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เรื่อง A Straight Line p/z Plot in Multilayer Tight Gas Reservoirs

โดย

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กิตติกรรมประกาศ

ผมต้องขอขอบคุณทางจุฬาลงกรณ์มหาวิทยาลัยที่ได้ให้เงินสนับสนุนพัฒนาอาจารย์ใหม่ กองทุนรัชดาภิเษกสมโภช นอกจากนั้นต้องขอขอบคุณความช่วยเหลือจากเพื่อนๆอาจารย์และหัวหน้าภาคภาควิชาวิศวกรรมเหมืองแร่และปิโตรเลียมคณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย จึงทำให้ผมสามารถทำงานวิจัยในชิ้นนี้เสร็จลุล่วงด้วยดี

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Introduction

Literature review

Schilthuis (1936) developed the zero-dimension material balance equation (MBE). The equation has long been regarded as one of the fundamental tools for interpreting and forecasting reservoir performance (Dake, 1978 and Hagoort et al., 2000). Havlena and Odeh (1963 & 1964) presented the technique of interpreting the general material balance equation as a straight line and applied the technique to several case studies. The general MBE states that volume of underground withdrawal, which results in pressure drop in a reservoir, equals the expansion of reservoir fluid(s) plus the reduction in hydrocarbon-pore volume (connate water expansion plus pore volume reduction). Material balance is an important performance-based tool in reservoir engineering which can be used to estimate the original volume of hydrocarbons-in-place in a reservoir (Fekete Associates Inc).

For a volumetric gas reservoir, the following assumtions are normally applied (Craft and Hawkins, 1991)

- The water and formation compressibilites are significantly less than the gas compressbility and are neglected.
- There is no water production from the reservoir.
- There is no water encroachment into the reservoir.
- The reservoir temperature is constant.
- At all locations in the reservoir, the pressure and fluid properties are the same.

The above assumptions imply that the reservoir behaves as a tank, and has a uniform pressure. Based on these assumptions, gas expansion is the only force driving production from the reservoir. After some period of production, the reservoir pressure declines from the initial pressure (p_i) to a lower average reservoir pressure (p). The general material balance for a volumetric gas reservoir can be rearranged as:

$$\frac{(p\ /z\ )}{(p_i\ /z_i)} = 1 - \frac{G_p}{G} \qquad (1)$$

The above equation implies that a plot of $(p/z)/(p_i/z_i)$ versus cumulative gas production (G_p) yields a straight line with a slope of (-1/G). It normally needs more than 2 years of gas production and static pressure data to have a reliable estimation of G (Craft and Hawkins, 1991). The technique is very popular in gas reservoir engineering (Payne, 1996) because its simplicity. The technique is independent of reservoir properties, well completion details, and production history.

Commingled wells behave very differently from conventional single layer completions. Conventional approaches – like (p/z) – can give very misleading results. The p/z plot underestimates gas-initially-in-place (GIIP) during the early production period and overestimates GIIP during the late production period (Kuppe et al., 2000). To properly apply the MBE for multi-layer gas reservoirs, one needs average reservoir pressure which requires an impractically long shut-in period due to differential depletion of the different layers. The average reservoir pressure is the most difficult piece of information to obtain.

During a typical production period, the higher productivity layer dominates the early-time production, which tends to cause greater pressure depletion in this layer. The pressure difference between the layers increases with time and reaches a maximum value. In a study by Ross (2014) the maximum value is reached at a recovery factor (RF) of 30%. Beyond this point, the lower-productivity layer dominates the late-time production period, and the pressure difference between the layers decreases with time.

Lefkovits et. al. (1959) discussed the characteristics of pressure build-up for a commingled production system. Cobb and Ramey (1972) showed that the time required for pressures to equilibrate between layers is impractically long because of the time required for the cross-flow from the lower productivity layer (less depleted) to the higher productivity layer (more depleted) to die away. This backflow or wellbore cross-flow is a very slow process which occurs under semi-steady-state conditions. This issue is more complicated when the permeability contrast is significantly high (Kuppe et al., 2000).

Without a sufficiently long shut-in period, the wellbore pressure tends to be significantly lower than the equilibrium system pressure. This is a critical problem for multi-layer tight gas reservoirs which have substantial pressure gradients. This issue violates the basic tank assumption (Payne, 1996). The p/z plot is generally curved and deviates from the theoretical straight line (Hagoort et al, 2000 and Ross, 2014). The p/z plot is sometime not stable. It leads to inaccurate production forecasts and underestimated reserve predictions. Payne (1996) did not recommend using the MBE method to analyze a tight gas reservoir.

Hagoort et al. (2000) proposed a method to quantitatively interpret the curved p/z plot using the concept of semi-steady-state time. By keeping the ratio ($t_{shut-in}(p)/t_{sss}(p)$) constant, the shut-in pressures are useful and comparable. The estimated GIIP is slightly lower than the true GIIP. The authors ignored the pressure dependence of the semi-steady-state time.

Kuppe et al. (2000) developed a simple spreadsheet model for multi-layer tight gas reservoirs. The (p/z) trends of lower-permeability and higher-permeability layers are weighted using their productivity indices (PI). They simultaneously matched both the MBE and the production profile. The model can be used to estimate GIIP, reserves, and productivity. Their work assumes that (1) there is no reservoir cross-flow; (2) there is no wellbore cross-flow (no extensive shut-in period); (3) the (kh) contrast between the 2 layers is less than one order of magnitude.

Ross (2014) combined pressure transient and production logging tools (PLT) data with a simple multi-tank model to illustrate the pressure behavior of a multi-layer reservoir. His work relies on a multi-block concept (Payne, 1996, Hagoort and Hoogstra, 1997, Fox et al., 1998, and Hagoort et al., 2000) to estimate the GIIP by layer. Last (2012) proposed to use a multi-rate PLT with petrophysical information and surveillance data to estimate the GIIP distribution. The author applied the technique to both synthetic and field data and discussed the limitations.

Previous efforts work on the curved p/z plot using wellbore build-up pressure. Normally, the shut-in periods are not long enough to reach pressure-equilibrium. There is no attempt to work on a straight line p/z plot.

Objective of this study

The objective of this study is to propose a new methodology for applying the MBE for a multi-layer gas reservoir. This study will present a new approach to calculate the average reservoir pressure using pressures from individual reservoirs, without a long shut-in period. The pressure calculated using this technique yields a straight line on the p/z plot, similar to one for a single layer gas reservoir. This study should be beneficial to engineers who are working on estimation of GIIP and reserves.

