Chapter II

Theoretical and Numerical model

1. Governing equations

The vertically integrated momentum and mass conservation equation of two-layer variable depth basin are, for the upper layer,

$$\frac{\partial \vec{M}_1}{\partial t} + f\vec{k} \times \vec{M}_1 + \rho_1 g H_1 \nabla(h_1 + h_2) = \vec{F}_1, \tag{1}$$

$$\rho_1 \frac{\partial h_1}{\partial t} + \nabla \cdot \vec{M}_1 = 0, \tag{2}$$

and for the lower layer,

$$\frac{\partial \vec{M}_2}{\partial t} + f\vec{k} \times \vec{M}_2 + gH_2 \nabla(\rho_1 h_1 + \rho_2 h_2) = \vec{F}_2 \tag{3}$$

$$\rho_2 \frac{\partial h_2}{\partial t} + \nabla \cdot \vec{M}_2 = 0, \tag{4}$$

where $\vec{M}$ is the mass transport per unit width, f is Coriolis parameter, $\vec{k}$ is vertical unit vector, ρ is water density, g is gravitational acceleration, H is the mean depth, $H+h$ is the stantaneous depth, $\vec{F}$ is the external forcing and distribution defined as

$$\vec{F}_1 = \vec{\tau}_s - \vec{\tau}_i \tag{5}$$

$$\vec{F}_2 = \vec{\tau}_i - \vec{\tau}_b \tag{6}$$

where $\vec{\tau}$ is the stress vector where subscripts s, i, and b stand for surface, interface, and bottom respectively.

By using conservation of energy theory, Banpapong et al., (1985) derived equations (1),(2),(3),and(4) to the form of normal mode equations, (7) and (8) for external and internal modes equations respectively.

$$\partial\vec{Q}_e / \partial t + f\vec{k} \times \vec{Q}_e + gD\nabla\psi_e - \varepsilon gD\psi_i \nabla(H_1 / D) = \vec{T}_s - \vec{T}_b ,$$

$$\partial\psi_e / \partial t + \nabla \cdot \vec{Q}_e - \varepsilon\vec{Q}_i \cdot \nabla(H_1 / D) = 0 , \qquad (7)$$

and

$$\partial\vec{Q}_i / \partial t + f\vec{k} \times \vec{Q}_i + g\Gamma_i \nabla\psi_i - \varepsilon g\Gamma_i \psi_e \nabla(H_1 / D) = (H_2 / D)\vec{T}_s + (H_1 / D)\vec{T}_b - \vec{T}_i ,$$

$$\partial\psi_i / \partial t + \nabla \cdot \vec{Q}_i - \varepsilon Q_e \cdot \nabla(H_1 / D) = 0 , \qquad (8)$$

where $\vec{Q}_e \equiv \vec{M}_e / \rho$, $\vec{Q}_i \equiv \vec{M}_i / \rho$, $\psi_e \equiv \phi_e / \rho$, $\psi_i \equiv \phi_i / \rho$, $\vec{T}_s \equiv \vec{\tau}_s / \rho$, $\vec{T}_i \equiv \vec{\tau}_i / \rho$, $\vec{T}_b \equiv \vec{\tau}_b / \rho$, $\varepsilon = (\rho_2 - \rho_1) / \rho_2$, $\Gamma_i \approx \varepsilon\left(H_1 H_2 / D\right)$. $\vec{M}$ and ϕ are mass transport per unit width and water elevation respectively, subscripts i, and e stand for internal and external modes.

In this study, the Gulf of Thailand is determined as the shallow water, thus vertically depth-averaged model is available to compute the current system. The computational model is operated for only external mode, so equation (8) will not be included in calculation. In equation (7), the coupling terms, the terms of opposite mode relative to the other terms, will be vanished, because ψ_i and Q_i are not computed.

