1$, that is the last term in the right hand side of both constraints, (2.65) and (2.66), are canceled out and the constraints perform as equality. In contrast, constraint (2.67) and (2.68) are defined for the extreme intervals. Constraint (2.67) is referred to the beginning of heat exchanger and the end of exchanger is expressed in constraint (2.68). Consider (2.67), at the end of exchanger, the last term in the right hand side is deleted. The last term in (2.68) can also be erased whenever there is a starting of exchanger.

However, the effect of stream splitting also needs to be concerned. The possibility of appearing two different heat exchangers in the same interval is used to construct the constraints for stream splitting.

Flow rate consistency for hot streams in extreme intervals - $i \in S^{H}$, $(i,j) \in B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(1 + \hat{K}_{ijm-1}^{z,H} + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H}\right) \cdot F_i && (2.69)\\
&\frac{\hat{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \geq \frac{\tilde{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} - \left(2 + \hat{K}_{ijm}^{z,H} - K_{ijm-1}^{z,H} - Y_{ijm-1}^{z,H}\right) \cdot F_i && (2.70)\\
&\frac{\hat{q}_{ijm}^{z,H} - \tilde{q}_{ijm}^{z,H}}{Cp_{im}(T_m^U - T_m^L)} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{Cp_{im-1}(T_{m-1}^U - T_{m-1}^L)} + \left(2 + K_{ijm-1}^{z,H} - \hat{K}_{ijm}^{z,H} - Y_{ijm}^{z,H}\right) \cdot F_i && (2.71)
\end{aligned}\right\} \begin{aligned} &z \in Z; m \in M^z \\ &i \in H_m^z \cap H_{m-1}^z \\ &j \in P_{im}^H \cap P_{im-1}^H \\ &i \in S^H; j \in C^z; (i,j) \in B \end{aligned}$$

When a heat exchanger starts at interval m-1, the constraint (2.69) is applied while the constraint (2.70) is used to identify when another heat exchanger between the same pair of hot and cold stream that ends at the interval m-1. Constraint (2.71) expresses at the end of a heat exchanger which the possibility of having two heat exchangers that start at the same interval is concerned. All constraints, (2.67) to (2.71), can be simplified for the case that stream split is not allowed because the flow rate for exchanger-internal intervals is equal to the actual flow rate.

Flow rate consistency for hot streams - $i \notin S^{H}$

$$\hat{q}_{ijm}^{z,H} \geq \left(Y_{ijm-1}^{z,H} + Y_{ijm}^{z,H} - Y_{ijm+1}^{z,H} - 2\right) \cdot \Delta H_{im}^{z,H} \qquad \begin{aligned} &z \in Z; m \in M^z; i \in H_{m-1}^z \cap H_m^z \cap H_{m+1}^z; i \notin S^H \\ &(i,j) \notin B; j \in C^z; j \in P_{im-1}^H \cap P_{im}^H \cap P_{im+1}^H \end{aligned} \quad (2.72)$$

$$\left.\begin{aligned}
&\hat{q}_{ijm}^{z,H} \geq \left(Y_{ijm}^{z,H} - K_{ijm}^{z,H} - \hat{K}_{ijm}^{z,H}\right) \cdot \Delta H_{im}^{z,H} && (2.73)\\
&\hat{q}_{ijm}^{z,H} \geq \left(Y_{ijm}^{z,H} + K_{ijm}^{z,H} + \hat{K}_{ijm}^{z,H} - 2\right) \cdot \Delta H_{im}^{z,H} && (2.74)
\end{aligned}\right\} \begin{aligned} &z \in Z; m \in M^z; i \in H_{m-1}^z \cap H_m^z \cap H_{m+1}^z; i \notin S^H \\ &j \in C^z; j \in P_{im-1}^H \cap P_{im}^H \cap P_{im+1}^H \end{aligned}$$

In case that only one exchanger is permitted, expressed in constraint (2.72), the heat flow is equivalent to the amount of enthalpy change for any internal interval. However, for the multiple exchangers, the variables Y is probably higher than one. Therefore, two following constraint, (2.73) and (2.74), are set to satisfy the concept of equivalent between heat flow and enthalpy change.

Consequently, flow rate consistency constraints for cold streams are shown below.

Definition of exchanger-internal intervals for cold streams $j \in S^C$

$$\left.\begin{aligned}
&\alpha_{ijn}^{z,C} \le 1 - K_{ijn}^{z,C} - K_{ijn-1}^{z,C} && (2.75)\\
&\alpha_{ijn}^{z,C} \le 1 - \hat{K}_{ijn}^{z,C} - \hat{K}_{ijn-1}^{z,C} && (2.76)\\
&\alpha_{ijn}^{z,C} \ge Y_{ijn}^{z,C} - K_{ijn}^{z,C} - K_{ijn-1}^{z,C} - \hat{K}_{ijn}^{z,C} - \hat{K}_{ijn-1}^{z,C} && (2.77)\\
&\alpha_{ijn}^{z,C} \ge 0 && (2.78)
\end{aligned}\right\} z\in Z; n\in M^z; j\in C_n^z \cap C_{n-1}^z; j\in S^C; i\in H^z; i\in P_{jn}^C \cap P_{jn-1}^C$$

Flow rate consistency for cold streams in exchanger-internal intervals-$j \in S^C$,$(i,j) \notin B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_n^U - T_n^L)} \le \frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^U - T_{n-1}^L)} + \left(1-\alpha_{ijn}^{z,C}\right)\cdot F_j && (2.79)\\
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_n^U - T_n^L)} \ge \frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^U - T_{n-1}^L)} - \left(1-\alpha_{ijn}^{z,C}\right)\cdot F_j && (2.80)
\end{aligned}\right\} \begin{array}{c} z\in Z; n\in M^z;\ j\in S^C \\ j\in C_n^z \cap C_{n-1}^z; i\in H^z;\ i\in P_{jn}^C \cap P_{jn-1}^C \end{array}$$