Methodology

MBAL for multi-layer gas reservoir

This section considers 2 volumetric gas reservoirs which are produced simultaneously. Layer 1 has a higher productivity than layer 2, with the following additional conditions. Both layers have the same gas properties but may have different hydrocarbon-pore volumes. The MBE for this system is

$$\frac{\left(\overline{p/z}\right)}{\left(\overline{p/z}\right)_i} = 1 - \frac{\left[G_{p1} + G_{p2}\right]}{\left[G_1 + G_2\right]} \tag{2}$$

Note that MBEs for a multilayer gas system and for a single layer system are similar. The cumulative production and total gas initially in place are the summation of cumulative gas production from layers 1 and 2, and the summation of gas initially in place from layers 1 and 2, respectively. $\left(\overline{p/z}\right)$ is average value of (p/z) from these two reservoirs. The latter quantity is the most difficult piece of information to obtain. For tight gas sands, it will take an impractically long shut-in period to reach the equilibrium pressure. There is still an existing pressure gradient even after a long shut-in period. It is the main contribution of this study to find a new approach to evaluate $(p/z)_{avg}$ without a long shut-in period.

Wellbore pressure during shut-in period

To obtain the average reservoir pressure, the normal practice is to shut in a well and wait for the pressure to reach equilibrium. Lefkovits et al. (1961) showed that it could potentially take many years for a multi-layer system to reach pressure equilibrium. In a tight commingled gas reservoir, transient flow could occur for a long time (Cobb et al., 1972, Raghawan et al., 1974, Hagoort et al, 2000 and Meehan and Verma, 1995, and Kuppe et al., 2000). Earlougher (1977) estimated the time required to reach semi-steady-state condition for a well in a bounded reservoir as

$$t_{sss} = \frac{\phi \mu c_t A}{k} t_{DAss} \tag{3}$$

t_{DAsss} is the dimensionless semi-steady-state time which is equal to 0.05 for a square reservoir with a well in the center. A 2-layer system, with $k_1/k_2 = 10$, requires shut-in period some 200 times longer than for 1-layer system, Cobb et al. (1972) and Kuppe et al. (2000). Therefore, the measured pressure from a pressure survey (without sufficiently long shut-in period) doesn't represent the average pressure for the drainage volumes of the well (Ross, 2014).

Lefkovits et al. (1961) discussed the typical characteristics of a pressure build-up for a 2-layer system, illustrated in Figure 1. After wellbore storage dies away, the semi-log straight line, Section EF, is observed. An analysis of this line yields (kh) of the whole system and p^* (a false pressure which is significantly lower than the average system pressure). Larsen (1981) and Prijambodo et al (1985) pointed out that the semi-log straight line may not even exist for some systems. Then the build-up pressure levels off, section FG. The pressure at this time reflects the PI-weighted system pressure, Chen et al. (1997). It is analogous to a single layer attaining equilibrium pressure. As the pressurization of layer 2 increases, the wellbore pressure rises again, section GH. Finally, the wellbore pressure reaches the average system pressure at point H, after the wellbore cross-flow diminishes. It reflects the pore-volume-compressibility-weighted system pressure, Chen et al. (1997).

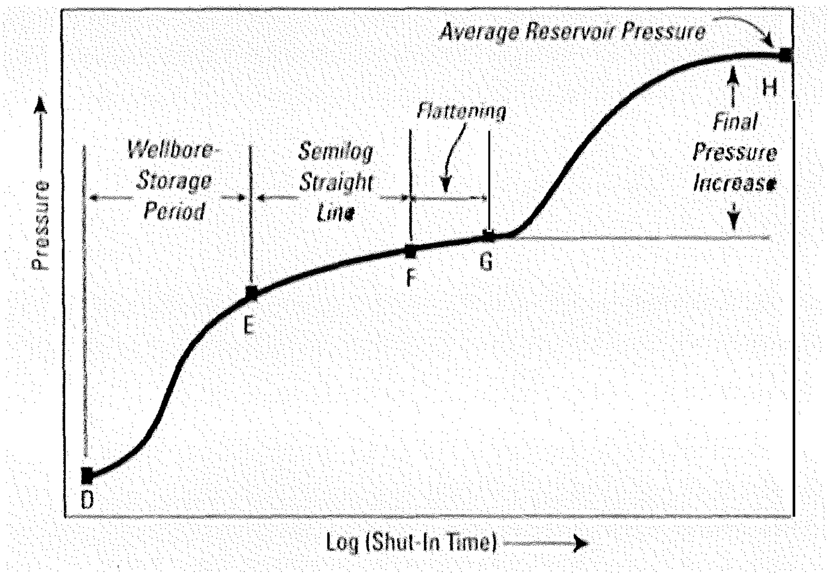

Figure 1: Pressure build-up for a 2-layer reservoir (after Kuppe and Chugh, 2000)

Hagoort et al. (2000) derived the analytical solution for pressure equilibration in 2-layer oil reservoir during a shut-in period. Their solution is similar to ones from Lefkovits et al. (1961) and Chen et al. (1993). They assumed semi-steady-state flow conditions. Their results match pretty well with simulation results. In Appendix B, this study derives the analytical solution for a 2-layer gas reservoir. During any shut-in period, the relationship between pressure in layer 1 (p_1), wellbore pressure (p_{wb}), the average reservoir pressure ($\overline{p}$), and pressure in layer 2 (p_2) is the following

$$p_1 < p_{wb} = \sqrt{\frac{J_1 p_1^2 + J_2 p_2^2}{J_1 + J_2}} < \overline{p} < p_2 \tag{4}$$

where J is the productivity index. Since layer 1 has higher productivity than layer 2; $J_1 > J_2$, based on the above equation, wellbore pressure is always closer to the pressure of the more permeable layer (p_1), Kuppe et al. (2000). While p_1 increases with time, p_2 decreases with time. This is due to the cross-flow of fluid from layer 2 via the wellbore into layer 1.When the cross-flow phenomenon stops, p_1, p_2, p_{wb}, and the averaged system pressure will be the same.

Averaged reservoir pressure:

This study proposes an alternative to estimate the averaged reservoir pressure without shutting in a well. Using the real gas law, the average system pressure is the weighted average of pressure of each layer. The weights are the relative hydrocarbon-pore volume.

$$(p/z)_{avg} = \frac{(p/z)_1 V_1 + (p/z)_2 V_2}{V_1 + V_2} = \frac{(p/z)_1 F_v + (p/z)_2}{F_v + 1} \quad (5)$$

where the hydrocarbon-pore-volume ratio (F_v) is defined as

$$F_v = \frac{V_1}{V_2} \quad (6)$$

In order to calculate the averaged reservoir pressure, one needs to find the pressures of individual layers, their gas compressibility factors, and the hydrocarbon-pore-volume ratio (F_v).

Selective Inflow Performance (SIP)

Stewart and Wittmann (1981) introduced the SIP method. For commingled producing wells, the layer static pressures are not available by direct measurement, not even by shutting in the well, unless all the reservoir layers are in strict hydraulic equilibrium. SIP can be applied to establish the Inflow Performance Relationship (IPR) of individual layers (Ilyas et al., 2012). The well is opened for production at different stabilized rates, assuming semi-steady state conditions. At each rate, a production log is run across all producing intervals to estimate the flow rates of, and flowing pressures at, each individual layer. PVT data is utilized to convert down-hole flow rates to the flow rates at standard conditions.

