CHAPTER III

NUMERICAL SOLUTION SCHEME

3.1 Determination of Flexibility Matrix

The development of a numerical solution scheme to determine quasi-static behavior of an axially loaded elastic bar embedded in a multilayered poroelastic half-space is considered in this chapter. Consider an elastic bar as shown in Fig. 4. The bar is discretized into N_t elements. Let f_{ij} denote the vertical displacement at a point $P_i(r_i,z_i)$ on the interface S due to body forces of unit intensity distributed in the volume of the j^{th} element. The displacement f_{ij} can be determined by solving the global equation system, eqn (2.45).

Since the global equations (2.45) are formulated by considering the continuity conditions at layer interfaces and the solutions of eqn (2.45) are obtained at those interfaces. Therefore, in order to evaluate f_{ij}, the thickness of the j^{th}element, Δt_j, must be discretized into a finite number of interfaces for placing an applied vertical patch load of unit intensity along the thickness Δt_j to represent the approximate action due to the body forces of unit intensity acting through the whole volume of the j^{th} element. The numerical integration scheme, namely the trapezoidal rule, is used in this step to accumulate the vertical displacements which obtain from placing the vertical patch load at each interface along Δt_j to obtain f_{ij}.

3.2 The Inverse Laplace-Hankel Integral Transform

The solutions of eqn (2.45) are the Laplace-Hankel transforms of displacements and pore pressure at layer interfaces for discrete values of ξ and s. The time domain response of a multilayered half-space is determined by numerically evaluating the inverse relationship of the integral appearing in eqn (2.17). The Laplace inversion is carried out numerically and the inversion of the integral with respect to ξ in eqn (2.17) is numerically evaluated by employing the trapezoidal rule.

There are two Laplace inversion methods which are widely used in poroelasticity problems(4),(5),(10),(15). The first one was proposed by Stehfest(16) and the other by Schapery(17). The formula due to Stehfest(14) is given by

$$f(t) \approx \frac{\ln 2}{t}\sum_{n=1}^{N} c_n \bar{f}\left(n\frac{\ln 2}{t}\right) \tag{3.1}$$

where $\bar{f}$ denotes the Laplace transform of f(t) and

$$c_n = (-1)^{n+N/2} \sum_{k=[(n+1)/2]}^{\min(n,N/2)} \frac{k^{N/2}(2k)!}{(N/2-k)!k!(k-1)!(n-k)!(2k-n)!} \tag{3.2}$$

and N is even. It was found(10) that the accurate time-domain solutions can be obtained from eqn (3.1) with $N \geq 6$ for general poroelasticity problems. It is important to note that the Stehfest scheme(14) is computationally quite demanding although it is accurate. A simpler and more computationally efficient scheme is given by Schapery(15) which can be expressed as

$$f(t) \approx \left[s\bar{f}\right]_{s=0.5/t} \tag{3.3}$$

where $\bar{f}$ denotes the Laplace transform of $f(t)$ and s is the Laplace transform parameter.