Flow rate consistency for cold streams in extreme intervals - $j \in S^C$,$(i,j) \notin B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_n^U - T_n^L)} \ge \frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn}(T_n^U - T_n^L)} - \left(1+\hat{K}_{ijn-1}^{z,C} + \hat{K}_{ijn}^{z,C} - K_{ijn-1}^{z,C}\right)\cdot F_j && (2.81)\\
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_n^U - T_n^L)} \le \frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^U - T_{n-1}^L)} + \left(1+K_{ijn-1}^{z,C} + K_{ijn}^{z,C} - \hat{K}_{ijn}^{z,C}\right)\cdot F_j && (2.82)
\end{aligned}\right\} \begin{array}{c} z\in Z; n\in M^z; (i,j)\notin B \\ j\in S^C; j\in C_n^z \cap C_{n-1}^z \\ i\in H^z;\ i\in P_{jn}^C \cap P_{jn-1}^C \end{array}$$

Flow rate consistency for cold streams in extreme intervals - $j \in S^C$,$(i,j) \in B$

$$\left.\begin{aligned}
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_n^U - T_n^L)} \ge \frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^U - T_{n-1}^L)} - \left(1+\hat{K}_{ijn-1}^{z,C} + \hat{K}_{ijn}^{z,C} - K_{ijn-1}^{z,C}\right)\cdot F_j && (2.83)\\
&\frac{\hat{q}_{ijn}^{z,C}}{Cp_{jn}(T_n^U - T_n^L)} \ge \frac{\tilde{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^U - T_{n-1}^L)} - \left(2+\hat{K}_{ijn}^{z,C} - K_{ijn-1}^{z,C} - Y_{ijn-1}^{z,C}\right)\cdot F_j && (2.84)
\end{aligned}\right\} \begin{array}{c} z\in Z; n\in M^z; (i,j)\in B \\ j\in S^C; j\in C_n^z \cap C_{n-1}^z \\ i\in H^z;\ i\in P_{jn}^C \cap P_{jn-1}^C \end{array}$$

$$\frac{\hat{q}_{ijn}^{z,C}-\tilde{q}_{ijn}^{z,C}}{Cp_{jn}(T_n^U-T_n^L)} \le \frac{\hat{q}_{ijn-1}^{z,C}}{Cp_{jn-1}(T_{n-1}^U-T_{n-1}^L)}+\left(2+K_{ijn-1}^{z,C}-\hat{K}_{ijn}^{z,C}-Y_{ijn}^{z,C}\right)\cdot F_j \qquad \begin{matrix} z\in Z\,;n\in M^z\,;(i,j)\in B \\ j\in S^C;\, j\in C_n^z \cap C_{n-1}^z \\ i\in H^z;\, i\in P_{jn}^C \cap P_{jn-1}^C \end{matrix} \tag{2.85}$$

Flow rate consistency for cold streams - $j \notin S^C$

$$\hat{q}_{ijn}^{z,C} \ge \left(Y_{ijn-1}^{z,C}-Y_{ijn}^{z,C}-Y_{ijn+1}^{z,C}-2\right)\cdot \Delta H_{jn}^{z,C} \qquad \begin{matrix} z\in Z\,;n\in M^z;\; j\in C_{n-1}^z \cap C_n^z \cap C_{n+1}^z\,;\, i\notin S^C \\ (i,j)\notin B;\; i\in H^z\,;\, i\in P_{jn-1}^C \cap P_{jn}^C \cap P_{jn+1}^C \end{matrix} \tag{2.86}$$

$$\left.\begin{aligned} &\hat{q}_{ijn}^{z,C} \ge \left(Y_{ijn}^{z,C}-K_{ijn}^{z,C}-\hat{K}_{ijn}^{z,C}\right)\cdot \Delta H_{jn}^{z,C} &(2.87)\\ &\hat{q}_{ijn}^{z,C} \ge \left(Y_{ijn-1}^{z,C}+K_{ijn}^{z,C}+\hat{K}_{ijn+1}^{z,C}-2\right)\cdot \Delta H_{jn}^{z,C} &(2.88)\end{aligned}\right\} \begin{matrix} z\in Z\,;n\in M^z;\; j\in C_{n-1}^z \cap C_n^z \cap C_{n+1}^z\,;\, i\notin S^C \\ (i,j)\in B;\; i\in H^z\,;\, i\in P_{jn-1}^C \cap P_{jn}^C \cap P_{jn+1}^C \end{matrix}$$

2.4.6 Temperature Difference Enforcing

This part is necessary to generate in order to assure the heat transfer feasible. Firstly, Figure 2.9, constraint (2.89) and (2.90) introduce the temperature difference of extreme interval for the condition that there are no splits are allowed. Additionally, constraint (2.91) to (2.96) further explain in case where stream splits are allowed.