For each layer, the flowing pressure is plotted against the flow rate to generate the IPR, illustrated in Figure 2 (Last, 2012, Ilyas et al., 2012 and Ross, 2014). There are many IPR relationships, including Fetkovich [C & n], LIT [$m(p_{avg}) - m(p_{wf}) = aq + bq^2$], and straight line methods [$p_{avg} - p_{wf} = aq$]. The estimated layer pressure is the pressure where the flow rate is zero. This methodology has become very popular, especially in gas wells. This is due to the shorter stabilization times. SIP overcomes a fundamental limitation of commingled producing wells (Petrowiki.org)

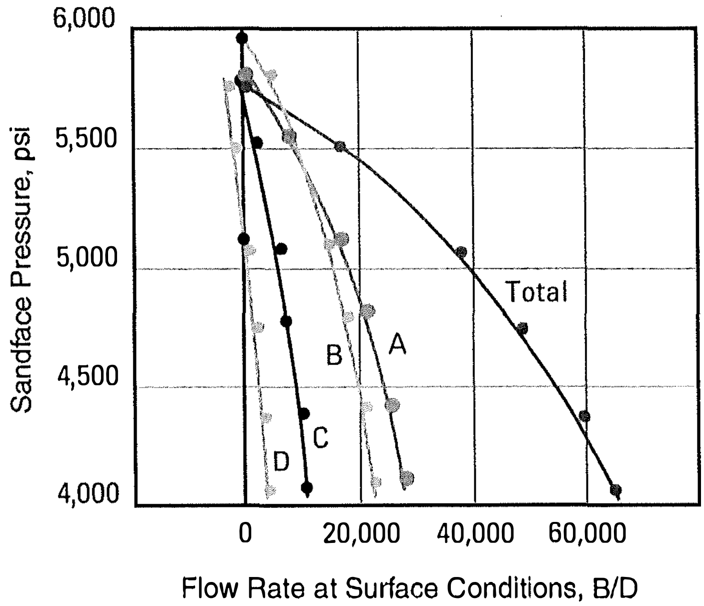

Figure 2: SIP analysis of a four-layer commingled reservoir with cross-flow (after Petrowiki.org)

Effect of hydrocarbon-pore-volume ratio (F_v)

In order to apply the proposed method, one needs to assume the hydrocarbon-pore-volume ratio (F_v). The value of F_v will affect the shape of the profile on a p/z plot, illustrated in Figure 3. If F_v is too small, the volume of the lower productivity layer is overestimated. The higher weight is assigned to the higher-pressured, lower productivity layer. Therefore, the (p/z) profile lies above the theoretical straight line. The (p/z) profile lies below the theoretical straight line if F_v is too big. Only the correct value of F_v yields a straight line on the p/z plot. The effects of F_v on the prediction of GIIP are summarized in Table 1.

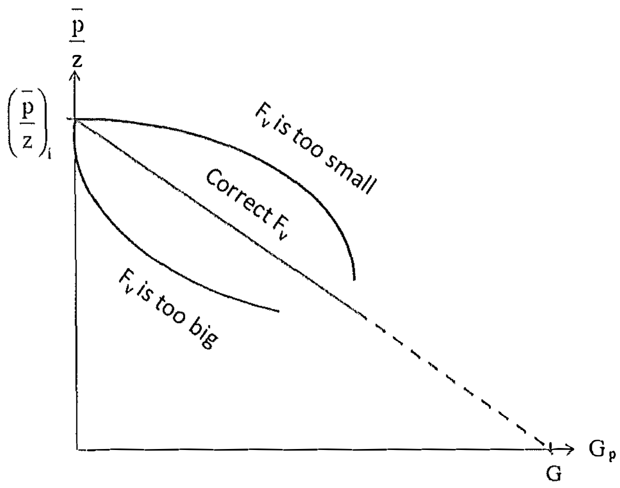

Figure 3: Effect of F_v on the p/z plot

Table 1: Effect of F_v on the prediction of GIIP

Value of F_V	Profile on the plot of (p/z) vs G_p	Prediction of GIIP	
		@ early time	@ late time
too small	concave: $\frac{d^2\left(\overline{p/z}\right)}{dG_p^2} < 0$	Overestimate	Correct
correct	straight line: $\frac{d\left(\overline{p/z}\right)}{dG_p} = -\frac{(p/z)_i}{G}$	Correct	Correct

too big	convex: $\dfrac{d^2\left(\overline{p/z}\right)}{d\,G_p^2} > 0$	Underestimate	Correct

With sufficient production data, a reliable value of F_v can be estimated. This is a critical piece of information for better reservoir management. For example, a well with layer 1 (higher productivity with a very small volume) and layer 2 (lower productivity with a very big volume). The pressure vs rate plot for each layer (Fetkovich, 1990) could help identify this issue. The production rate will be high with a short production period. After that, the well will produce at a low rate for a very long time. The proper well intervention is probably to stimulate layer 2 with a significant reserve to increase the well productivity.

Figure 4 illustrates the effects of F_v and the initial flow rate ratio (q_{iR}) on the pressure/recovery factor plot. When F_v = 1.0 and q_{iR} = 1.0, the two layers are perfectly identical. They are depleted at the same rate and behave similarly to a single-layer system. When F_v < 1, the hydrocarbon-pore-volume of layer 1 is smaller. Therefore, the pressure in layer 1 will be depleted faster. Consequently, the area between the profile of layer 1 and the theoretical straight line increases. When F_v > 1, the area between the theoretical straight line and the profile of layer 2 increases. When q_{iR} > 1.0, the production rate from layer 1 is higher. Therefore, the pressure in layer 1 will be depleted faster. Consequently, the area between the profile of layer 1 and the theoretical straight line increases.

