Chapter IV
The Effects of Drifts and Diffusion on the Propagation of Solar Cosmic Rays

Part of this work is to study the effects of drifts and diffusion perpendicular to the magnetic field on the propagation of the particles released from the Sun. "Drifts" refer to the systematic propagation of the particles perpendicular to the magnetic field because of the curvature or gradient of the magnetic field. Diffusion is the random propagation of the particles due to random irregularities in the magnetic field; here we will consider such motions perpendicular to the mean field. If perpendicular drifts or diffusion have effects on the transport, then we must consider them in the simulations.

Perpendicular drifts

Drifts are the systematic motions of particles perpendicular to the magnetic field. There are two important types of drifts, as follows:

Curvature drift

Curvature drift is a systematic motion perpendicular to the magnetic field, due to the curvature of the magnetic field. The curvature of the magnetic field lines makes the drift velocity of particles increase when the radius of curvature of magnetic field lines of force, R, is longer than the gyroradius, a, of the particle. The guiding center of a particle will tend to follow a straight line tangent to a curving field line. This leads to an effective centrifugal acceleration $v_{\|}^2/R$, which can also be viewed as due to an effective electric field (Jackson 1975):

$$\vec{F}_{\text{eff}} = q\vec{E}_{\text{eff}} = (\gamma m)\vec{a}_{\text{cent}}. \tag{4.1}$$

where $\vec{a}_{\text{cent}}$ is the centrifugal acceleration. Then

$$\vec{E}_{\text{eff}} = \frac{\gamma m}{q}\left(\frac{v_{\|}^2}{R}\right)\hat{R}_0 = \frac{\gamma m}{q}\left(\frac{\vec{R}}{R^2}\right)v_{\|}^2. \tag{4.2}$$

where $\vec{R}$ points from the center of curvature to the point of interest on the field line. The effective electric field is always perpendicular to the magnetic field. The drift velocity of a particle moving perpendicular to electric and magnetic fields is

$$\vec{v}_d = \frac{c(\vec{E}\times\vec{B})}{B^2}. \tag{4.3}$$

We find the velocity $\vec{v}_d$ for the curvature drift velocity perpendicular to the magnetic field to be

$$\begin{aligned}\vec{v}_c &= \frac{c(\vec{E}_{\text{eff}}\times\vec{B}_0)}{B_0^2}\\ &\approx \frac{c\gamma m}{q}v_{\|}^2\frac{\vec{R}\times\vec{B}_0}{R^2B_0^2}.\end{aligned} \tag{4.4}$$

Gradient drifts

Gradient drifts arise from the perpendicular gradient of the mean magnetic field. This gradient is very small compared to the mean field strength. The gradient drift velocity is

$$\vec{v}_g = \frac{a^2\omega_B}{2B^2}(\vec{B}\times\vec{\nabla}_{\perp}B). \tag{4.5}$$

where ω_B is the gyration frequency, or the frequency with which the particle gyrates around the field line,

$$\omega_B = \frac{qB}{\gamma mC}, \tag{4.6}$$

and a is the gyroradius of the particle (Tuska 1990).

The particle motions in the interplanetary magnetic field have drifts in a direction orthogonal to both $\vec{\nabla} B$ and $\vec{B}$, and the drift of the guiding center is slower than the particle orbit.

Perpendicular Diffusion

Diffusion is a characteristic of the particle motion in a random magnetic field. For the transport of particles released from the Sun, this is described by a tensor, κ. The diffusion of these particles along the direction of the interplanetary spiral magnetic field is called "parallel diffusion," with a coefficient $\kappa_{\|}$, and that across the interplanetary magnetic field is called "perpendicular diffusion," with a coefficient $\kappa_{\perp}$. A relationship based on these diffusion coefficients is

$$\kappa_r(r, t) = \kappa_{\|} \cos^2 \psi + \kappa_{\perp} \sin^2 \psi, \tag{4.7}$$

where $\kappa_r(r, t)$ is a diffusion coefficient that depends on radius and energy, ψ is the angle between the mean magnetic field and the radial direction,

$$\psi = \tan^{-1} \left(\frac{\Omega r}{v_{sw}} \right) \tag{4.8}$$

(see Figure 2.5), r is the distance from the Sun and v_{sw} is the speed of the solar wind. The variation of κ_r with r is often parameterized as $\kappa_r \approx r^b$.

Result of Previous Work

Table 4.1 summarizes the opinions expressed in various papers in the literature. Column 3 also shows whether the expressions are based on reasoned assumption (a), theoretical calculation (c), or fits to observations (o). This is far from a complete survey of the

literature, but it summarizes important papers that have addressed the issue of how to choose $\kappa_{\perp}$ and b for transport models.

Table 4.1: Assumptions, calculations, and observations of κ and b in previous work.