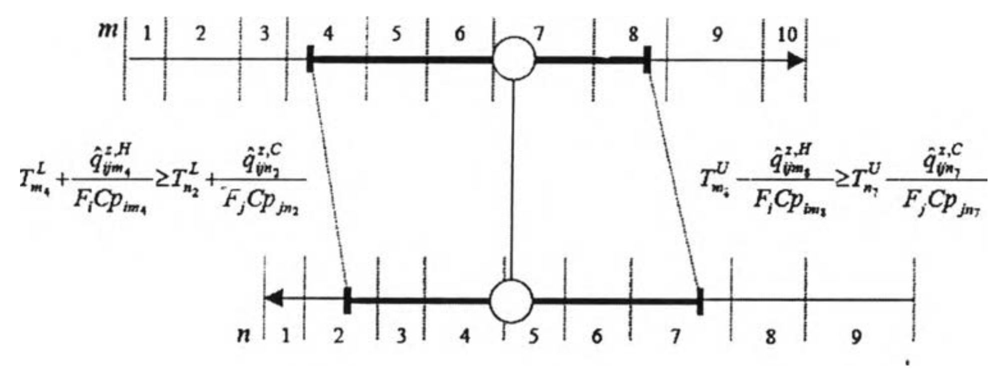


Figure 2.9 Temperature difference assurance when splits are not allowed.

Temperature feasibility constraints - $i\notin S^H, j\notin S^C$

$$\left.\begin{aligned} &T_m^L+\frac{\hat{q}_{ijm}^{z,H}}{F_iCp_{im}} \ge T_n^L+\frac{\hat{q}_{ijn}^{z,C}}{F_jCp_{jn}}-\left(2-K_{ijm}^{z,H}-K_{ijn}^{z,C}\right)\cdot T_n^U &(2.89)\\ &T_m^U-\frac{\hat{q}_{ijm}^{z,H}}{F_iCp_{im}} \ge T_n^U-\frac{\hat{q}_{ijn}^{z,C}}{F_jCp_{jn}}-\left(2-\hat{K}_{ijm}^{z,H}-\hat{K}_{ijn}^{z,C}\right)\cdot T_n^U &(2.90)\end{aligned}\right\} \begin{matrix} z\in Z; m,n\in M^z\,; T_n^L \le T_m^U; T_n^U \ge T_m^L \\ i\in H_m^z; j\in C_n^z; i\notin S^H; j\notin S^C; i\in P_{jn}^C; j\in P_{im}^H \end{matrix}$$

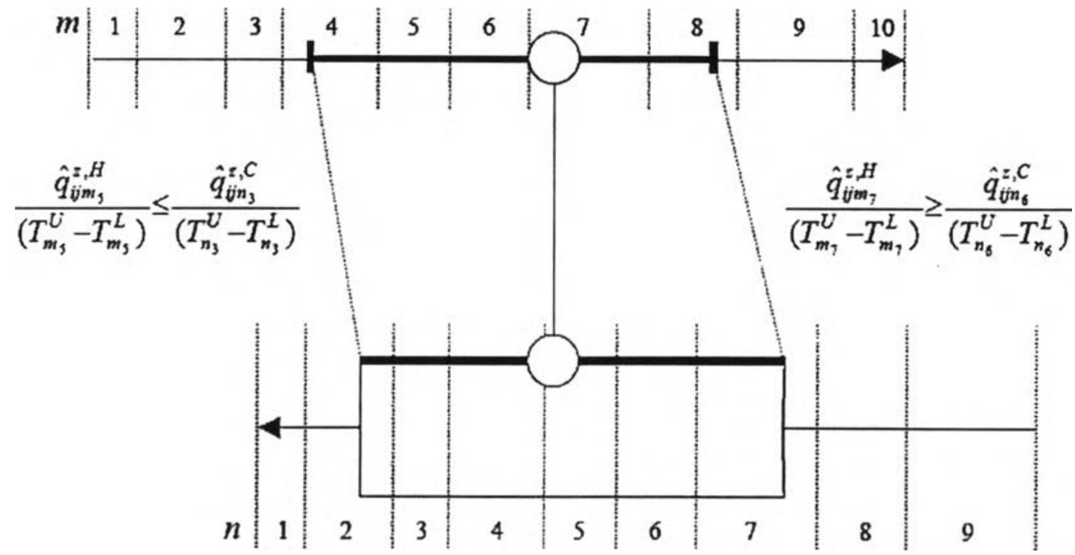


Figure 2.10 Temperature difference assurance when splits are allowed.

Temperature feasibility constraints - $i \in S^H, j \in S^C, (i,j) \notin B$

$$\hat{K}^{z,C}_{ijn} \le 2 - K^{z,H}_{ijm} - K^{z,C}_{ijn} \tag{2.91}$$

$$\frac{\hat{q}^{z,C}_{ijn}}{T^U_m - T^L_n} \le \frac{\hat{q}^{z,C}_{ijn+1}}{T^U_{n+1} - T^L_{n+1}} \frac{Cp_{jn}}{Cp_{jn+1}} + \left(2 - K^{z,H}_{ijm} - K^{z,C}_{ijn}\right) \cdot \frac{\Delta H^{z,C}_{jn}}{T^U_m - T^L_n} \tag{2.92}$$

$$\frac{\hat{q}^{z,H}_{ijm}}{Min\{T^U_m ; T^U_n\} - T^L_m} \ge \frac{\hat{q}^{z,H}_{ijm+1}}{T^U_{m+1} - T^L_{m+1}} \frac{Cp_{im}}{Cp_{im+1}} - \left(2 - K^{z,H}_{ijm} - K^{z,C}_{ijn}\right) \frac{\Delta H^{z,H}_{im+1}}{T^U_{m+1} - T^L_{m+1}} \tag{2.93}$$