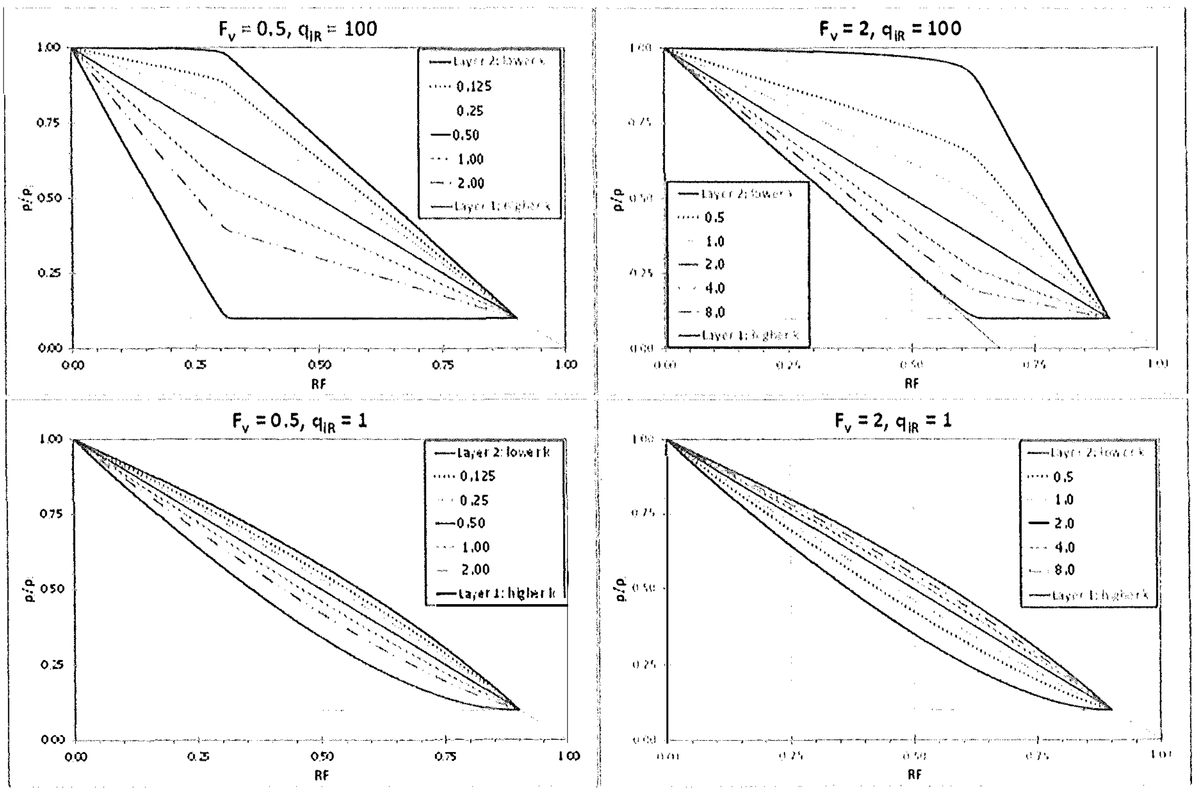

Figure 4: The effects of F_v and q_{iR} on the pressure/recovery factor plot

The effects of F_v and q_{iR} on production contribution are illustrated in Figure 5. When F_v = 1.0 and q_{iR} = 1.0, the two layers are perfectly identical. Each layer contributes 50% of the production through the well life. For q_{iR} = 1.0, the production contributions from individual layers are about the same in the short term. In the longer term, the production contribution from each layer depends on its hydrocarbon-pore-volume. The large reservoir dominates the production in the longer term. When q_{iR} > 1.0, the short-term production is dominated by the higher-productivity layer 1. Layer 2, with lower productivity dominates the long-term production, regardless of the value of F_v.

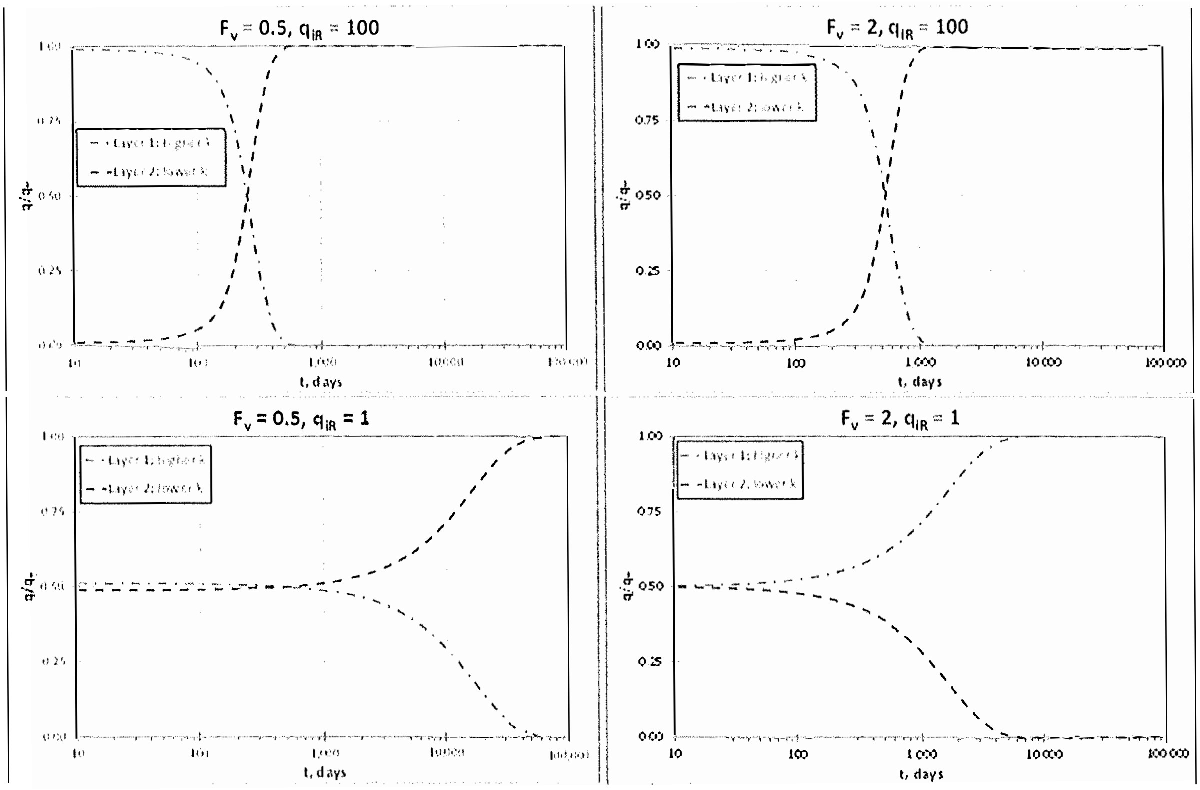

Figure 5: The effects of F_v and q_{iR} on production contribution

Payne (1996) pointed out that, in addition to differential depletion in a multi-layer system, the following factors could cause the p/z plot to be non-linear and to deviate from the theoretical straight line.