After cutting unused terms and subscript of mode, equation (7) as governing equation can be written in the form

$$\partial\vec{Q} / \partial t + f\vec{k} \times \vec{Q} + gD\nabla\psi = \vec{T}_s - \vec{T}_b ,$$

$$\partial\psi / \partial t + \nabla \cdot \vec{Q} = 0 . \qquad (9)$$

2. Grid system

A space-staggered computational mesh is employed in the numerical analogs of governing equations. The grid spacing is taken as 6 × 6 minutes in latitude (Δφ) and longitude (Δλ), covering 37×79 grid blocks for the Gulf of Thailand. In this grid system, the transport per unit width, represented by U and V are located for north and east components at the mid point of the appropriate side of each grid block, while the water elevation, ψ, is located at the middle of each grid block. Figure 1 illustrates location of these variables whose location are identified by I and J. The grid spacing is taken as the distance between the same variable.

The grid covering in the Gulf is illustrated in Figure 2. The depth in the Gulf of Thailand are digitized from the navigation chart at each ψ position. However, depths at U and V position are calculated from depth at ψ. Figure 3 illustrated the computer plotted contours of depth field in the Gulf of Thailand.

3. Numerical integration scheme

The multi-operational alternating direction implicit (ADI) algorithm developed by Lendertse (1967) was used for time integration of the finite difference equations.

The spatial average of a field variable X is written as

$$\bar{\bar{X}}^n(I,J) = \frac{1}{4}[X(I-\frac{1}{2},J-\frac{1}{2})+X(I+\frac{1}{2},J-\frac{1}{2})+X(I-\frac{1}{2},J+\frac{1}{2})+X(I+\frac{1}{2},J+\frac{1}{2})], \tag{10}$$

where $X^n(I,J) \equiv X(\lambda_0 + I\Delta\lambda,\ \varphi_0 + J\Delta\varphi, t_0 + n\Delta t)$. This spatial averages is used for U and V required in representing appropriate spatial centered estimates of Coriolis terms.

Time derivative are represented by the standard centered differences. For space derivative, a spherical coordinate system is employed, thus the grid spacing in latitude is different between lines of J shown in (12).

$$\frac{\partial X}{\partial t} = (1/2\Delta t)[X^{n+1}(I,J) - X^{n-1}(I,J)], \tag{11}$$

$$\frac{\partial X}{\partial \lambda} = (1/\Delta\lambda a\theta(J))[X^n(I+\frac{1}{2},J) - X^n(I-\frac{1}{2},J)], \tag{12}$$

$$\frac{\partial X}{\partial \varphi} = (1/a\Delta\varphi)[X^n(I,J+\frac{1}{2}) - X^n(I,J-\frac{1}{2})], \tag{13}$$

where X is any scalar field variable, a is average radius of the earth (6.37×10^6 m), $\theta(J)=COS(\varphi_0+J\,\Delta\varphi)$, where φ_0 is the reference latitude (5.6 ^{0}N).

For ADI algorithm, the cycle of circulation is separated into two operations. During the first-half cycle, at odd time step, ψ and U are computed implicitly along lines of constant latitude, then V is computed explicitly. In the second-half step, at even time step, ψ and V are computed implicitly along lines of constant longitude, and U is computed explicitly.