$z \in Z ; m,n \in M^z ; i \in S^H$
$j \in S^C ; T^L_n < T^U_m ; T^U_n > T^L_m$
$i \in H^z_m \cap H^z_{m+1} ; j \in C^z_n \cap C^z_{n+1} ;$
$i \in P^C_{jn} \cap P^C_{jn+1} ; j \in P^H_{im} \cap P^H_{im+1}$

$$K^{z,H}_{ijm} \le 2 - \hat{K}^{z,H}_{ijm} - \hat{K}^{z,C}_{ijn} \tag{2.94}$$

$$\frac{\hat{q}^{z,H}_{ijm}}{T^U_m - T^L_n} \le \frac{\hat{q}^{z,H}_{ijm-1}}{T^U_{m-1} - T^L_{m-1}} \frac{Cp_{im}}{Cp_{im-1}} + \left(2 - \hat{K}^{z,H}_{ijm} - \hat{K}^{z,C}_{ijn}\right) \frac{\Delta H^{z,H}_{im}}{T^U_m - T^L_n} \tag{2.95}$$

$$\frac{\hat{q}^{z,C}_{ijn}}{T^U_n - Max\{T^L_m ; T^L_n\}} \ge \frac{\hat{q}^{z,C}_{ijn-1}}{T^U_{n-1} - T^L_{n-1}} \frac{Cp_{jn}}{Cp_{jn-1}} + \left(2 - \hat{K}^{z,H}_{ijm} - \hat{K}^{z,C}_{ijn}\right) \frac{\Delta H^{z,C}_{jn-1}}{T^U_{n-1} - T^L_{n-1}} \tag{2.96}$$

$z \in Z ; m,n \in M^z ; i \in S^H$
$j \in S^C ; T^L_n < T^U_m ; T^U_n > T^L_m$
$i \in H^z_m \cap H^z_{m+1} ; j \in C^z_n \cap C^z_{n+1} ;$
$i \in P^C_{jn} \cap P^C_{jn+1} ; j \in P^H_{im} \cap P^H_{im+1}$

All these next constraints are performed only for overlapping pairs of intervals where $T_n^L < T_m^U$ and $T_n^U > T_m^L$ which m and n are the overlapping intervals of hot and cold stream at the hot end of heat exchanger. Constraint (2.91) is generated to guarantee that the cold end of the cold stream of heat exchanger will not be located at the same interval with the hot end. Feasible heat transfer forces the constraint (2.92) in valid. That is the hot end temperature for the cold stream is less than the hot stream. Moreover, constraint (2.93) stated that the hot end temperature of the hot stream equal to $Min\ \{T_m^U ; T_n^U\}$ as illustrated in Figure 2.11. Finally, the constraints for the case of multiple heat exchangers are presented next.

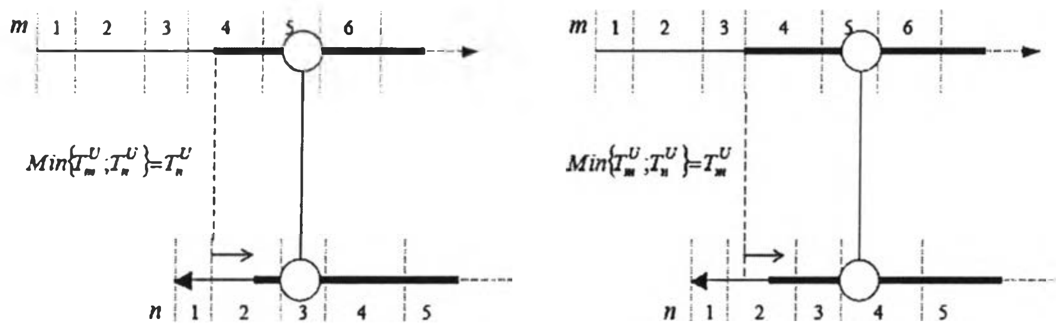


Figure 2.11 Temperature difference assurance at the hot end of an exchanger - $i \in S^H, j \in S^C, (i,j) \notin B$.

Temperature feasibility constraints $i \in S^H, j \in S^C, (i,j) \in B$

$$\left.\begin{aligned}
&\hat{K}_{ijn}^{z,C} \leq 1 + Y_{ijn}^{z,C} - K_{ijm}^{z,H} - K_{ijn}^{z,C} && (2.97)\\
&\frac{\hat{q}_{ijn}^{z,C}}{T_m^U - T_n^L} \leq \frac{\hat{q}_{ijn+1}^{z,C}}{T_{n+1}^U - T_{n+1}^L}\frac{Cp_{jn}}{Cp_{jn+1}} + \left(1 + Y_{ijn}^{z,C} - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right)\cdot\frac{\Delta H_{jn}^{z,C}}{T_m^U - T_n^L} && (2.98)\\
&\frac{\tilde{q}_{ijn}^{z,C}}{T_m^U - T_n^L} \leq \frac{\hat{q}_{ijn+1}^{z,C}}{T_{n+1}^U - T_{n+1}^L}\frac{Cp_{jn}}{Cp_{jn+1}} + \left(2 - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right)\cdot\frac{\Delta H_{jn}^{z,C}}{T_m^U - T_n^L} && (2.99)
\end{aligned}\right\}
\begin{aligned}
&z \in Z; m,n \in M^z; i \in S^H;\\
&j \in S^C;\ T_n^L < T_m^U; T_n^U > T_m^L\\
&i \in H_n^z \cap H_{m+1}^z; j \in C_n^z \cap C_{n+1}^z\\
&i \in P_m^C \cap P_{jn+1}^C; j \in P_{im}^H \cap P_{im+1}^H
\end{aligned}$$

$$\frac{\hat{q}_{ijm}^{z,H}}{Min\{T_m^U ; T_n^U\} - T_m^L} \geq \frac{\hat{q}_{ijm+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L}\frac{Cp_{im}}{Cp_{im+1}} - \left(2 - K_{ijm}^{z,H} - K_{ijn}^{z,C}\right)\cdot\frac{\Delta H_{im+1}^{z,H}}{T_{m+1}^U - T_{m+1}^L} \qquad (2.100)$$