- Low permeability
- Heterogeneity
- Condensate dropping in a reservoir when the pressure is lower than the dew-point pressure.
- An aquifer
- An oil leg
- Rock compressibility

Recovery factor at the maximum differential pressure ($p_2 - p_1$)

Fetkovich (1990) presented a new approach to analyze 2-layer gas systems, without pressure communication. The higher permeability reservoir (layer 1) dominates the production during the early production period. Consequently, the pressure of layer 1 (p_1) will be less than the pressure of layer 2 (p_2). For each layer, (p/z) is plotted against the system cumulative gas production. The profiles of layers 1 and 2 are concave up and concave down, respectively. The separation between these two profiles is dependent on the hydrocarbon-pore volume ratio (F_v) and on the initial flow-rate ratio (q_{iR}). The differential pressure ($p_2 - p_1$) is increasing with production time and will reach a maximum value at some point. Ross (2014) performed a simulation study and showed that the maximum differential pressure is at about 30% RF. Beyond this RF, the differential pressure is decreasing with production time. During this later production period, the production from layer 1 is significantly decreased and the lower permeability reservoir (layer 2) dominates the system production. [It's important to note that the production profile Ross simulated is based on a period of plateau production, followed by a longer period of declining production -similar to what one would expect from a well producing against a constant back pressure. In the case where total well production remains constant throughout the period of study, the differential pressure $p_2 - p_1$ remains roughly constant].

This study derives the recovery factor (RF_m) in which the differential pressure is maximum. The details are in Appendix B. The plot of (p/p_i) versus RF is illustrated in the Figure 6. The characteristics of the plot of (p/pi) versus RF are summarized in Table 2. In practice, the value of F_v is unknown. The information in the table helps guide us about the estimation of F_v. For example, if the slope of the profile is increasing (decreasing) with production, it implies that the value of F_v is overestimated (underestimated). Only the profile of the correct F_v yields the constant slope.

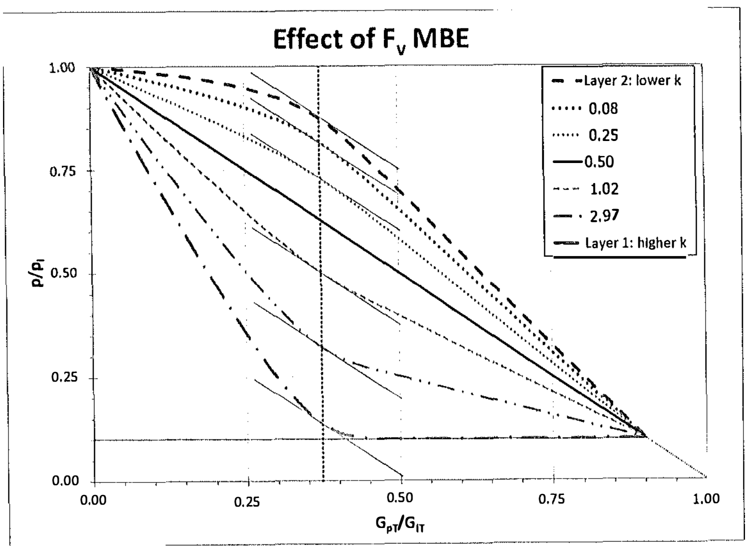

Figure 6: (p/p_i) versus RF

Table 2: Characteristics of the plot of (p/p_i) versus RF

RF	Characteristics
RF < RF_m	• For underestimated F_v, slope > -1 • For correct F_v, slope = -1 • For overestimated F_v, slope < -1
RF = RF_m	• All profiles have the slope of -1.
RF > RF_m	• For underestimated F_v, slope < -1 • For correct F_v, slope = -1 • For overestimated F_v, slope > -1

Applications

Example 1: synthetic data

This exercise consider a 2-layer system with the properties summarized in Table 3 below. Layer 1 has higher permeability (rate) with a smaller hydrocarbon-pore volume while Layer 2 has lower permeability with a bigger volume. Therefore, differential depletion is expected in this well performance.

Table 3: Reservoir properties for Example 1

Parameters	Layer 1	Layer 2
G, MMscf	500	1,000
q_i, MMscf/d	0.80	0.08
b, dimensionless	0.1	0.4
p_i, psia	5,000	5,000
p_{wf}, psia	500	500

The results for the base case are illustrated in Figure 7. Note that the effects of fluid crossflow during shut-in, and differences in z factor, are neglected. The key characteristics of the p/z plot for a multi-layer system are the following:

- All profiles, regardless of the length of shut-in, are below the theoretical straight line which represents the infinite shut-in. The wellbore pressure is likely to track the higher permeability layer pressure (layer 1) than to the lower permeability one (layer 2).
- The longer the shut-in, the less the deviation from the theoretical straight line. The results are consistent with Hagoort et al. (2000). It takes a very long shut-in period to get the reliable average system pressure.
- The RF at the point where (p_2-p_1) reaches a maximum is about 36% for this specific case. This RF depends on F_v, q_{iR}, and b. The details of the derivation are in Appendix B. Ross (2014) found that this RF is about 30% for the case in his work.
- When RF < RF_m, GIIP is underestimated. The shorter the shut-in period, the greater the severity of the underestimation.
- When RF > RF_m, all profiles tend to yield the correct GIIP, regardless of the shut-in period. Therefore, it is not necessary to have a long shut-in period to get the reliable system pressure, as long as we have a consistent shut-in period. This is probably the major finding in this exercise.

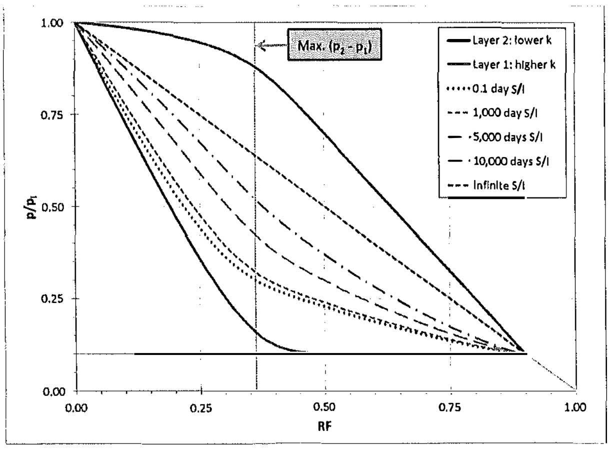

Figure 7: Results of Example 1, base case

The effects of F_v and q_{iR} are illustrated in the following Figures.

- When the size of G_1 (or F_v) increases, the area between the profiles of layer 1 and the theoretical straight line is reduced. Since the hydrocarbon-pore volume in layer 1 is increased, the pressure in this layer will be depleted more slowly. The effect of the length of shut-in period is less critical than in the base case.
- When the size of q_{i2} increases (or q_{iR} decreases), the area between the profiles of layer 2 and the theoretical straight line is reduced. Since layer 2 is depleted faster, the pressure in this layer will be reduced faster. The effect of the length of shut-in period is less critical than the base case.