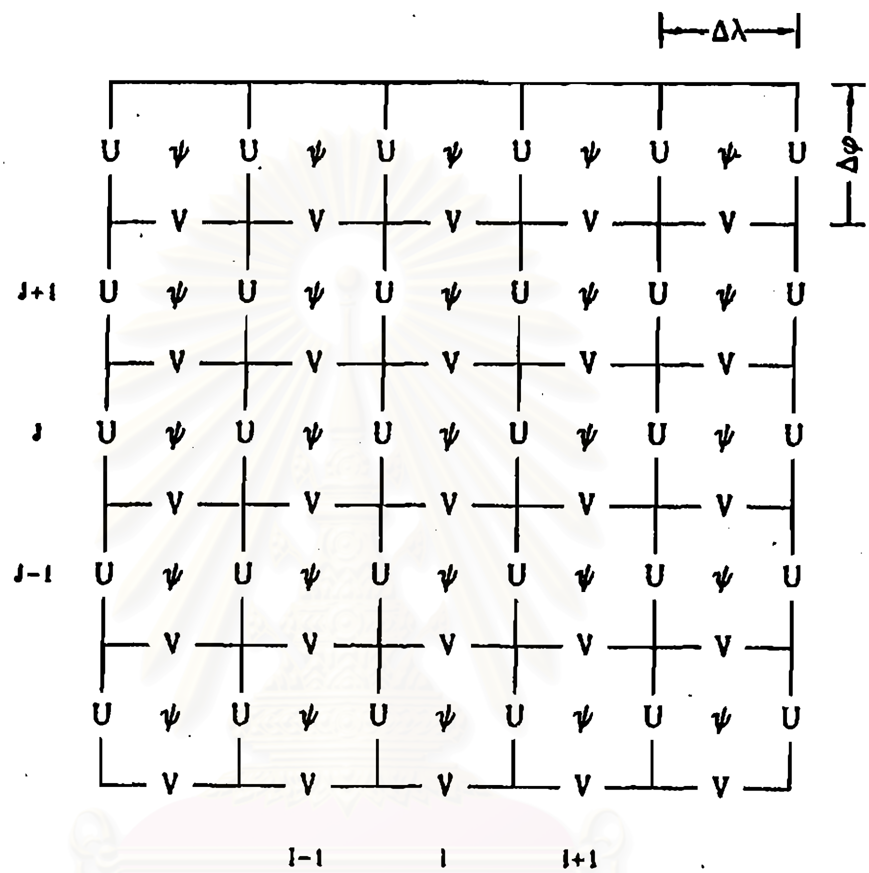

Figure 1. Computational grid showing location where U, V, and ψ are evaluated.