Diffusion coefficients	Characteristics of $\kappa_{\parallel}$ and $\kappa_{\perp}$	Radial index (b)	References
$\kappa_r \approx \kappa_{\parallel}$	$\kappa = \kappa_{\parallel} \cos^2 \psi, \kappa_{\perp} = 0$		Parker 1963
$\kappa_r \propto r^b$ $\kappa_r \approx 10^{21}$ to 10^{22} cm^2/s		0.0 $\rightarrow$ 0.5(c) (κ isotropic) 2.0 to 2.5 (κ very anisotropic)	Parker 1965
$\kappa_r = \kappa_0 r$	$\kappa_{\parallel} = \kappa_0 r / \cos^2 \psi$, $\kappa_{\perp} = 0$	1(a)	Axford 1965 Ng & Gleeson 1971 Völk et al. 1974
$\kappa_\theta, \kappa_\phi = \kappa_{\perp}$ $\kappa_r = \kappa_{\parallel}$ =const. $\kappa_r \approx (2 \rightarrow 8) \times 10^{20}$cm^2/s (2.7 AU)	$\kappa_{\parallel} = \kappa_0 r / \cos^2 \psi$, $\kappa_{\perp} = 0$	(a) (o)	Lupton & Stone 1973
$\kappa_r \approx \kappa_{\parallel}$, $\kappa_{\parallel} \propto r^{2.4}$ $\kappa_r \propto r^{0.4}$ ($\approx$5 AU)	$\kappa_{\perp} \sim 5 \times 10^{20}$cm^2/s $\kappa_{\perp}/\kappa_{\parallel} \approx 0.01 \rightarrow 0.02$	0.4(o)	Conlon 1978
$\kappa_r = M(r/r_e)^b$ near Earth M is local diffusion coeff. $M \approx (2.5 \rightarrow 5) \times 10^{21}$cm^2/s $M = \kappa_{\parallel} \cos^2 \psi = \frac{1}{3} v \lambda_r$	$\kappa_{\parallel} \approx \kappa_r$ $\kappa_{\perp}$ neglected to first order	0 $\rightarrow$ 1(o)	Schulze, Richter, & Wibberenz 1977
$\kappa_r \approx \kappa_{\parallel} \cos^2 \psi$ (1 AU) $\kappa_r \approx 3 \times 10^{21}$ cm^2/s	$\kappa_{\parallel} \approx \kappa_r$	(o)	Hamilton 1977
$\kappa_r \propto r^b$ (1-5 AU),4Mev,26Mev protons		0.0 ± 0.3	Zwickl & Webber 1977
		0.6 ± 0.1	Beeck et al. 1987
Estimates of $\kappa_{\perp}/\kappa_{\parallel}$, cited by Ng 1987			
$\kappa_{\perp} \approx \beta \cdot 4 \times 10^{20}$ cm^2/s, $\kappa_{\perp}/\kappa_{\parallel} = 0.07$ (rigidity 1 GV)		(c)	Forman, Jokipii, & Owens 1974
$\kappa_{\perp} = 1.2 \times 10^{19}$ cm^2/s, $\kappa_{\perp}/\kappa_{\parallel} = 0.024$ (1 MeV protons) $\kappa_{\perp} = 3.5 \times 10^{19}$ cm^2/s, $\kappa_{\perp}/\kappa_{\parallel} = 0.012$ (10 MeV protons)		(o)	Toptygin 1985
$\kappa_{\perp} = 3.5 \times 10^{19}$ cm^2/s, $\kappa_{\perp}/\kappa_{\parallel} = 0.061$ (1 MeV protons) $\kappa_{\perp} = 1.5 \times 10^{20}$ cm^2/s, $\kappa_{\perp}/\kappa_{\parallel} = 0.075$ (10 MeV protons)		(o)	Moussas et al. 1982a,b

Result for Drifts and Diffusion of Solar Cosmic Rays

Gradient and Curvature Drifts

It is important to estimate the effects of gradient and curvature drifts before correcting for their effects or neglecting them entirely. The effect of these drifts is to add another term to the transport equation;

$$\frac{\partial F}{\partial t} = \ldots - (\vec{v}_g + \vec{v}_c)\vec{\nabla} F, \tag{4.9}$$

where F is the density of particles in the "mixed frame," and $\vec{v}_g$ and $\vec{v}_d$ are the drift velocities due to gradient and curvature drifts, respectively.

Now we will calculate the magnitude of the drift velocities. The gradient drift velocity is given by equation:

$$\vec{v_g} = \frac{a^2 \omega_B}{2B^2}(\vec{B} \times \vec{\nabla}_\perp B). \tag{4.10}$$

Let us calculate the magnitude of this for a typical proton energy of 33 MeV and interplanetary magnetic field strength of 5×10^{-5} G. The various quantities in this formula are found to be

$$\begin{aligned}
\omega &\equiv \frac{qB}{\gamma mc} = \frac{4.80 \times 10^{-10} \text{ esu} \cdot 5 \times 10^{-5} \text{ G}}{1.04 \cdot 1.67 \times 10^{-24} \text{ g} \cdot 3.00 \times 10^{10} \text{ cm/s}} \\
&= 0.479 \text{ Hz} \\
v &= 7.75 \times 10^9 \text{ cm/s} = 0.259 \text{ c} \\
v_\perp &= v\sqrt{1-\mu^2}, \text{where } \mu \text{ is the pitch} - \text{angle cosine} \\
a &= v_\perp/\omega = 1.61 \times 10^{10} \text{ cm}\sqrt{1-\mu^2} = 1.08 \times 10^{-3}\text{AU}\sqrt{1-\mu^2} \\
& \quad \text{(at most about 0.4 times the Earth to moon distance).}
\end{aligned} \tag{4.11}$$