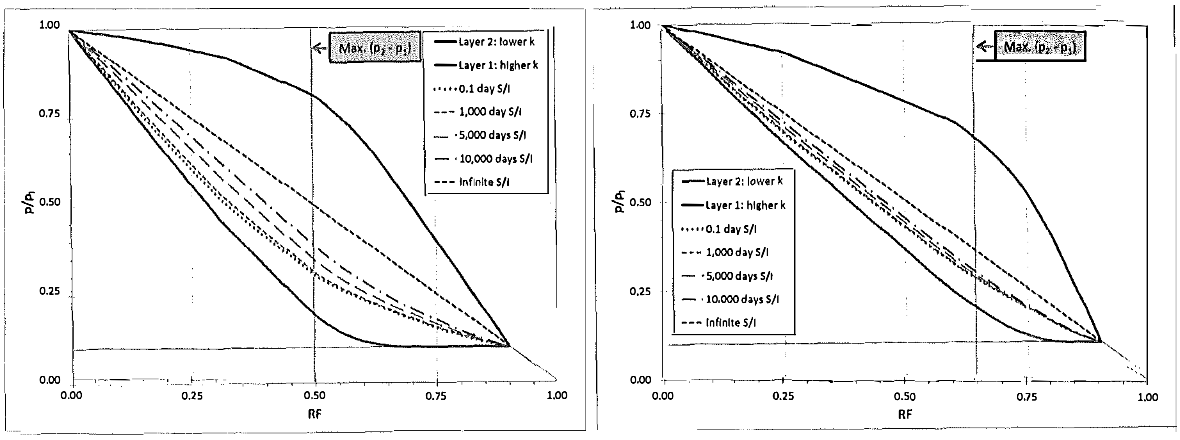

Figure 8: Results of Example 1,G_1 = 1,000 MMscf and 2,000 MMscf

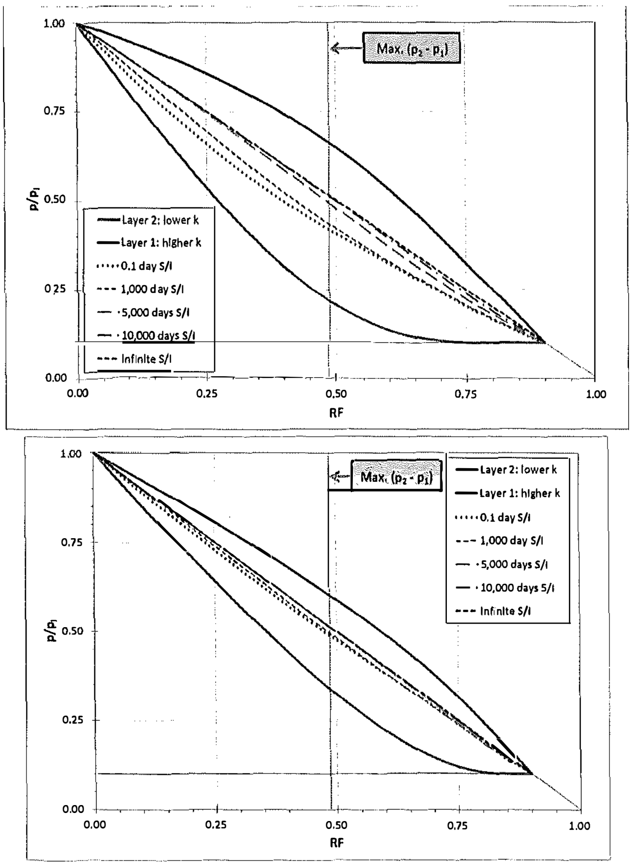

Figure 9: Results of Example 1, q_{i2} = 0.4 MMscf/d and 0.8 MMscf/d

Discussion

SIP drawbacks

SIP has several drawbacks. It reflects the averaged pressure around a wellbore, not the whole reservoir. This is probably an issue for tight reservoirs. In addition, there are PL interpretation uncertainties, especially for multi-phase flow.

Group multi layers into 2 layers based on (q_i/G)

For a multi-layer system, it is recommended by Fetkovich et al. (1996) to combine layers with similar value of (qi/G) into a single layer. Several studies confirmed that a 2-layer system is in most cases sufficient to replicate the performance of a multi-layer system.

Fluid cross-flow during shut-in period

Because the shut-in periods required by the technique are short, the perturbation of the long-term material balance trend due to inter-layer crossflow during the shut-in can be neglected. If, however, there are longer unplanned shut-down periods (as often occurs in practice) then the crossflow must be accounted for.

Conclusions

1. Application of the material balance equation to analyze production data from multi-layer gas reservoirs yields misleading long-term performance forecasts and inaccurate reserve estimation.
2. This study proposes a new approach for utilizing the material balance equation to analyze production data from multi-layer gas reservoirs. It doesn't require a long shut-in period. The average pressure of the multilayer system is estimated from production logging data. With this new average reservoir pressure, material balance methods will give a straight line, similar to one for a single layer gas reservoir.
3. The new methodology was validated using synthetic data for a multilayer gas reservoir. Accurate long-term performance prediction and reserves estimation were obtained.

Nomenclature

A	:	reservoir area, acre
b	:	Arp's decline exponent, dimensionless
c_t	:	system compressibility, psi^{-1}
F_v	:	layer volume ratio, dimensionless

G : gas initially in place, MMscf

G_p : cumulative gas production, MMscf

h : thickness, ft

J : productivity index, MMscf/d/psi^2

k : effective permeability, md

p : pressure, psia

$\overline{p}$: average reservoir pressure, psia

q : gas production rate, MMscf/d

RF : recovery factor, dimensionless

r_w : wellbore radius, ft

r_e : reservoir radius, ft

s : skin factor, dimensionless

t : time, day

V_i : hydrocarbon pore volume of layer i, ft^3

V_R : correct layer volume ratio, dimensionless

z : gas compressibility, dimensionless

ϕ : porosity, dimensionless

μ : viscosity, cp

Subscript

1 : layer 1 (higher permeability)

2 : layer 2 (lower permeability)

a : abandonment condition or apparent

D_{Ass} : dimensionless semi-steady-state

i : initial condition

sss : semi-steady-state condition

w : well

wf : bottom-hole flowing

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Appendix

Appendix A. Pressure build-up during shut-in period

This section considers a producing gas well with 2 layers. These two layers are commingling produced. After a period of production, the pressures in layers 1 and 2 are p_1 and p_2, respectively. The layer 1 is assumed to have a higher productivity than layer 2. Consequently, p_1 will be less than p_2. During the shut-in period, gas will cross flow from layer 2 via the wellbore into layer 1. This phenomenon is illustrated in Figure A-1.