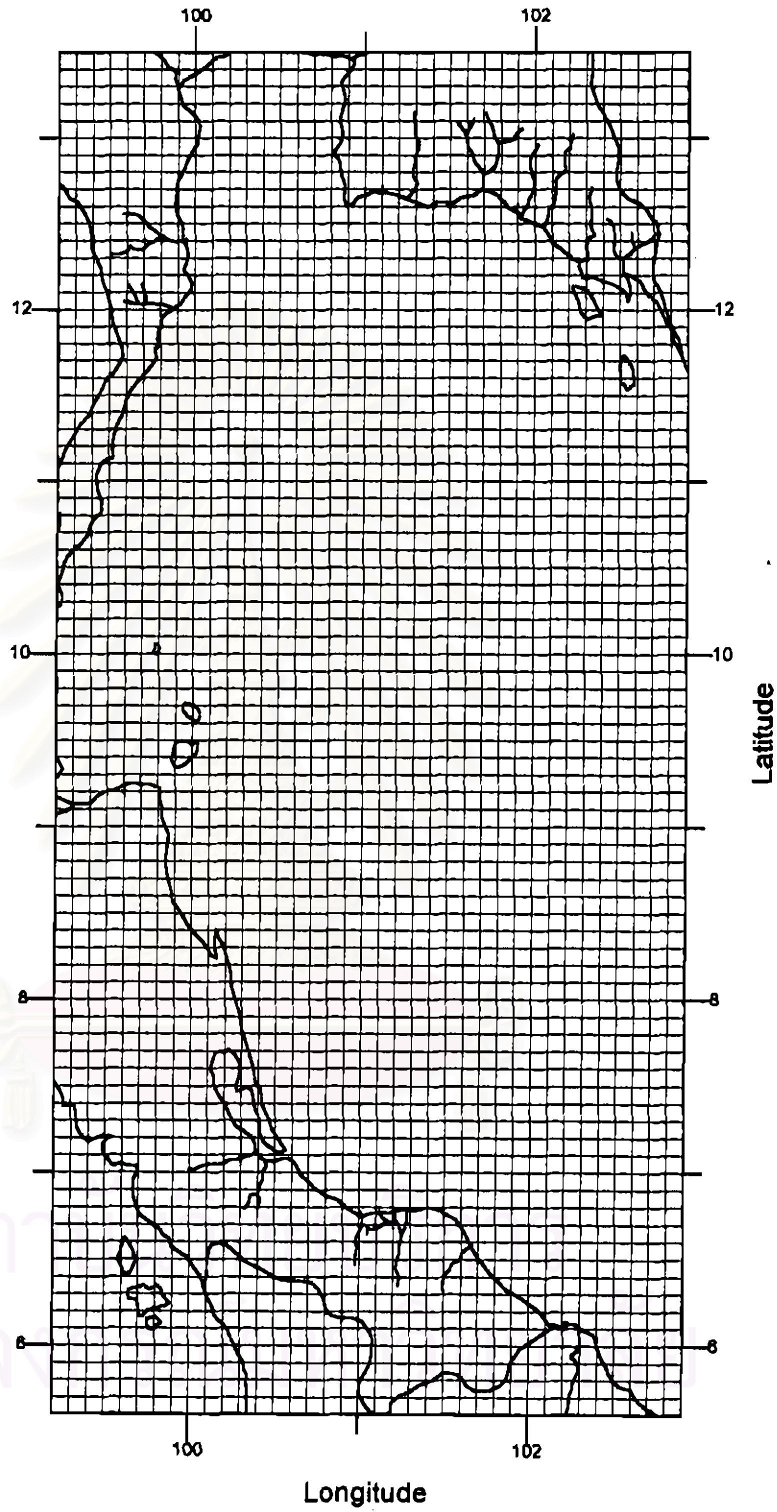

Figure 2. Grid system for the Gulf of Thailand. The grid increment are 6 minutes in latitude and longitude.