Since $\vec{B}$ is perpendicular to $\vec{\nabla}_\perp B$, we now have

$$v_g = 6.21 \times 10^{19} \text{ cm}^2/\text{s} \cdot (1-\mu^2)\frac{1}{B}\frac{dB}{dl}, \tag{4.12}$$

where l is along the direction of the perpendicular gradient of $\vec{B}$, which, for points in the solar equatorial plane, is in the plane perpendicular to $\vec{B}$ and directed toward the Sun. For an Archimedean spiral field, there is a radius $R \approx 1$ AU such that $B \approx \sqrt{(R/r)^2 + (R/r)^4}$. Then

$$\frac{1}{B}\frac{dB}{dr} = -\frac{1}{r}\frac{r^2 + 2R^4}{r^2 + R^2}, \tag{4.13}$$

Note also that $dr/dl = -\sin\psi = -r/\sqrt{r^2 + R^2}$. Assuming $R = r = 1$ AU, we have

$$\begin{aligned}
\frac{1}{B}\frac{dB}{dl} &= \frac{1}{B}\frac{dB}{dr}\frac{dr}{dl} = \frac{3}{2\sqrt{2}\text{ AU}} \\
&= 7.09 \times 10^{-14}\text{ cm}^{-1} \\
v_g &= 4.40 \times 10^6\text{ cm/s}(1 - \mu^2).
\end{aligned}$$

We can perform a similar calculation to find the curvature drift velocity. Since $\vec{R}$ is perpendicular to $\vec{B}$, with a magnitude of about 1 AU, and $v_{\|} = \mu v$, we use equation (4.4) and $\omega = qB/\gamma mc$ to get

$$\begin{aligned}
v_c &= \frac{v_{\|}^2}{\omega R} = \frac{(7.75 \times 10^9\text{ cm/s})^2}{0.479\text{ Hz} \cdot 1.50 \times 10^{13}\text{ cm}}\,\mu^2 \\
&= 8.36 \times 10^6\text{ cm/s}\,\mu^2.
\end{aligned}$$

Since $-1 < \mu < 1$, we see that the maximum total drift velocity, $\vec{v}_g + \vec{v}_c$, occurs when $\mu = \pm 1$, yielding a total velocity of 8.36×10^6 cm/s. Now we typically analyze flare data for up to 1 day after the flare occurrence, so the maximum distance drifted over that time is 7.22×10^{11} cm. At a radius of 1 AU, this corresponds to an angle of only 2.77°. Given that the particle distribution is spread out laterally over $\sim 60°$, we can expect that the change in the flux at a given point due to the southward shift of this distribution by less than 3° will be quite small. Given that the lateral distribution is not actually known, and probably varies from flare to flare, it is not possible to accurately model this small effect for any given flare. Therefore, it is justifiable to neglect the effect of gradient and curvature drifts.

Perpendicular Diffusion.

The effect characteristic of the perpendicular diffusion in the transport equation is

$$\frac{\partial F}{\partial t} = \cdots + \kappa_\perp \nabla_\perp^2 F, \tag{4.14}$$

From previous work, there have been have the different measurements of $\kappa_\perp$. Palmer (1982) used $\kappa_\perp = (v/c) \cdot 10^{21}$ cm^2/s. Thus we will use a value of $\kappa_\perp \approx 2.5 \times 10^{20}$ cm^2/s. The transverse Laplacian of F can be roughly estimated by assuming a Gaussian lateral distribution with a width of σ. Assume that the detector is near the peak in the distribution. In this case, the second derivative in each lateral direction yields $-F/\sigma^2$, for a total of $-2F/\sigma^2$. If we take σ to be the distance corresponding to 60°, we get roughly 1 AU. Therefore, we have

$$\begin{aligned} \frac{\partial F}{\partial t} &= \cdots - \frac{5 \times 10^{20}\ \mathrm{cm}^2/\mathrm{s}}{(1.5 \times 10^{13}\ \mathrm{cm})^2} F \\ &= \cdots - 2.2 \times 10^{-6}\ \mathrm{s}^{-1} F, \end{aligned}$$

which makes F decrease with a time constant of 5.2 days. The flux decay over 5.2 days is smaller than the other terms in the transport equation, which typically make the flux decay with a time constant of at most 20 hours. It would be nice to estimate the particle density, F, due to perpendicular diffusion, so as to improve the accuracy of the fits. However, the magnitude of the correction is again highly uncertain because $\kappa_\perp$ and the shape of the lateral distribution of particles are very poorly measured.

Therefore, we conclude that the effects of both drifts and perpendicular diffusion can be neglected.