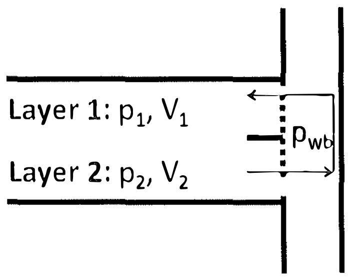

Figure A-1: wellbore condition during shut-in period

During pseudo-steady-state (pss) flow, the gas flow rate can be expressed in term of pressure squared as the following (Chaudhry (2003) and Juell and Whitson (2011))

$$q = J\left(p_R^2 - p_{wf}^2\right) \qquad \text{(A-1)}$$

where the productivity index (J) is defined as

$$J = \frac{kh}{1{,}422{,}000\left(\overline{\mu z}\right)T\left[\ln\left(\frac{0.472\, r_e}{r_w}\right) + s\right]} \qquad \text{(A-2)}$$

Because of changing (increasing) wellbore pressure, the shut-in flow rate will never reach pss condition. However, several studies (Hagoort et al. (2000), Lefkovits et al. (1961) and Chen et al. (1993)) suggested that the pss solution can be used to approximately estimate the flow rate/pressure during the transient flow. Then the gas cross-flow rate during shut-in is

$$q_g = J_1\left[p_{wb}^2 - p_1^2\right] = J_2\left[p_2^2 - p_{wb}^2\right] = \frac{J_1 J_2}{J_1 + J_2}\left(p_2^2 - p_1^2\right) \qquad \text{(A-3)}$$

The shut-in wellbore pressure can be expressed as

$$p_{wb} = \sqrt{\frac{J_1 p_1^2 + J_2 p_2^2}{J_1 + J_2}} \qquad \text{(A-4)}$$

Several studies (Kuppe et al. (2000), Hagoort et al. (2000), and Ross (2014)) used the PI approach to estimate the static wellbore pressure. Since layer 1 has higher productivity than layer 2; $J_1 > J_2$, based on the above equation, wellbore pressure is always closer to the pressure of the more permeable layer (p_1).

The initial and boundary conditions are

- Initial condition: at $t = 0$, $p_1 = p_1(0)$ and $p_2 = p_2(0)$
- Boundary condition: as $t \rightarrow \infty$, $p_1 = p_2 = \overline{p}$

The material balance equation for a single-layer reservoir during shut-in period, ignoring the gas compressibility factor (z), is

$$\frac{p_i}{p_i(0)} = 1 - \frac{G_{pi}}{G_{i,0}} \tag{A-5}$$

Note that G_p for layers 1 and 2 are negative (taking) and positive (producing), respectively. Therefore, the pressure in layers 1 and 2 will be increasing and decreasing, respectively, with time. These phenomena cause the gas cross-flow rate decreasing with time and approach zero as the equilibrium condition is reached. Differentiating Eq. (A-5) for both layers and using Eq. (A-3) yield

$$\frac{dp_1}{dt} = \frac{p_1(0)}{G_{1,0}} q_g = \frac{p_1(0)}{G_{1,0}} \frac{J_1 J_2}{J_1 + J_2}\left(p_2^2 - p_1^2\right) \tag{A-6}$$

$$\frac{dp_2}{dt} = -\frac{p_2(0)}{G_{2,0}} q_g = -\frac{p_2(0)}{G_{2,0}} \frac{J_1 J_2}{J_1 + J_2}\left(p_2^2 - p_1^2\right) \tag{A-7}$$

Dividing Eq. (A-6) by Eq. (A-7) yields

$$\frac{dp_1}{dp_2} = -\frac{p_1(0)/p_2(0)}{G_{1,0}/G_{2,0}} = -\frac{1}{F_v} \tag{A-8}$$

Integrating the above equation from (t = 0) to any time during shut-in yields

$$p_1 = \left[p_1(0) + \frac{p_2(0)}{F_v}\right] - \frac{p_2}{F_v} \tag{A-9}$$

Substituting Eq. (A-9) into Eq. (A-7), integrating from (t = 0) to any time, and applying the initial and boundary conditions yields

$$\frac{p_2}{\overline{p}} = \frac{1 + \dfrac{p_2(0) - \overline{p}}{\left|\left(\dfrac{F_v - 1}{F_v + 1}\right) p_2(0) + \overline{p}\right|} \exp\left[-\overline{p}\left(1 + \dfrac{1}{F_v}\right)\left(\dfrac{p_2(0)}{G_{2,0}}\right)\left(\dfrac{J_1 J_2}{J_1 + J_2}\right) t\right]}{1 - \left(\dfrac{F_v - 1}{F_v + 1}\right) * \dfrac{p_2(0) - \overline{p}}{\left|\left(\dfrac{F_v - 1}{F_v + 1}\right) p_2(0) + \overline{p}\right|} \exp\left[-\overline{p}\left(1 + \dfrac{1}{F_v}\right)\left(\dfrac{p_2(0)}{G_{2,0}}\right)\left(\dfrac{J_1 J_2}{J_1 + J_2}\right) t\right]} \tag{A-10}$$

At any time during the shut-in period, the averaged system pressure is constant and can be expressed as

$$\bar{p} = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2} = \frac{p_1 F_v + p_2}{F_v + 1} \quad \forall\, t \geq 0 \tag{A-11}$$

The value of p_1 can be estimated from the above equation once the value of p_2 is available from Eq. (A-10). While the averaged system pressure remains constant, the values of p_1 and p_2 are increasing and decreasing, respectively. When the equilibrium condition is reached, the values of pressures in layer 1, layer 2, wellbore, and the average pressure are the same.

Appendix B. Recovery factor at the maximum value of differential pressure ($P_2 - P_1$)

Gas reserve at the initial condition or ultimate recovery (UR) is

$$UR = (RF) * G \tag{B-1}$$

Based on the material balance equation (MBE) for gas reservoir, the recovery factor (RF) at the abandonment condition is defined as

$$RF = 1 - \frac{(p/z)_a}{(p/z)_i} \tag{B-2}$$

Subscripts a and i represent the abandonment and initial conditions, respectively. Let's consider a 2-layer gas system. Both layers are assumed to have the same initial pressure, fluid properties, and abandonment condition. The initial gas-rate ratio and the hydrocarbon-pore-volume ratio at the initial condition are, respectively, defined as

$$q_{iR} = \frac{q_{i1}}{q_{i2}} \tag{B-3}$$

$$F_v = \frac{G_1}{G_2} \tag{B-4}$$

The MBEs for the 2-layer gas system and for individual layers are the following:

$$\frac{(p/z)_{avg}}{(p/z)_i} = 1 - \frac{[G_{p1} + G_{p2}]}{[G_1 + G_2]} = 1 - \frac{G_{pT}}{G_T} \tag{B-5}$$

$$\frac{(p/z)_j}{(p/z)_{i,j}} = 1 - \frac{G_{pj}}{G_j} \quad \text{for } j = 1, 2 \tag{B-6}$$

Normally, the actual value of F_v is unknown. An engineer has to guess the initial value of F_v which is called the apparent hydrocarbon-pore-volume ratio (F_{va}) in this study. Their relation is the following.