$$K_{ijm}^{z,H} \leq 1 + Y_{ijm}^{z,H} - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C} \tag{2.101}$$

$$\frac{\hat{q}_{ijm}^{z,H} - \tilde{q}_{ijm}^{z,H}}{T_m^U - T_n^L} \leq \frac{\hat{q}_{ijm-1}^{z,H}}{T_{m-1}^U - T_{m-1}^L}\frac{Cp_{im}}{Cp_{im-1}} + \left(2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}\right)\cdot\frac{\Delta H_{im}^{z,H}}{T_m^U - T_n^L} \tag{2.102}$$

$$\frac{\hat{q}_{ijm}^{z,H} - \tilde{q}_{ijm}^{z,H}}{T_n^U - Max\{T_m^L, T_n^L\}} \leq \frac{\hat{q}_{ijn-1}^{z,C}}{T_{n-1}^U - T_{n-1}^L}\frac{Cp_{jn}}{Cp_{jn-1}} - \left(2 - \hat{K}_{ijm}^{z,H} - \hat{K}_{ijn}^{z,C}\right)\cdot\frac{\Delta H_{jn-1}^{z,C}}{T_{n-1}^U - T_{n-1}^L} \tag{2.103}$$

$z \in Z; m,n \in M^z; i \in S^H;$
$j \in S^C; T_n^L < T_m^U; T_n^U > T_m^L$
$i \in H_m^z \cap H_{m-1}^z; j \in C_n^z \cap C_{n-1}^z$
$i \in P_{jn}^C \cap P_{jn-1}^C; j \in P_{im}^H \cap P_{im-1}^H$

2.4.7 Heat Exchanger Area Calculation

The area of heat exchanger can be determined by considering the heat transfer of any stream match.

Heat transfer area for one heat exchanger is permitted

$$A_{ij}^z = \sum_{m \in M_i^z} \sum_{\substack{n \in N_j^z; T_n^L < T_m^U \\ j \in P_{im}^H; i \in P_{jn}^C}} \left[\frac{q_{im,jn}^z (h_{im} + h_{jn})}{\Delta T_{mn}^{ML} h_{im} h_{jn}}\right] \qquad z \in Z;\ i \in H^z; j \in C^z; (i,j) \in P \tag{2.104}$$

For multiple heat exchangers between streams i and j are allowed, each exchanger area can be formulated by this following constraints.

Heat transfer area for multiple heat exchangers

$$\hat{A}_{ij}^{z,k} \leq \sum_{\substack{l \in M_i^z \\ l \leq m}} \sum_{\substack{n \in N_j^z \\ T_n^L < T_m^U \\ j \in P_{im}^H \\ i \in P_{jn}^C}} \left[\frac{(q_{il,jn}^z - \breve{q}_{il,jn}^z)\cdot(h_{il} + h_{jn})}{\Delta T_{ln}^{ML}\cdot h_{il}\cdot h_{jn}}\right] - \sum_{h=1}^{k-1} A_{ij}^{z,h} + A_{ijmax}^z\left(2 - \hat{K}_{ijm}^{z,H} - G_{ijm}^{z,k}\right) \tag{2.105}$$

$$\hat{A}_{ij}^{z,k} \geq \sum_{\substack{l \in M_i^z \\ l < m}} \sum_{\substack{n \in N_j^z \\ T_n^L < T_m^U \\ j \in P_{im}^H \\ i \in P_{jn}^C}} \left[\frac{(q_{il,jn}^z - \breve{q}_{il,jn}^z)\cdot(h_{il} + h_{jn})}{\Delta T_{ln}^{ML}\cdot h_{il}\cdot h_{jn}}\right] - \sum_{h=1}^{k-1} A_{ij}^{z,h} - A_{ijmax}^z\left(2 - \hat{K}_{ijm}^{z,H} - G_{ijm}^{z,k}\right) \tag{2.106}$$

$z \in Z; m \in M^z$
$i \in H_m^z; j \in C^z$
$j \in P_{im}^H; (i,j) \in B$
$k = 1,\ldots,k_{max} - 1$

$$\hat{A}_{ij}^{z,k} \geq A_{ij}^z - \sum_{h=1}^{k-1} \hat{A}_{ij}^{z,h} \tag{2.107}$$

$$\sum_{h=1}^{k_{max}} h \cdot G_{ijm}^{z,h} = \sum_{\substack{l \in M_i^z : l \leq m \\ j \in P_{im}^H}} K_{ijl}^{z,H} + 1 - Y_{ijm}^{z,H} \tag{2.108}$$

$$\left.\begin{array}{l}\displaystyle\sum_{\substack{n\in N_j^z : T_n^L < T_m^U \\ j\in P_{im}^H : i\in P_{jn}^C}} \tilde{q}_{im,jn}^{z} = \tilde{q}_{ijm}^{z,H} \qquad (2.109) \\ \\ \bar{q}_{im,jn}^{z} \le q_{im,jn}^{z} \qquad (2.110)\end{array}\right\} \begin{array}{l} z\in Z ; m\in M^z \\ i\in H_m^z ; j\in C^z \\ j\in P_{im}^H ; (i,j)\in B \\ k = 1,\dots,k_{max}-1 \end{array}$$

The maximum number of heat exchangers allowed per match, k_{max}, is required for area calculation. The heat exchanger area of the k-th heat exchanger is calculated by subtracting the area of the former exchangers, k-1, from the total accumulated area until the end of the k-th exchanger. The binary variables, $\hat{X}_{ijm}^{z,h}$, are used to specify which exchanger is present at a certain temperature interval. Obviously, all constraints (2.105) to (2.110) are constructed for hot stream intervals only because hot and cold stream intervals can generate the same heat exchanger area.