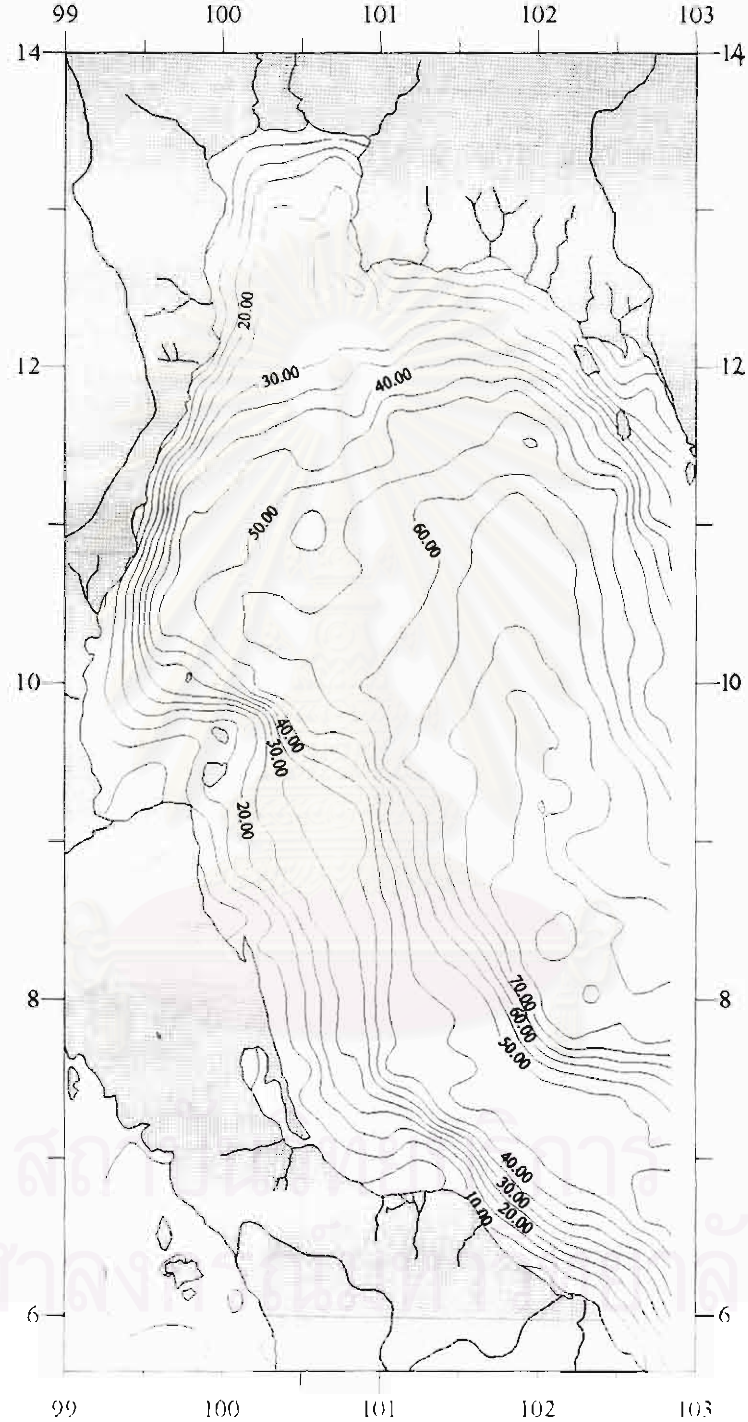

Figure 3. Computer plotted of the digitized water depth (in meters) in the Gulf of Thailand

The implicit formulation of finite difference analogs of momentum and mass conservation equations, respectively are, for odd time steps,

$$-\gamma_x(I-\tfrac{1}{2},J)\psi^{n+1}(I-\tfrac{1}{2},J) + U^{n+1}(I,J) + \gamma_x(I+\tfrac{1}{2},J)\psi^{n+1}(I+\tfrac{1}{2},J)$$
$$= U^n(I,J) + \Delta t\, f(J)\overline{V}^n(I,J) + \Delta t\, F_x \qquad ,(14)$$

$$-\upsilon_x(J)U^{n+1}(I-\tfrac{1}{2},J) + \psi^{n+1}(I,J) + \upsilon_x(J)U^{n+1}(I+\tfrac{1}{2},J)$$
$$= \psi^n(I,J) + \upsilon_y\delta_y V^n(I,J) \qquad ,(15)$$

and for even time steps

$$-\gamma_y(I,J-\tfrac{1}{2})\psi^{n+1}(I,J-\tfrac{1}{2}) + V^{n+1}(I,J) + \gamma_y(I,J+\tfrac{1}{2})\psi^{n+1}(I,J+\tfrac{1}{2})$$
$$= V^n(I,J) + \Delta t\, f(J)\overline{\overline{U}}^n(I,J) + \Delta t\, F_y \qquad ,(16)$$

$$-\upsilon_y(J-\tfrac{1}{2})V^{n+1}(I,J-\tfrac{1}{2}) + \psi^{n+1}(I,J) + \upsilon_y(J+\tfrac{1}{2})V^{n+1}(I,J+\tfrac{1}{2})$$
$$= \psi^n(I,J) + \upsilon_x\delta_x U^n(I,J) \qquad ,(17)$$

where

$$\gamma_x(I,J) = (\Delta t / \Delta\lambda)\,[gD(I,J) / a\theta(J)],$$

$$\gamma_y(I,J) = (\Delta t / \Delta\varphi)\,[gD(I,J) / a],$$

$$\upsilon_x(J) = (\Delta t / \Delta\lambda)\,[1 / a\theta(J)],$$

$$\upsilon_y(J) = (\Delta t / \Delta\varphi)\,[1 / a],$$

$$\delta_x U^n(I,J) = U^n(I+\tfrac{1}{2},J) - U^n(I-\tfrac{1}{2},J),$$

$$\delta_y V^n(I,J) = V^n(I,J+\tfrac{1}{2}) - V^n(I,J-\tfrac{1}{2}),$$

a = radius of the Earth,

$f(J) = 2\Omega SIN(\varphi_0 + J\Delta\varphi)$,

F = friction terms.