$$F_v = \frac{G_1}{G_2} = F_{va} + (F_v - F_{va}) \tag{B-7}$$

The averaged reservoir pressure, ignoring the gas compressibility factor (z), is

$$\bar{p} = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2} = \frac{p_1 F_v + p_2}{F_v + 1} = \bar{p}_a \frac{(1 + F_{va})}{(1 + F_v)} + \frac{p_1 (F_v - F_{va})}{1 + F_v} \tag{B-8}$$

where $\bar{p}_a = (p_1 F_{va} + p_2)/(1 + F_{va})$ is the apparent averaged system pressure based on F_{va}. Rearrange the above equation as

$$\bar{p}_a = \frac{(1 + F_v)}{(1 + F_{va})}\left[\bar{p} - \frac{p_1 (F_v - F_{va})}{1 + F_v}\right] \tag{B-9}$$

Using the above equation together with MBEs for the whole system and layer 1, it can be shown that

$$\frac{d[\bar{p}_a / p_i]}{d[G_{pT} / G_{iT}]} = \frac{(1 + F_v)}{(1 + F_{va})}\left[\frac{(F_v - F_{va})}{F_v}\left(\frac{q_1}{q_T}\right) - 1\right] \tag{B-10}$$

On the plot of $[\bar{p} / p_i]$ vs $[G_{pT} / G_{iT}]$, when $(p_2 - p_1)$ is maximum, we have the following condition, illustrated in the Figure A-1, regardless of the value of F_{va}.

$$d[\bar{p}_a / p_i] / d[G_{pT} / G_{iT}] = -1 \;\forall\; F_{va} \tag{B-11}$$

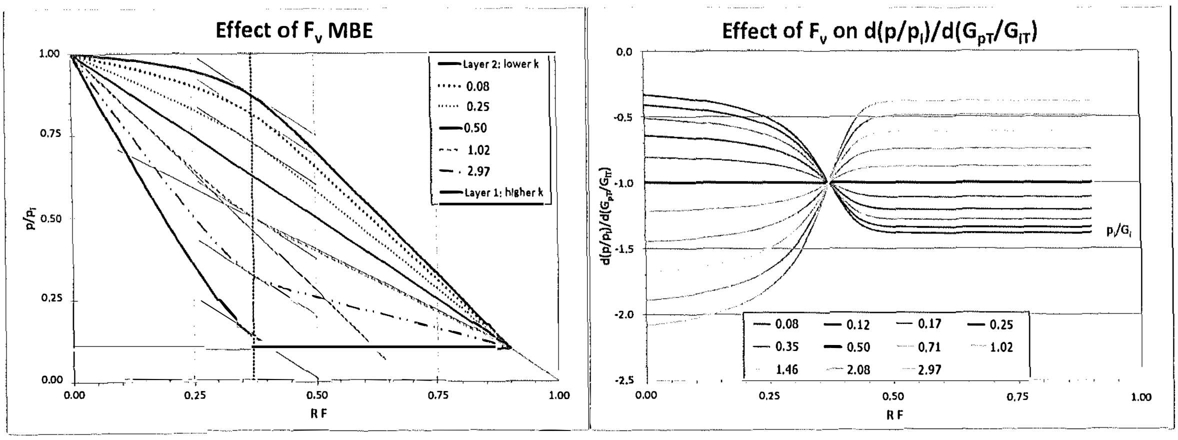

Figure B-1: Effects of F_v on p/z plot

At this level of RF, all profiles have the same slope of (-1), regardless the assuming value of F_{Va}. Solving Eq. (B-10) yields

$$\frac{q_1}{q_2} = F_v \tag{B-12}$$

The instantaneous flow-rate ratio is proportional to the initial hydrocarbon-pore-volume. The result is consistent with the work of Chen et al. (1997). It occurs during pseudo-steady-state flow.

Case I: for an exponential decline (b = 0)

Fetkovich et al. (1996) derived the following rate/time equation for a gas well producing against a constant wellbore pressure with an exponential decline as:

$$\frac{q}{q_i} = \exp\left[-\left(\frac{q_i}{G}\right)t\right] \tag{B-13}$$

The condition in Eq. (B-12) is satisfied when the production time is

$$t = \frac{\ln\left[\left(\frac{q_{i2}}{G_2}\right)/\left(\frac{q_{i1}}{G_1}\right)\right]}{\left[\left(\frac{q_{i2}}{G_2}\right)-\left(\frac{q_{i1}}{G_1}\right)\right]} \tag{B-14}$$

The recovery factor (RF) at any production time is

$$\frac{RF}{RF_a} = 1-\left(\frac{F_v}{1+F_v}\right)\exp\left[-\left(\frac{q_{i1}}{G_1}\right)t\right]-\left(\frac{1}{1+F_v}\right)\exp\left[-\left(\frac{q_{i2}}{G_2}\right)t\right]$$

(B-15)

where $(RF)_a = (G_1/G_{i1}) = (G_2/G_{i2})$ is the recovery factor at the abandonment condition. Substitute t from Eq. (B-14) into the above equation yields

$$\frac{RF_m}{RF_a} = 1-\left(\frac{F_v}{1+F_v}\right)\left(\frac{F_v}{q_{iR}}\right)^{\frac{q_{iR}}{[q_{iR}-F_v]}}-\left(\frac{1}{1+F_v}\right)\left(\frac{F_v}{q_{iR}}\right)^{\frac{F_v}{[q_{iR}-F_v]}} \tag{B-16}$$

Case II: for hyperbolic decline b $\in$(0.0, 1.0)

Fetkovich et al. (1996) derived the following rate/time equation for a gas well producing against a constant pressure with hyperbolic decline

$$\frac{q}{q_i} = \left[1+\left(\frac{b}{1-b}\right)\left(\frac{q_i}{G}\right)t\right]^{-\frac{1}{b}} \tag{B-17}$$

Both layers are assumed to have the same Arps' decline exponent; $b_1 = b_2 = b$. The condition in Eq. (B-12) is satisfied when the production time is

$$t = \left(\frac{1-b}{b}\right)\frac{\left(\frac{q_{i1}}{G_1}\right)^b - \left(\frac{q_{i2}}{G_2}\right)^b}{\left(\frac{q_{i1}}{G_1}\right)\left(\frac{q_{i2}}{G_2}\right)^b - \left(\frac{q_{i1}}{G_1}\right)^b\left(\frac{q_{i2}}{G_2}\right)} \tag{B-18}$$

Based on Fetkovich et al. (1996), the cumulative gas production can be expressed as

$$G_p = G - G\left(\frac{q}{q_i}\right)^{1-b} \tag{B-19}$$

Similarly, it can be shown that

$$\frac{RF_m}{RF_a} = 1 - \frac{\left[\frac{F_v - q_{iR}}{F_v^{1-b} - q_{iR}^{1-b}}\right]^{\frac{(b-1)}{b}}\left[F_v^{2-b} + q_{iR}^{1-b}\right]}{(F_v + 1)} \tag{B-20}$$

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