2.4.8 Number of Shells

The variable U_{ij}^z is used to define as the number of shells.

Maximum Shell Area

$$A_{ij}^z \le A_{ij\,max}^z U_{ij}^z \qquad z\in Z;\ i\in H^z ; j\in C^z ; (i,j)\in P ; (i,j)\notin B \qquad (2.111)$$

$$\hat{A}_{ij}^{z,k} \le A_{ij\,max}^z \hat{U}_{ij}^{z,k} \qquad z\in Z;\ i\in H^z ; j\in C^z ; (i,j)\in P ; (i,j)\in B \qquad (2.112)$$

2.4.9 Objective Function

The objective function of the MILP model is to minimize the annualized total cost, this is composed of the operating and capital cost. The simply assumption of linear relation is used to approximate the total cost. The equation applied to calculate the objective value is indicated below. The first term represents the cost of hot utility, the second referred to cooling utility cost, followed by the fixed cost for heat exchanger and end up with the area cost.

$$
\begin{aligned}
Min \quad Cost = & \sum_{z} \sum_{i \in HU^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} c_i^H F_i^H \Delta T_i + \sum_{z} \sum_{j \in CU^z} \sum_{\substack{i \in H^z \\ (i,j) \in P}} c_j^C F_j^C \Delta T_j + \left[\sum_{z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} \left(c_{ij}^F U_{ij}^z + c_{ij}^A A_{ij}^z \right) \right]_{(i,j) \notin B} \\
& + \left[\sum_{k} \left\{ \sum_{z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} \left(c_{ij}^F \hat{U}_{ij}^{z,k} + c_{ij}^A \hat{A}_{ij}^{z,k} \right) \right\} \right]_{(i,j) \in B}
\end{aligned}
\tag{2.113}
$$

2.5 Model for Retrofit Heat Exchanger Network

Not only designing an optimal heat exchanger network, but the problem of heat exchanger network analysis is also play attention in the retrofit part. The MILP model is extended by adding some constraints for being the retrofit configuration. An Existing heat exchanger network is necessarily identified into the model, the location of the presented exchanger units are needed to introduce. A certain reconstruction and financial investment of adding new exchangers or area expanding in an existing process can considerably reduce the total cost of the existing plant. These options are targeted to decrease the total cost by enhancing the heat integration among process streams.

2.5.1 Area Additions for Existing and New Heat Exchanger Units

The number of heat exchanger unit in each match is considering for the additional area. Firstly, for the case where only one heat exchanger unit is allowed per matching, $(i,j) \notin$ B, both possibility of adding the exchanger area in the same shell and a new one are proposed. However, when $(i,j) \in$ B, there are more than one exchanger exists in the same pair of hot and cold stream matching, the area expansion possibility can be generated by adding area to the existing exchangers and also set up the new units. The following set of constraints is used to identify when a heat exchanger unit is equipped with the existing network.

Area addition to the existing heat exchangers – $(i,j) \notin B$

$$\left.\begin{aligned}
&A_{ij}^{z} \le A_{ij}^{z^{0}} + \Delta A_{ij}^{z^{0}} + A_{ij}^{z^{N}} && (2.114)\\
&\Delta A_{ij}^{z^{0}} \le \Delta A_{ij\max}^{z^{0}} && (2.115)\\
&A_{ij}^{z^{N}} \le A_{ij\max}^{z^{N}} \cdot \left(U_{ij}^{z} - U_{ij}^{z^{0}}\right) && (2.116)\\
&U_{ij}^{z} \le U_{ij\max}^{z} && (2.117)
\end{aligned}\right\} \quad z \in Z;\ i \in H^{z};\ j \in C^{z};\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} \ge 1$$

The area of exchanger per match (i,j) which presented only one exchanger should not over a summation of the existing area ($A_{ij}^{z^{0}}$), the area added to the existing shells ($\Delta A_{ij}^{z^{0}}$) and the area placed into the new shells ($A_{ij}^{z^{N}}$). The extended area into the existing shells and number of new shell need to be assigned as maximum. Additionally, a new shell is counted whenever the area is increased that shown in constraint (2.116). However, another set of equations is presented for the case in which there is no exchanger unit settled between a pair of hot and cold process streams.

Area required for new matches – (i,j) $\notin$ B

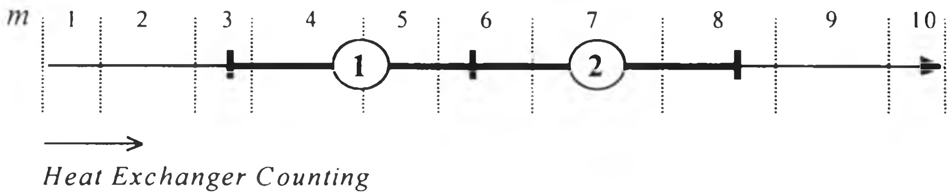
$$A_{ij}^{z^{N}} \le A_{ij\max}^{z^{N}} \cdot U_{ij}^{z} \qquad z \in Z;\ i \in H^{z};\ j \in C^{z};\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} = 0 \qquad (2.118)$$

$$U_{ij}^{z} \le U_{ij\max}^{z} \qquad z \in Z;\ i \in H^{z};\ j \in C^{z};\ (i,j) \in P;\ (i,j) \notin B;\ U_{ij}^{z,0} = 0 \qquad (2.119)$$