The forcing terms, surface and bottom stress, are described in the next section.

The explicit coding of the momentum equation at odd time step is

$$V^{n+1}(I,J) = V^{n}(I,J) - \frac{1}{2}\Delta t\, f(J)[\overline{U}^{n+1}(I,J)+\overline{U}^{n}(I,J)] - \gamma_y(I,J)\delta_y\psi^{n}(I,J) + \Delta t F_y \quad (18)$$

and for even time step

$$U^{n+1}(I,J) = U^{n}(I,J) - \frac{1}{2}\Delta t\, f(J)[\overline{V}^{n+1}(I,J)+\overline{V}^{n}(I,J)] - \gamma_x(I,J)\delta_x\psi^{n}(I,J) + \Delta t F_x \quad (19)$$

4. Surface and bottom stresses

The forcing terms consisted of the surface and bottom stress, separately written in x and y component. That is

$$F_x = \vec{T}_{sx} - \vec{T}_{bx}, \quad (20)$$

$$F_y = \vec{T}_{sy} - \vec{T}_{by}. \quad (21)$$

Generally, the stress terms are presented in the form

$$\vec{T} = k\left|\vec{W}\right|\vec{W}, \quad (22)$$

where $\vec{W}$, in the case of surface stress, is the wind velocity at 10 meters above water surface, but for bottom stress, $\vec{W}$ is the depth averaged current velocity. The dimensionless constant k is not the same value between two terms.

For surface stress, Reid and Bodine (1968) quoted by Banpapong et al., (1985) considered k as a function of wind speed in the form

$$k = k_1, \qquad \text{for } \left|\vec{W}\right| \le W_c \quad (23)$$

$$k = k_1 + k_2(1 - W_c/\left|\vec{W}\right|)^2, \qquad \text{for } \left|\vec{W}\right| \ge W_c \quad (24)$$

where k_1 and k_2 are taken as 1.1×10^{-6} and 2.5×10^{-6}, respectively, and W_c is a critical wind speed which is taken as 7.0 m/s.

In case of Thailand, the whole gulf is located in the zone of weak wind, with a typical wind 2 to 8 m/s (Vongvisessomjai et al., 1978). Thus, value 1.1×10^{-6} for constant k is selected.

The wind stress terms in x and y direction are coded as

$$T^{n}_{sx}(I,J) = 1.1\times 10^{-6}[(W_x^2(I,J)+W_y^2(I,J))^{\frac{1}{2}}]^n[W_x^n(I,J)]\,, \qquad (25)$$

$$T^{n}_{sy}(I,J) = 1.1\times 10^{-6}[(W_x^2(I,J)+W_y^2(I,J))^{\frac{1}{2}}]^n[W_y^n(I,J)], \qquad (26)$$

where W_x and W_y are wind speed in x and y direction respectively.

For bottom stress terms, the coefficient k is taken as 2.5×10^{-3}. This value of k is referred and employed for hydrodynamic computation in Bunpapong et al., (1985) and Vongvisessomjai et al., (1978).The model coding of bottom stress terms are shown as follow.

$$T^{n+1}_{bx}(I,J) = 2.5\times 10^{-3}\{[(1/D)(U^n(I,J))]^2+[(1/D)(\bar{\bar{V}}^n(I,J))]^2\}^{\frac{1}{2}}\{(1/D)U^{n+1}(I,J)\} \qquad (27)$$

$$T^{n+1}_{by}(I,J) = 2.5\times 10^{-3}\{[(1/D)(U^n(I,J))]^2+[(1/D)(\bar{\bar{V}}^n(I,J))]^2\}^{\frac{1}{2}}\{(1/D)V^{n+1}(I,J)\} \qquad (28)$$