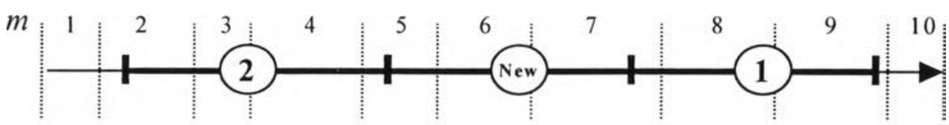
On the other hand, when there is more than one exchanger unit presented in the same pair of streams, $(i,j) \in B$, the position and order of each unit is necessary to record. A variable δ_{hk} is used to identify the exchanger location, an example is shown in Figure 2.12. For example, variable $\delta_{13} = 1$ indicate that the exchanger presented in the first location in the original network and it has been equipped in the third position in the retrofitted design network. Definition of variable δ_{hk} is defined below

$$\delta_{hk} = \begin{cases} 1 & \text{If the } h\text{-th original heat exchanger is placed in the } k\text{-th position in the retrofitted netwo:} \\ 0 & \text{Otherwise} \end{cases}$$

Before Retrofit



After Retrofit



δ_{hk}		k = 1	2	3
h	1	0	0	1
	2	1	0	0

Figure 2.12 Area computation when $(i,j) \in B$.

The area of the k-th existing exchanger between streams i and j after retrofit should smaller or equal to the combination of original area of h-th exchanger ($\sum_{h=1}^{k_c} A_{ij}^{z,h^{II}} \delta_{ij}^{z,hk}$), the area added to the existing shells ($\Delta A_{ij}^{z,k^{II}}$) and the area for new shells ($A_{ij}^{z,k^{N}}$). Whenever an existing h-th exchanger unit is analyzed to relocate into k-th position, $\sum_{h=1}^{k_c} \delta_{ij}^{z,hk} = 1$, there is no new heat exchanger unit for the retrofit network, therefore the retrofit exchanger area will be the original area combine with the addition area. On the contrary, original area term in constraint (2.121) for retrofit match will be canceled where as the new heat exchanger unit is placed, $\sum_{h=1}^{k_c} \delta_{ij}^{z,hk} = 0$.

Area addition to existing and new heat exchangers when $(i,j)\in B$

$$\left.\begin{aligned}
&A_{ij}^{z,k} \leq \sum_{h=1}^{k_e} A_{ij}^{z,h^0}\delta_{ij}^{z,hk} + \Delta A_{ij}^{z,k^0} + A_{ij}^{z,k^N} && (2.120)\\
&\Delta A_{ij}^{z,k^0} \leq \sum_{h=1}^{k_e}\left(\Delta A_{ij\max}^{z,h}\delta_{ij}^{z,hk}\right) && (2.121)\\
&A_{ij}^{z,k^N} \leq A_{ij\max}^{z^N}\cdot\left(1-\sum_{h=1}^{k_e}\delta_{ij}^{z,hk}\right) && (2.122)\\
&\sum_{h=1}^{k_e}\delta_{ij}^{z,hk} \leq 1 && (2.123)
\end{aligned}\right\} \quad z\in Z;\ i\in H^z;\ j\in C^z;(i,j)\in P;(i,j)\in B;1\leq k\leq k_{\max}$$

$$\sum_{k=1}^{k_{\max}}\delta_{ij}^{z,hk} \leq 1 \qquad z\in Z;\ i\in H^z;\ j\in C^z;(i,j)\in P;(i,j)\in B;1\leq h\leq k_e \tag{2.124}$$

$$\sum_{k=1}^{k_{\max}}\ \sum_{h=1}^{k_e}\delta_{ij}^{z,hk} = k_e \qquad z\in Z;\ i\in H^z;\ j\in C^z;(i,j)\in P;(i,j)\in B \tag{2.125}$$

In addition, the number of new heat exchanger unit placed into the existing network would be specified as the following constraint.

$$\sum_{z}\sum_{i\in H^z}\sum_{\substack{j\in C^z\\(i,j)\in P}}\left(U_{ij}^{z}-U_{ij}^{z^0}\right)\leq U_{\max}^{N} \tag{2.126}$$

2.5.2 Objective Function

In retrofit situation, the exchanger investment cost-functions are different from the grassroot design. The objective function for the retrofit heat exchanger network structure also subjects to minimize the total annualized cost but the retrofit programming model has complicated functions for the area cost. Not only count for the number of exchanger unit, but there are also the existing units which need to optimize for area addition or new able place an exchanger. Therefore, the exchanger area for the retrofit target is consisted of area addition to the initial structure and the new exchanger area. All other terms, the hot and cold utility cost, seem to be the same as the grassroot design model. However, fixed charge for the exchanger unit is need to count as the increasing number of unit which correspond to minus the number of exchanger unit, U_{ij}^{z}, with the initial unit, $U_{ij}^{z^0}$.

$$Min\ \ Cost = \sum_{z} \sum_{i \in HU^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} c_i^H F_i^H \Delta T_i + \sum_{z} \sum_{j \in CU^z} \sum_{\substack{i \in H^z \\ (i,j) \in P}} c_j^C F_j^C \Delta T_j + \sum_{z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} c_{ij}^F \left(U_{ij}^z - U_{ij}^{z^0} \right)$$

$$+ \sum_{z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \notin B}} \left(c_{ij}^{A^0} \Delta A_{ij}^{z^0} + c_{ij}^{A^N} A_{ij}^{z^N} \right) + \sum_{z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \in B}} \sum_{k=1}^{k_{max}} \left(c_{ij}^{A^0} \Delta A_{ij}^{z,k^0} + c_{ij}^{A^N} A_{ij}^{z,k^N} \right) \tag{2.127}$$

2.6 Additional Topics in Heat Exchanger Networks Retrofit

2.6.1 Limitation of Repiping

The restriction of changing the exchanger unit is needed to identify. When settling two exchangers in series, an exchanger must be settled after the former unit ends.

$$\sum_{\substack{l \in M_i^z \\ l \le m \\ j_2 \in P_{ij}^{H}}} K_{ij_2 l}^{z,H} \le \sum_{\substack{l \in M_i^z \\ l \le m \\ j_1 \in P_{ij}^{H}}} \hat{K}_{ij_1 l}^{z,H} \qquad z \in Z\,; m \in M^z\,;\ i \in H_m^z; (i,j_1) \in P\,; (i,j_2) \in P\,; (j_1, j_2) \in \Theta_i \tag{2.128}$$

2.6.2 Relocation of Existing Heat Exchangers

Generally, in the proposed MILP model, wherever an existing exchanger addressed between matching of hot stream i and cold stream j, it also be equipped in the same pair of hot and cold stream for the retrofitted structure. However, relocation the existing exchanger unit from the original match (i,j) to the different match (i',j') of hot and cold stream would be needed to consider.

Relocation possibility can be calculated by the binary variables, such as 1000 binary variables are used to define the possible relocations for the network composed of 10 hot, 10 cold streams and 10 original heat exchanger units. A very large number of integers will effect to the model performance. Thus, this algorithm is considered the exchanger relocation for the case where highly reducing cost occurs. So, the designer should define which exchanger is relocated and the following constraints are used to figure out the exchanger area after repositioning.

Area requirement for existing, rclocated and new heat exchangers – $(i,j) \notin$ B

$$\left.\begin{aligned}
&A_{ij}^{z} \leq A_{ij}^{z^0} + \Delta A_{ij}^{z^0} + A_{ij}^{z^N} && (2.129)\\
&A_{ij}^{z^0} = \sum_{k=1}^{k_c^l} A^k \delta_{ij}^{z,k} && (2.130)\\
&\Delta A_{ij}^{z^0} \leq \sum_{k=1}^{k_c^l} \Delta A_{\max}^k \delta_{ij}^{z,k} && (2.131)\\
&A_{ij}^{z^N} \leq A_{ij\max}^{z^N} \cdot \left(U_{ij}^{z} - U_{ij}^{z^0} \cdot \sum_{h=1}^{k_c^l} \delta_{ij}^{z,k} \right) && (2.132)\\
&U_{ij}^{z} \leq U_{ij\max}^{z} && (2.133)\\
&\sum_{h=1}^{k_c} \delta_{ij}^{z,k} \leq 1 && (2.134)
\end{aligned}\right\} \quad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \notin B$$

Where A^k is the area of original exchanger that has been relocated to the new match (i',j'). Whenever the original k-th exchanger is utilized to serve in a new match, $\sum_{k=1}^{k_c} \delta_{ij}^{z,k}$ is equal to one, and then relocation constraint (2.130) forces that the existing exchanger area at the new match (i',j'), $A_{ij}^{z^0}$, also equals to the unit area of the original match, A^k. Maximum area addition for the existing unit which served to relocate is also required.

Area requirement for existing, relocated and new heat exchangers – $(i,j) \in$ B

$$\left.\begin{aligned}
&A_{ij}^{z,k} \leq A_{ij}^{z,k^0} + \Delta A_{ij}^{z,k^0} + A_{ij}^{z,k^N} && (2.135)\\
&A_{ij}^{z,k^0} = \sum_{h=1}^{k_c^l} A^h \delta_{ij}^{z,hk} && (2.136)\\
&\Delta A_{ij}^{z,k^0} \leq \sum_{h=1}^{k_c^l} \Delta A_{\max}^h \delta_{ij}^{z,hk} && (2.137)\\
&A_{ij}^{z,k^N} \leq A_{ij\max}^{z^N} \cdot \left(1 - \sum_{h=1}^{k_c^l} \delta_{ij}^{z,hk} \right) && (2.138)\\
&\sum_{h=1}^{k_c^l} \delta_{ij}^{z,hk} \leq 1 && (2.139)
\end{aligned}\right\} \quad z \in Z;\ i \in H^z;\ j \in C^z;\ (i,j) \in P;\ (i,j) \in B;\ 1 \leq k \leq k_{\max}$$

For the exchanger relocation, the objective function would be

$$
\begin{aligned}
Min \quad Cost = & \sum_{z} \sum_{i \in HU^z} \sum_{\substack{j \in C^z \\ (i,j) \in P}} c_i^H F_i^H \Delta T_i + \sum_{z} \sum_{j \in CU^z} \sum_{\substack{i \in H^z \\ (i,j) \in P}} c_j^C F_j^C \Delta T_j \\
& + \sum_{z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \notin B}} \left(c_{ij}^F \left(U_{ij}^z - U_{ij}^{z^0} \cdot \sum_{k=1}^{k_z'} \delta_{ij}^{z,k} \right) + c_{ij}^{A^0} \Delta A_{ij}^{z^0} + c_{ij}^{A^N} A_{ij}^{z^N} \right) \\
& + \sum_{z} \sum_{i \in H^z} \sum_{\substack{j \in C^z \\ (i,j) \in P \\ (i,j) \in B}} \left(c_{ij}^F \left(U_{ij}^z - \sum_{k=1}^{k_{\max}} \sum_{h=1}^{k_z'} \delta_{ij}^{z,hk} \right) + \sum_{k=1}^{k_{\max}} \left(c_{ij}^{A^0} \Delta A_{ij}^{z,k^0} + c_{ij}^{A^N} A_{ij}^{z,k^N} \right) \right)
\end{aligned}
\tag{2.140}
$$